Decomposing the Variance of Consumer Ratings and the Impact on Price and Demand

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Consumer ratings play a decisive role in purchases by online shoppers. Although the effect of the average and the number of consumer ratings on future product pricing and demand have been studied with some conclusive results, the effects of the variance of these ratings are less well understood. We develop a model where we decompose the variance of consumer ratings in two sources: taste differences about search and experience attributes of a durable good, and quality differences among instances of this good in the form of product failure. We find that (i) optimal price increases and demand decreases in variance caused by taste differences, (ii) optimal price and demand decrease in variance caused by quality differences, and (iii) when holding the average rating as well as the total variance constant, for products with low total variance both price and demand increase in the relative share of variance caused by taste differences. Counter to intuition, we demonstrate that risk averse consumers may prefer a higher priced product with a higher variance in ratings when deciding between two similar products with the same average rating.

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1. Introduction

Most products on the market can be described by search and experience attributes (Nelson 1981). Search attributes can be determined by inspection or by examining product specifications from
the manufacturer without the necessity of use (Shapiro 1983). Examples of search attributes for a set of headphones are the color or technical features such as active noise cancellation. In contrast, experience attributes such as the wearing comfort or the sound characteristics of headphones can hardly be known before using a product (Klein 1998, Nelson 1981, Wei and Nault 2013). Through web 2.0, the need to use a product to assess experience attributes has fundamentally changed due to consumer ratings gathered and presented on e-commerce platforms. In particular, consumer ratings offer a form of peer learning also called electronic word-of-mouth (see, e.g., Dellarocas 2003) among consumers by enabling prospective consumers to learn from other consumers’ experiences (Wu et al. 2015). Consequently, experience attributes are transformed by consumer ratings into attributes that can be searched. Thus, prospective consumers can learn how a given product performs on experience attributes by examining consumer ratings without the necessity of use (Chen and Xie 2008, Hong et al. 2012, Kwark et al. 2014). An example is the sound characteristics of headphones. Without consumer ratings, assessing this attribute requires using the actual device. By having consumer ratings available, the sound characteristics of headphones can be inferred from the experiences of other consumers (e.g., whether a specific type of headphones have a strong or a light bass).

Even if consumers can infer experience attributes from consumer ratings, there still remains uncertainty if products are characterized by inconsistent quality (i.e., inconsistent quality goods). Inconsistent quality goods are characterized by the fact that for some instances of a product the quality of search and experience attributes deviate from their intended quality. Such inconsistent quality can be observed by browsing through consumer ratings. For instance, by browsing through consumer ratings of a specific type of headphones (see Amazon (2015)), consumers can observe that for some instances the cord broke after a relatively short period of use while for other instances it did not break. This means that consumers can make inferences from consumer ratings about the probability of product failure. What they cannot learn from consumer ratings is whether their individual instance of the product (i.e., the actual set of headphones they buy) will fail. Thus, the actual quality of an individual instance of an inconsistent quality good – regardless of the number
Table 1 Different Types of Products and Product Attributes

<table>
<thead>
<tr>
<th>Attribute Type</th>
<th>Definition</th>
<th>Consistent Quality Goods</th>
<th>Inconsistent Quality Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search attributes</td>
<td>The set of attributes that can be determined without product use through examining the product specifications provided by the manufacturer.</td>
<td>The quality of search attributes is consistent among instances of a consistent quality good (i.e., search attributes do not fail).</td>
<td>The quality of search attributes differs among instances of an inconsistent quality good (i.e., search attributes may fail).</td>
</tr>
<tr>
<td>Experience attributes</td>
<td>The set of attributes that can only be determined through product use. As soon as meaningful consumer ratings are available, these attributes can be determined without use through examining these ratings.</td>
<td>The quality of experience attributes is consistent among instances of a consistent quality good (i.e., experience attributes do not fail).</td>
<td>The quality of experience attributes differs among instances of an inconsistent quality good (i.e., experience attributes may fail).</td>
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of available ratings – cannot be determined through examining consumer ratings. The distinctions we make between different types of products and their attributes is shown in Table 1.

