Risk and Rationality: The Relative Importance of Probability Weighting and Choice Set Dependence

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Innsbruck 2019

Introduction

Expected Utility Theory (EUT) fails as a descriptive model for choice under risk

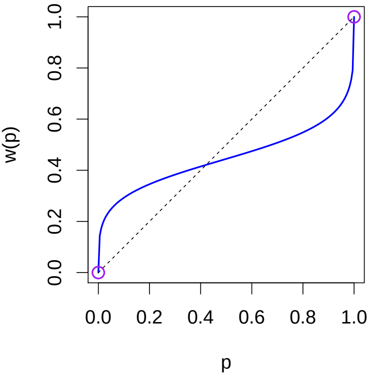
For example, many people

- are both risk-seeking and risk-averse at the same time
- buy lottery tickets and damage insurance

⇒ The many violations of EUT have spurred the development of various alternative decision theories

Introduction

One major class of decision theories assumes **probability weighting** to explain violations of EUT



- Cumulative Prospect Theory (CPT) is the most prominent example (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992)
- Rank Dependent Utility (RDU) is another example (Quiggin, 1982)

Introduction

Another major class of decision theories uses **choice set dependence** to explain violations of EUT

- Salience Theory of Choice Under Risk (ST) is a recent example (Bordalo, Gennaioli & Shleifer, BGS, 2012)
- ST is based on local thinking and salience
- Due to cognitive limitations, a local thinker focuses her attention on some but not all states of the world
- Salience shifts the focus of attention to states of the world in which one payoff stands out relative to the payoffs of the alternatives
- Depending on her degree of local thinking, the decision maker overweights salient states

- Another example is Regret Theory (Loomes & Sugden, 1982)

Introduction

Although probability weighting and choice set dependence often lead to similar predictions, discriminating between them is important

- Sheds light on the cognitive processes behind risky choices
- ST also applies to deterministic consumer choice (BGS, 2012b, 2013a; Dertwinkel-Kalt et al., 2017)
- ST explains the counter-cyclicality of risk premia (BGS, 2013b)
- ST explains how legally irrelevant information sometimes influences judicial decisions (BGS, 2015)
- Choice set dependence can describe some types of preference reversals as it allows for violations of transitivity

Contributions

We exploit the choice set dependence of experimental choices to discriminate between these two classes of decision theories

1. We provide non-parametric evidence at the aggregate level
2. We account for heterogeneity in a structural model and use a finite mixture approach to classify subjects into CPT-, ST-, and EUT-types
3. We perform out-of-sample predictions to assess the validity of our classification of subjects into types

Main Contribution

The paper presents the first joint test of the relative importance of probability weighting and choice set dependence

Empirical Strategy

To discriminate between the two classes of decision theories, we

1. use a series of binary choices between lotteries that may trigger the Allais paradox
2. manipulate the choice set by making the payoffs of the same lotteries either independent of each other or perfectly correlated

When can the Allais paradox occur? (BGS, 2012) Example

	Lottery Payoffs	
	independent	correlated
EUT	✗	✗
CPT (probability weighting)	✓	✓
ST (choice set dependence)	✓	✗

Experimental Design

The experiment uses 283 student subjects and exposes them to two incentivized parts

1. **Main part:** Subjects make a series of 45 binary choices between lotteries that may trigger the Allais paradox

   Subjects face the lotteries of each binary choice twice, once with independent and once with perfectly correlated payoffs

$\Rightarrow$ Used for discriminating between CPT-, ST-, and EUT-types

2. **Additional part:** Subjects face 6 additional binary choices between lotteries that may lead to preference reversals

$\Rightarrow$ Used for making out-of-sample predictions

Experimental Design: Main Part

We presented the lottery choices in the 'canonical presentation' and the 'states of the world presentation' to half of he subjects each

Canonical Presentation

Independent payoffs:

Probability	67%	33%
Option X	0	2500

Probability	34%	66%
Option Y	2400	0

Perfectly correlated payoffs:

Probability	1%	66%	33%
Option X	0	0	2500
Option Y	2400	0	2400

Experimental Design: Main Part

We presented the lottery choices in the ‘canonical presentation’ and the ‘states of the world presentation’ to half of he subjects each

States of the World Presentation

Independent payoffs:

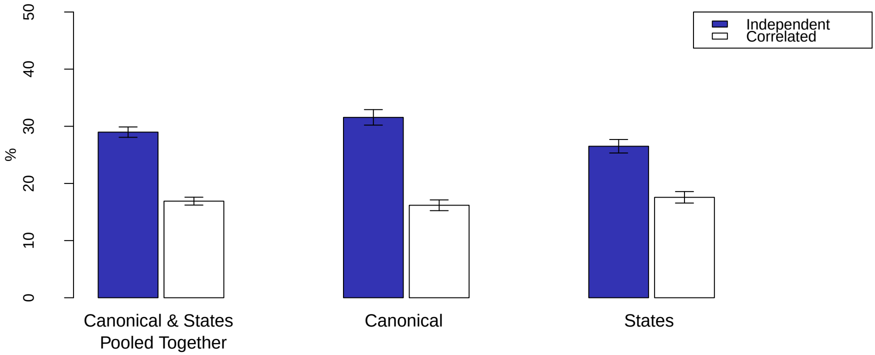
Probability	11.22%	22.78%	44.22%	21.78%
Option X	2500	0	0	2500
Option Y	2400	2400	0	0

Perfectly correlated payoffs:

Probability	1%	66%	33%
Option X	0	0	2500
Option Y	2400	0	2400

Aggregate Choices

Frequency of Allais paradoxes (APs) in binary lottery choices



Result 1: Non-parametric evidence from aggregate choices

- APs occur frequently with both independent and correlated payoffs
- However, APs are more frequent with independent payoffs

Aggregate Choices

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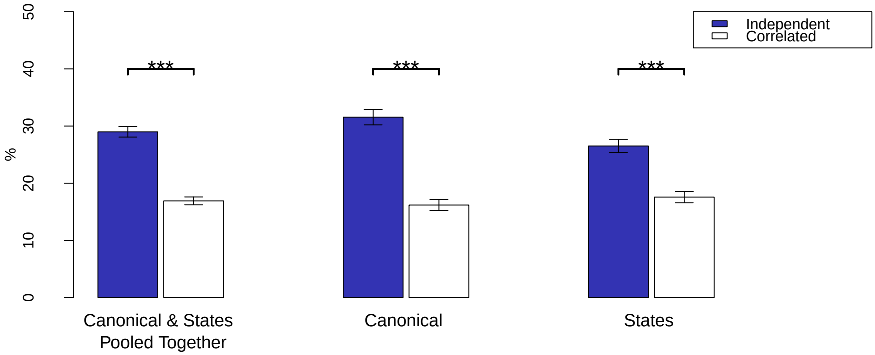


Result 1: Non-parametric evidence from aggregate choices

⇒ EUT is clearly rejected in aggregate choices

⇒ Both probability weighting and choice set dependence play a role

Type-Specific Choices: Structural Model

We estimate a finite mixture model to classify subjects into types

- The finite mixture model allows us to account for individual heterogeneity in a parsimonious way
- Instead of estimating every subjects' parameters separately, it assumes the population to consist of CPT-, ST-, and EUT-types
- It identifies the three types and characterizes them by their relative sizes and type-specific parameters
- After estimating the types' sizes and parameters, we can classify each subject into the type she most likely belongs to given her behavior

Type-Specific Choices: Structural Model

The likelihood contribution of subject i with choices C_i is

$$\ell\left(\Psi; x\right) = \sum_{m \in \mathcal{M}} \pi_m \, f_m(C_i, \theta_m)\,,$$

where

- $f_m(C_i, \theta_m)$ is the type-specific density that i exhibits the choices C_i if she belongs to type $m \in \mathcal{M} = \{CPT, ST, EUT\}$
- π_m represents the relative size of type m
- $\Psi = (\theta_{CPT}, \theta_{ST}, \theta_{EUT}, \pi_{CPT}, \pi_{ST})$ is the vector of parameters that need to be estimated

Type-Specific Choices: Structural Model

After estimating the finite mixture model, we apply Bayes' rule to obtain ex-post probabilities of individual type-membership

$$\tau_{iM} = \frac{\hat{\pi}_M \, f_M(C_i, \hat{\theta}_M)}{\sum_{m \in \mathcal{M}} \hat{\pi}_m \, f_m(C_i, \hat{\theta}_m)}$$

Based on τ_{iM}, we can

1. classify each subject into the type M she most likely stems from, given the fit of the model and given her choices

2. assess the quality of the classification of subjects into types

- If the classification is clean and the types are well separated, almost all subjects exhibit τ_{iM} close to 0 or 1
- If the classification is ambiguous and the types overlap, many subjects exhibit $\tau_{iM} \approx 1/3$

Type-Specific Choices: Results

Type	CPT	ST	EUT
Relative size	0.379^{***}	0.337^{***}	0.284^{***}
Concavity of utility function	0.572^{***}	0.870^{***}	0.080^{**}
Likelihood sensitivity	$0.469^{\circ\circ\circ}$		
Degree of local thinking		$0.924^{\circ\circ\circ}$	
Choice sensitivity	0.302^{***}	2.756^{***}	0.010^{***}
AIC		22,937.41	
BIC		23,017.98	

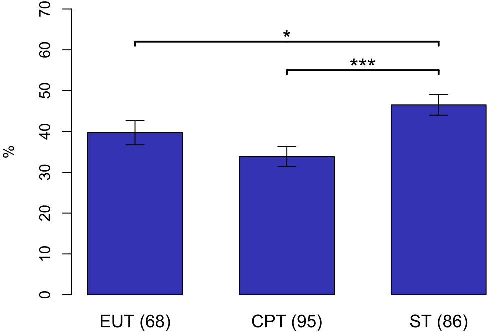
Significantly different from 0 (1) at $\alpha = 1\%$: *** ($^{\circ\circ\circ}$)

Result 2: Heterogeneity Detials

Clean classification into 38% CPT-, 34% ST-, and 28% EUT-types

Out-of-Sample Predictions

Frequency of preference reversals per type in the 6 additional binary choices



Result 3: Out-of-sample predictions

Choice set dependence matters for all types, but ST-types exhibit more preference reversals than CPT- and EUT-types

Conclusion

- At the aggregate level, both probability weighting as well as choice set dependence matter
- The finite mixture model uncovers substantial heterogeneity and classifies subjects into 38% CPT-, 34% ST-, and 28% EUT-types
- The out-of-sample analysis shows that choice set dependence matters for all types, but the ST-types exhibit most preference reversals

⇒ No theory alone can describe the risky choices of all subjects

⇒ Probability weighting and choice set dependence drive the preferences of roughly an equal share of subjects

⇒ Beyond choice under risk, the methodology and its result may help to better understand consumer, investor, and judicial decisions

No Links between Types and Observable Characteristics

Multinomial Logit with Baseline: EUT

	CPT	ST
Female	0.186	-0.029
CA	0.093	-0.025
Big 5: Extraversion	-0.005	-0.021
Big 5: Agreeableness	0.069	0.060
Big 5: Conscientiousness	0.017	0.036
Big 5: Neuroticism	-0.035	0.007
Big 5: Openness	-0.018	-0.106***
Constant	-0.957	1.233
Joint tests (p-values)		
Overall significance	0.753	0.151
Big 5 joint significance	0.708	0.064*

Significantly different from 0 at $\alpha = 1\%$: ***, 5%: **, 10%: *

Empirical Strategy: The Allais Paradox

Consider the choice between the following two lotteries with independent payoffs and a common consequence z

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 = 0.01 \\ z & p_3 = 0.66 \end{cases} \qquad \text{vs.} \qquad Y = \begin{cases} 2400 & p_1 + p_2 = 0.34 \\ z & p_3 = 0.66 \end{cases}$$

According to EUT the choice between X and Y does not depend on z

$$\underbrace{p_1\, u(2500) + p_2\, u(0) + \cancel{p_3\, u(z)}}_{\text{EU}(X)} \overset{?}{\lesseqgtr} \underbrace{(p_1 + p_2)\, u(2400) + \cancel{p_3\, u(z)}}_{\text{EU}(Y)}$$

Empirical Strategy: The Allais Paradox

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However, the empirical evidence contradicts EUT

- If $z = 2400$ many prefer the safe option Y over the risky lottery X
- If $z = 0$ many prefer the risky lottery X over the safer lottery Y

$\Rightarrow$ The switch from relatively risk averse to relatively risk seeking behavior contradicts EUT

Empirical Strategy: CPT can describe the Allais Paradox

If $z = 2400$ the choice is

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 = 0.01 \\ 2400 & p_3 = 0.66 \end{cases} \qquad \text{vs.} \qquad Y = 2400$$

- Subjects overweight the small probability $p_2 = 0.01$ of getting 0

⇒ The risky lottery X becomes less attractive compared to Y

If $z = 0$ the choice is

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 + p_3 = 0.67 \end{cases} \qquad \text{vs.} \qquad Y = \begin{cases} 2400 & p_1 + p_2 = 0.34 \\ 0 & p_3 = 0.66 \end{cases}$$

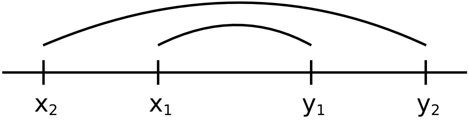
- Subjects underweight the large probability $p_2 + p_3 = 0.67$ of getting 0

⇒ The risky lottery X becomes more attractive compared to Y Details

Empirical Strategy: What about ST?