As consumer ratings help consumers “to mitigate the uncertainty about the quality of a product and about its fit to consumers’ needs” (Kwark et al. 2014, p. 93), it is not surprising that 90% of all purchase decisions are influenced by consumer ratings (Drewnicki 2013) and the most popular feature of Amazon.com is its consumer ratings (New York Times 2004). Consumer ratings are most commonly provided in the form of a star (or comparable numerical) rating system (indicating the valence of the consumer rating) and a textual review. The information contained in the textual reviews is summarized by the star rating system typically ranging from one (lowest rating) to five (highest rating) on most e-commerce websites. Bar charts often show the distribution of the star rating, with the average rating displayed prominently beneath the product name (e.g., amazon.com, bestbuy.com, target.com, walmart.com). Thus, consumers can see at a glance the average rating from other consumers and the extent to which opinions about the product differ (variance). Elements of this variance can be caused by taste differences about search and experience attributes such as the color or the sound of headphones (i.e., some consumers like the color or the sound
while others do not) or by quality differences among instances of a product in the form of product failure (i.e., some instances of the considered headphones fail while others do not).

Among the literature that has recently emerged on consumer ratings several studies find that both the absolute number of posted consumer ratings and the average consumer rating increase demand. Fewer studies (e.g., Clemons et al. 2006, Hong et al. 2012, Sun 2012) explicitly analyze the effect of the variance of online consumer ratings on price and demand and, to the best of our knowledge, none explicitly decomposes the variance into different sources. This is important because what information is encoded in this variance is still an open research question (Markopoulos and Clemons 2013).

We consider durable goods where variance in consumer ratings can be caused by taste and quality differences in order to answer the following research question: Does the variance of consumer ratings caused by taste differences and quality differences differentially affect price and demand?

To determine the effect of the different sources of variance of consumer ratings on price and demand we construct a model featuring a monopoly retailer and consumers that differ in taste and risk aversion. We analyze two types of durable goods (see Table 1).

1. **Consistent quality goods**: To connect with the existing literature, in particular with Sun (2012), we analyze consistent quality goods where the variance of consumer ratings is solely caused by taste differences.

2. **Inconsistent quality goods**: More importantly, we analyze inconsistent quality goods where the variance of consumer ratings is caused by taste differences and quality differences in the form of product failure.

Our analysis yields the following main results: First, a higher variance caused by taste differences always signals that a product is liked by some consumers but less liked by others, and results in a higher price and lower demand. Second, a higher variance caused by quality differences signals that there is higher failure risk associated with the product resulting in a lower price and lower demand. Third, holding the average rating as well as the total variance constant, increasing the
relative share of variance caused by taste differences may lead to an increase in both price and demand. Through this mechanism, price and demand can increase with an increasing total variance. We demonstrate, therefore, that risk averse consumers may prefer a higher priced product with a higher total variance in ratings when deciding between two similar products with the same average rating.

2. Related Literature

A substantial portion of the related literature on the effects of consumer ratings empirically examines the effect of average consumer ratings and the number of consumer ratings on sales of products from different product categories. Some authors have found that an increase in average ratings has a positive effect on the sales of books (e.g., Chevalier and Mayzlin 2006, Sun 2012, Li and Hitt 2008), hotel bookings (e.g., Ye et al. 2011), and movies (e.g., Dellarocas et al. 2007). Whereas others fail to find such an effect both for books (e.g., Chen et al. 2004) and for movies (e.g., Duan et al. 2008). For the total number of ratings, the literature shows a positive effect on sales (e.g., Chen et al. 2004, Chevalier and Mayzlin 2006, Duan et al. 2008), whereas Godes and Mayzlin (2004) do not find any such effect. A more comprehensive review of this portion of related literature can be found in Babic Rosario et al. (2016).

There are several studies that find consumer ratings impact prices, mostly in the services industry. In the hotel industry, Lewis and Zervas (2016) find high-rated hotels increase prices and low-rated hotels decrease prices. In online accommodation sharing – AirBnB – prices have been found to increase after an accumulation of positive ratings, where the positive ratings are understood to establish good reputations (Gutt and Herrmann 2015, Ikkala and Lampinen 2014, 2015, Teubner et al. 2017). For a sample of Chicago restaurants, Bai et al. (2017) find that local restaurants with high ratings and a high number of reviews were more likely to initiate daily deals. There is also speculation that the addition of consumer reviews of airlines on TripAdvisor may lead to higher customer willingness to pay for a seat rated as excellent (The Economist 2016).

Fewer studies have empirically studied the effect of the variance of consumer ratings on prices and sales (Babic Rosario et al. 2016), and the results are inconclusive. Clemons et al. (2006) find
that the variance of consumer ratings is associated with higher growth in sales in hyperdifferentiated markets such as the craft beer industry. Taking the number of published reviews as a proxy for sales, Lu et al. (2014) also find a significant positive correlation between the variance of consumer ratings and sales of hotel rooms via online travel agencies. In contrast, Ye et al. (2009) find a significant negative correlation, and Ye et al. (2011) find no significant correlation between the variance of consumer ratings and hotel bookings. Also Chintagunta et al. (2010) do not find a significant correlation between sales and the variance of consumer ratings for movie box office sales.

In an experiment and in an empirical study using data from Amazon and eBay from products in the electronics category, Wu et al. (2013) find that if a consumer is risk averse toward product uncertainty, then a consumer’s willingness to pay for a product with a higher variance of consumer ratings is lower.

One of the very few studies that analyzes the effects of the variance of consumer ratings on sales analytically is Hong et al. (2012). The authors distinguish search and experience products and find that, for a pure search product, when the number of consumer ratings increases, the variance of ratings decreases. In contrast, for a product that is primarily characterized by experience attributes, when the number of consumer ratings increases, the variance of ratings may increase depending on how dominant these experience attributes are.

Most closely related to our approach, Sun (2012) analytically models the informational role of the variance of consumer ratings on price and demand. In this model, consumers are risk neutral and all products can be described by product quality and mismatch costs. Products with a high mismatch cost are products for which only some consumers have a strong liking while others substantially dislike it, whereas products with a low mismatch cost appeal to a broad audience. In Sun (2012) a high average rating indicates a high product quality, whereas a high variance of ratings is associated with a high mismatch cost where the variance of consumer ratings is solely caused by taste differences. The variance of ratings can help consumers to determine whether a product’s average rating is low because of its low product quality or because of its high mismatch
cost. In case of a low rating due to a high mismatch cost some consumers still buy the product because they know that the product matches their taste and that they do not incur a mismatch cost. In this way, a higher variance can increase the demand for a product. Sun (2012) empirically tests the predictions from her analytical model using data for books sold on amazon.com and barnesandnoble.com, finding a positive effect of the variance of consumer ratings for books with a low average rating.

In our analytical model, we build on the results from Sun (2012) and others that have a similar model setup (e.g., Chen and Xie 2008, Li and Hitt 2010). We depart from the extant literature by distinguishing taste differences and quality differences as separate sources of the variance of consumer ratings and analyze how they differentially affect price and demand for durable goods.

3. Notation and Assumptions

Our assumptions pertain to a number of different factors relating to, first, consumer heterogeneity, second, product characteristics, and third, consumer rating behavior. These are presented in turn.

Assumption 1 (Consumer Heterogeneity). Consumers are heterogeneous in taste and in risk aversion. Taste and risk aversion are independent.