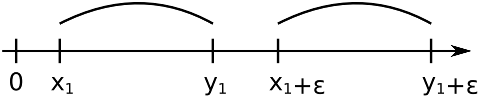
In ST, subjects rank each payoff state according to its salience, $\sigma(x_s, y_s)$, which satisfies two intuitive properties

Ordering



$\sigma(x_1, y_1) < \sigma(x_2, y_2)$

Diminishing Sensitivity



$\sigma(x_1, y_1) > \sigma(x_1+\varepsilon, y_1+\varepsilon)$

Empirical Strategy: ST with Independent Payoffs

If $z = 2400$ the choice is

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 = 0.01 \\ 2400 & p_3 = 0.66 \end{cases} \qquad \text{vs.} \qquad Y = 2400$$

- Ranking: $\sigma(0, 2400) > \sigma(2500, 2400) > \sigma(2400, 2400)$

$\Rightarrow$ The downside of lottery X is salient and makes it less attractive

If $z = 0$ the choice is

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 + p_3 = 0.67 \end{cases} \qquad \text{vs.} \qquad Y = \begin{cases} 2400 & p_1 + p_2 = 0.34 \\ 0 & p_3 = 0.66 \end{cases}$$

- Ranking: $\sigma(2500, 0) > \sigma(0, 2400) > \sigma(2500, 2400) > \sigma(0, 0)$

$\Rightarrow$ The upside of lottery X is salient and makes it more attractive

Details

Empirical Strategy: ST with Correlated Payoffs

Now, consider the case in which the payoffs of the two lotteries

$$X = \begin{cases} 2500 & p_1 = 0.33 \\ 0 & p_2 = 0.01 \\ z & p_3 = 0.66 \end{cases} \quad \text{vs.} \quad Y = \begin{cases} 2400 & p_1 + p_2 = 0.34 \\ z & p_3 = 0.66 \end{cases}$$

are perfectly correlated, i.e.

Probability	0.33	0.01	0.66
Payoff x_s	2500	0	z
Payoff y_s	2400	2400	z

- The salience ranking $\sigma(0, 2400) > \sigma(2500, 2400) > \sigma(z, z)$ no longer depends on the common consequence z

$\Rightarrow$ ST cannot explain the Allais paradox when payoffs are correlated Back

Appendix: Valuation of Lotteries in CPT

In CPT, the value of a lottery X with payoffs $x_1 \geq \ldots \geq x_J$ is

$$V(X) = \sum_{j=1}^{J} \pi_j \, u(x_j) \, ,$$

where π_j is the non-linear decision weight attached to the utility of payoff x_j. The decision weight is given by

$$\pi_j = \begin{cases} w(p_1) & \text{for } \; j = 1 \\ w\left(\sum_{k=1}^{j} p_k\right) - w\left(\sum_{k=1}^{j-1} p_k\right) & \text{for } \; 1 < j \leq J \end{cases} ,$$

where p_k is payoff x_k's probability and $w(\cdot)$ is the probability weighting function

Appendix: Valuation of Lotteries in CPT

We use the one-parameter version of Prelec's (1998) probability weighting function

$$w(p) = \exp(-(-\ln(p))^{\alpha}),$$

where $0 < \alpha \leq 1$ measures sensitivity to probability changes Back

Appendix: Valuation of Lotteries in ST

In ST, the value of a lottery X is

$$V(X) = \sum_{s \in S} \pi_s \, u(x_s) \, ,$$

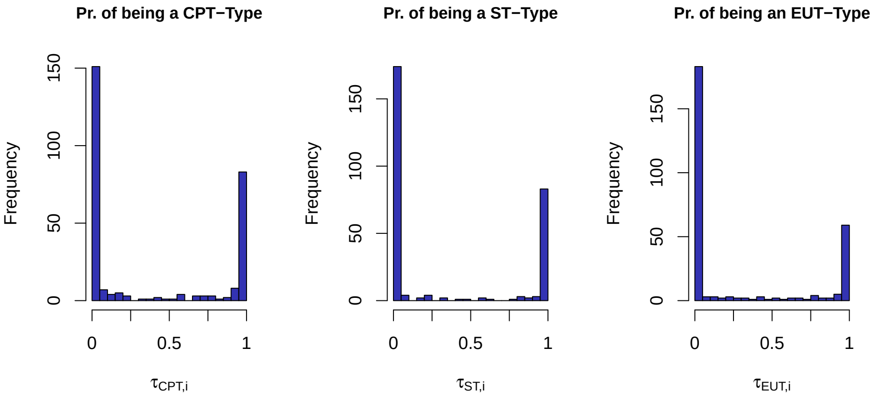
where π_s is the non-linear decision weight attached to the utility of payoff x_s realized in state $s \in S$. The decision weight is given by

$$\pi_s = p_s \, \frac{\delta^{r_s}}{\sum_{m \in S} \delta^{r_m} \, p_m} \, ,$$

where

- p_s is the probability that state s is realized
- $r_s \in \{1, \ldots, |S|\}$ is the salience-rank of state s, with lower ranks indicating higher salience
- $\delta \leq 1$ is the decision maker's degree of local thinking Back

Appendix: Clean Classification of Subjects into Types



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