In line with Sun (2012) and Herrmann et al. (2015), we assume that consumers are heterogeneous in their taste for specific product attributes. We represent consumer taste by $\tau$ which is uniformly distributed between zero and one, i.e., $\tau \sim U[0,1]$.

We further assume that consumers in e-commerce are risk averse. This assumption is justified by results from laboratory experiments (e.g., Holt and Laury 2002) as well as from surveys among online shoppers (e.g., Bhatnagar et al. 2000). For example, Bhatnagar et al. (2000) find that “the likelihood of purchasing on the Internet decreases with product and financial risk”. Consumers are also heterogeneous in risk aversion. We denote consumer risk aversion through a risk premium $\theta$ which is uniformly distributed between zero and one, i.e., $\theta \sim U[0,1]$. Independent tastes and risk premiums are represented by a square with edge length 1 (see Figure 1) where the line segment [AB] represents consumers’ tastes and the line segment [AC] represents consumers’ risk premia.
A consumer’s taste is equal to the position on the taste-axis and consumer’s risk premium is equal to the position on the risk premium-axis. For example, a consumer located in A is risk neutral and the taste matches perfectly with the product, whereas a consumer located in E has a high risk premium and the taste is slightly mismatched with the product.

Our model has two periods and in each period a unit mass of consumers is uniformly distributed within this square. In the product diffusion literature, the widely recognized Bass (1969) model and the even more widely recognized Rogers (1962) diffusion of innovations work separates innovators from imitators (Bass), or innovators and early adopters from early majority, late majority, and laggards (Rogers). Following this prior seminal research and using the terminology from Bass, in our model first-period consumers are innovators that have a strong preference to adopt new products early, rely primarily on their own expectations, and are unaffected by word of mouth through the number or prior purchasers or by consumer ratings. Second-period consumers are imitators that prefer to mitigate their product uncertainty through examining peer opinions such as consumer ratings of innovators (see, e.g., Li and Hitt 2010) that serve as an informed version of word of mouth.

To keep our analysis tractable, innovators that do not purchase in the first period exit the market and do not spill over to the second period in our main model (subsection 4.1 and 4.2). This formulation is similar to other literature analyzing consumer ratings such as Chen and Xie (2008).
or Li and Hitt (2008). In subsection 4.3 we extend our main model by numerically analyzing the effect of innovators that spill over to the second period.

According to Bass and Rogers, the number of imitators is typically higher than the number of innovators. Consequently, we introduce a scaling factor \( k \in (1, \infty) \) that we use to scale the unit mass of imitators relative to the unit mass of innovators. Bass and Rogers also suggest that imitators have a lower risk tolerance (i.e., a higher risk premium) than innovators. Hence, we introduce a scaling factor \( z \in (1, \infty) \) that we use to scale the risk premium of imitators.

Although we refer to the Bass terminology and definitions to separate innovators from imitators, our mathematical model is fundamentally different from the Bass model. The Bass model is a model of exposure, consistent with its roots in epidemiology as a model of the spread of disease where in each time \( t \) some proportion of innovators and of imitators are exposed to disease and falls ill. Asymptotically the population is exposed and falls ill, or in the Bass sense purchases the product. Price and promotion can be used to change the time path, but the asymptotic property remains. In contrast, we model consumer choice where some innovators in the first period and some imitators in the second period after learning from consumer ratings, find the price too high to purchase and exit the market.

Assumption 2 (Product Characteristics). Each product is characterized by a positive matched quality, positive or zero mismatch costs, and a failure rate between zero and one. Mismatch costs are limited by the matched quality of a product.

Assumption 2 defines products by three characteristics: matched quality, mismatch costs, and failure rate. Matched quality represents the general product quality and determines how much an ideal consumer (i.e., a consumer with a perfectly matched taste) enjoys a product that does not fail during its typical period of usage. Matched quality reflects search attributes that can be obtained from product specifications such as the availability of an active noise cancellation feature in a set of headphones, and experience attributes that can be obtained from consumer ratings such as how well the noise cancellation works while worn in an airplane or train. Both influence
the general product quality. We denote matched quality as $v$ and assume that $v \in \mathbb{R}^+$. Mismatch costs are the same as in Sun (2012) and reflect search attributes such as the color of headphones and experience attributes such as the sound characteristics of headphones “that would have an influence on how much consumers would differ in their enjoyment of the product” (Sun 2012, p. 697). We denote mismatch costs as $x$ and assume that $x \in [0, v]$. Mismatch costs negatively affect consumers’ enjoyment of a product depending on individual consumer tastes. For example, some consumers love the sound of headphones that have a strong bass while others dislike a strong bass. Products with attributes that result in mismatch costs of close to zero are a perfect fit for all consumers (i.e., typical mass market products such as blank paper) while products with attributes that result in high mismatch costs are a perfect fit for just a small group of consumers (i.e., typical niche products such as Management Information Systems textbooks). In contrast to Sun (2012), we assume that mismatch costs are limited by the matched quality of a product. Thus, even consumers that maximally dislike all attributes that result in mismatch costs receive non-negative enjoyment from the product if they were to obtain it for free.

Finally, we allow for inconsistent quality goods. As defined in the Introduction, inconsistent quality goods are characterized by some product instances that fail and create unacceptable experiences (Sridhar and Srinivasan 2012) while other product instances do not fail. This fact is captured by the third product characteristic – failure rate, $f \in [0, 1]$, that accounts for the likelihood of failure during a product’s typical useful life (Bardey 2004). Products with a failure rate of zero never fail during their typical useful life (distinctive of consistent quality goods), and products with a failure rate of one always fail during this period.

In the first period, publicly available product specifications from the product manufacturer provide the dominant source of product information. These specifications represent certain search attributes and are used by innovators to build expectations about the uncertain experience attributes and potential quality issues of a product resulting in expected matched quality $v_e$, expected mismatch costs $x_e$, and expected failure rate $f_e$. As we do not consider screening mechanisms or reputational effects of the manufacturer, all innovators and the retailer have the same
information from the product manufacturer and any remaining information asymmetries are negligible. Consequently, innovators and the retailer are homogeneous in their expectations of product characteristics and we do not assume any relationship between $v_r$, $x_r$, and $f_r$. In the second period, imitators and the retailer learn from consumer ratings of innovators and can determine the product characteristics realized matched quality $v_r$, realized mismatch costs $x_r$, and realized failure rate $f_r$ that may differ from the expectations.

Assumption 3 (Consumer Rating Behavior). All purchasing innovators with extreme experiences (positive or negative) publish honest consumer ratings. Purchasing innovators that experience product failure publish a consumer rating of zero.

Since the 1960s marketing researchers reported that innovators are very keen to talk about their experiences with a product. For example, Engel et al. (1969) write that “there seems to be no question that the first users of a new product or service are active in the word-of-mouth channel” (p. 15). In contrast to Sun (2012), we do not require that all purchasing innovators express their experiences with a product via consumer ratings. However, we suppose that soon after a product launch at least all innovators with extreme experiences publish a consumer rating. This is consistent with the empirically observed under-reporting bias (Hu et al. 2017) meaning that “consumers with extreme ratings (positive or negative) are more likely to report their reviews than consumers with moderate ratings” (Hu et al. 2017, p. 450). Further, the published consumer ratings are honest and correspond to the actual utility derived from the consumption of the product. Consequently, there is no external manipulation of consumer ratings as discussed in Mayzlin (2006) or Luca and Zervas (2016) and consumer ratings are continuous in our model. The typically provided star rating systems on e-commerce platforms represent a discretization of our model.

Our latter part of Assumption 3 can be justified by Sridhar and Srinivasan (2012). They propose that a consumer who experiences product failure (i.e., unacceptable product experience) in the face of other consumers’ positive ratings, experiences high normative conflict. Paraphrasing Sridhar and Srinivasan (2012), in such a situation, the consumer, already dissatisfied because of the product
failure, may be motivated to provide an even lower rating to rectify the “incorrect” (according to personal experience) rating on the review website. We further analyzed different inconsistent quality goods using a text mining approach and found that the vast majority of consumers post a 1-star rating (representing a rating of zero in our model) if the textual review is associated with failure. Details of this analysis are available from the authors.

We summarize our notation in Table 2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$v$</td>
<td>Matched quality, $v \in R^+$</td>
</tr>
<tr>
<td>$x$</td>
<td>Mismatch costs, $x \in [0, v]$</td>
</tr>
<tr>
<td>$f$</td>
<td>Failure rate, $f \in [0, 1]$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Consumer taste, $\tau \sim U[0, 1]$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Consumer risk premium, $\theta \sim U[0, 1]$</td>
</tr>
<tr>
<td>$k$</td>
<td>Scaling factor to scale the unit mass of imitators, $k \in (1, \infty)$</td>
</tr>
<tr>
<td>$z$</td>
<td>Scaling factor to scale the risk premium of imitators, $z \in (1, \infty)$</td>
</tr>
</tbody>
</table>

4. **Model Analysis**

We consider a two period model with a monopoly retailer and risk averse consumers. The sequence of events is illustrated in Figure 2.

In the first period, a unit mass of innovators enters the market. Each innovator demands at most one unit, as is the case with durable goods. The retailer sets a profit maximizing price $p_1$ and innovators decide whether to purchase based on their expectations of the uncertain product characteristics. Finally, purchasing innovators post honest ratings.

In the second period, $k$ unit masses of imitators enter the market. Imitators and the retailer observe the ratings of innovators and learn about the product characteristics. Based on this information, the retailer sets a profit maximizing price $p_2$ that may differ from $p_1$ and imitators decide
whether to purchase. We take retailers as myopic in the first period as they cannot forecast con-
sumer ratings and, thus, how consumer ratings of innovators may influence second-period price.
Dynamic pricing can be observed for almost all products sold on Amazon.com (see price track-
ers such as Keepa.com or camelcamelcamel.com) and there is evidence that retailers dynamically
adjust prices in response to consumer ratings (see Related Literature section).

In the following we separately analyze consistent quality goods (i.e., products that do not fail) and inconsistent quality goods (i.e., products that may fail).

4.1. Consistent Quality Goods

Consistent quality goods are solely characterized by matched quality and mismatch costs (i.e.,
failure rate equals zero) and the variance of consumer ratings is solely caused by taste differences
on search and experience attributes.

First Period: Innovators make their purchase decisions based on their expectations of \( v \) and \( x \),
which are denoted by \( v_e \) and \( x_e \), respectively. The expected net utility of innovators is

\[
u_1 = v_e - x_e \tau - p_1.\]
Note that the uncertainty associated with experience attributes could be modeled as a separate risk component in the utility function. As this uncertainty is eliminated through consumer ratings for imitators, it has no effect on the qualitative nature of our results. For the sake of simplicity, we do not consider this uncertainty in the utility function of innovators for consistent quality goods nor for inconsistent quality goods later.

Solving \( v_e - x_e \tau - p_1 = 0 \) for \( \tau \) yields the taste of the indifferent innovator which we denote with \( \tilde{\tau}_1 = (v_e - p_1)/x_e \). All innovators with \( \tau \leq \tilde{\tau}_1 \) purchase, and all innovators with \( \tau > \tilde{\tau}_1 \) do not. As \( \tau \) is uniformly distributed between zero and one and there is a unit mass of innovators in the market, first-period demand \( D_1 \) is equal to \( \tilde{\tau}_1 \).

Knowing this demand, the retailer can maximize profits by solving \( \max_{p_1} p_1 D_1 \). This leads to the optimal first-period price and demand:

\[
p^*_1 = \frac{v_e}{2} \quad \text{and} \quad D^*_1 = \frac{v_e}{2x_e}.
\]

At least all purchasing innovators with extreme experiences (i.e., all the innovators with \( \tau = 0 \) and \( \tau = D^*_1 \)), and possibly all other purchasing innovators, publish an honest rating. Our ratings are based on the experienced gross utility \( v_r - \tau x_r \) of innovators as proposed by Sun (2012).

Knowing that tastes are uniformly distributed in \([0, D^*_1]\), imitators can resolve the under-reporting bias and infer all potential ratings that are uniformly distributed between \([v_r - D^*_1 x_r, v_r]\). Given this uniform distribution of ratings, the average rating \( M \), and the variance of ratings \( V \) can be computed, respectively, as

\[
M = v_r - 0.5D^*_1 x_r \quad \text{and} \quad V = \frac{D^*_1 x_r^2}{12}.
\]

Second Period: By considering the average and the variance of ratings, imitators learn about experience attributes and mitigate their uncertainty about product characteristics. Imitators can directly derive the realized product characteristics \( v_r \) and \( x_r \) by rearranging (2):
\[ v_r = M + \sqrt{3V} \text{ and } x_r = \frac{\sqrt{12V}}{D_1^*}. \] (3)

After deriving \( v_r \) and \( x_r \), imitators have no remaining uncertainty about experience attributes. Given this information, the net utility for imitators is

\[ u_2 = v_r - x_rr - p_2. \]

Based on \( u_2 \) the retailer can derive the taste of the indifferent imitator as a function of the second-period product price \( p_2 \): \( \tilde{\tau}_2 = (v_r - p_2)/x_r \). As taste is uniformly distributed among imitators and the mass of imitators is scaled by the factor \( k \), second-period demand \( D_2 \) is equal to \( k\tilde{\tau}_2 \). Knowing this demand, the retailer can again maximize profits by solving: \( \max_{p_2} p_2 D_2 \). This leads to the optimal second-period levels of price and demand:

\[ p_2^* = \frac{v_r}{2} \text{ and } D_2^* = \frac{kv_r}{2x_r}. \] (4)

Using (3), optimal price and demand can be rewritten in terms of \( M \) and \( V \):

\[ p_2^* = \frac{M}{2} + \frac{\sqrt{3V}}{2} \text{ and } D_2^* = \frac{kD_1^*}{4} \left( \frac{M}{\sqrt{3V}} + 1 \right). \] (5)

Based on these representations of \( p_2^* \) and \( D_2^* \), the following proposition details the effects of \( M \) and \( V \) on optimal price and demand for consistent quality goods (all proofs of the propositions are in the appendix).

**Proposition 1.** For consistent quality goods, price and demand both increase with the average rating, price increases and demand decreases with the variance of ratings.

The intuition behind Proposition 1 is as follows. First, a high average rating is a credible signal of a high product quality (see (3)). Thus, the retailer charges a higher price and consumers have a higher demand for a product with a higher quality (see (4)). The first part of Proposition 1 represents a theoretical confirmation of the empirical findings of Chevalier and Mayzlin (2006), Sun (2012), Li and Hitt (2008), and Dellarocas et al. (2007) that found a positive impact of average
consumer ratings on sales for books and movies, both of which are consistent quality goods that are characterized by search and experience attributes, and typically do not fail.

Second, a high variance of ratings indicates a high product quality and high mismatch costs (see (3)). This means that an imitator with taste that closely matches the product enjoys such a product more than a product with a low variance of ratings. The retailer charges a higher price to all imitators to skim the higher willingness to pay of imitators with tastes that closely match the product. This higher price deters imitators with tastes that do not closely match the product, resulting in a lower demand (see (4)). Figure 3 illustrates the response of second-period price and demand to changes in the variance of ratings.

In contrast to Sun (2012), we do not find that a higher variance of ratings may also increase second-period demand. In Sun’s model, a necessary condition for such an effect is that the average rating $M$ is negative. From (2), we know that a negative average rating means that $x_r > 2v_r/D_1$. $D_1$ has a maximum of 1 which implies that $x_r > 2v_r$. This would mean that the enjoyment of an innovator with taste $\tau = 1$ is at most $-v_r$ if $p_1 = 0$. As most products do not exhibit such characteristics, our second assumption rules out the possibility of $M$ being negative by assuming $x$ is non-negative, $x \in [0, v]$.

Comparing prices across periods, our analysis for consistent quality goods further shows that a discounted second-period price in response to the average rating and variance of ratings results from an overestimation of matched quality, $v_c > v_r$ (see (1) and (4)), in the first period.
4.2. Inconsistent Quality Goods

Inconsistent quality goods are not only characterized by matched quality and mismatch costs, but additionally by a failure rate. For these products, the variance of consumer ratings is not only caused by taste differences but also by quality differences in the form of product failure.

First Period: Innovators make their purchase decisions based on expected matched quality $v_e$, expected mismatch costs $x_e$, and expected failure rate $f_e$. Innovators with taste $\tau$ and risk premium $\theta$ have the expected net utility

$$u_1 = (v_e - x_e \tau)(1 - f_e) - p_1 - f_e \theta. \quad (6)$$

The first part of the right hand side of (6) is equal to the expected net utility of a risk neutral innovator. The last term in (6) captures a risk averse innovators's negative utility caused by the risk that the product fails. Our modeling of consumer risk does not make any assumptions about the specific form of risk aversion such as hyperbolic absolute risk aversion or constant absolute risk aversion. Our only assumption is that consumers do not like the possibility of their product failing.

Given $u_1$, we can derive the taste of a risk-neutral innovator that is indifferent between purchasing and not purchasing the product, $\tilde{\tau}_{1\theta=0}$, and the risk premium of an indifferent innovator with a perfectly matched taste, $\tilde{\theta}_{1\tau=0}$.

$$\tilde{\tau}_{1\theta=0} = \frac{v_e(1 - f_e) - p_1}{x_e(1 - f_e)} \quad \text{and} \quad \tilde{\theta}_{1\tau=0} = \frac{v_e(1 - f_e) - p_1}{f_e}.$$

Due to the independence of taste and risk premium, first-period demand $D_1$ equals $0.5\tilde{\tau}_{1\theta=0}\tilde{\theta}_{1\tau=0}$ (i.e., the area of the triangle $[A, \tilde{\tau}_{1\theta=0}, \tilde{\theta}_{1\tau=0}]$ in Figure 4) meaning that all innovators with a taste/risk premium pair that is located below the linear function of indifferent innovators purchase the product and all others do not and exit the market.

In terms of $v_e$, $x_e$, and $f_e$, first-period demand can be written as:

$$D_1 = \frac{(v_e(1 - f_e) - p_1)^2}{2f_e x_e (1 - f_e)}.$$
Based on this demand the retailer maximizes profits by choosing first-period price: \( \max_{p_1} D_1 \). This results in an optimal first-period price and demand:

\[
p_1^* = \frac{v_e(1 - f_e)}{3} \quad \text{and} \quad D_1^* = \frac{2v_e^2(1 - f_e)}{9f_e x_e}.
\]

(7)

Innovators that purchase and publish a rating base this rating on their experienced gross utility \( v_r - x_r \tau \) if the consumed product does not fail and a rating of zero if it does. If all purchasing innovators publish a rating, then this results in a rating distribution where \( 1 - f_r \) percent of purchasing innovators publish a rating of \( v_r - x_r \tau \) for products that do not fail and \( f_r \) percent publish a rating of zero for products that fail. For products that do not fail, ratings are triangularly distributed between \( v_r - \tilde{\tau}_1^{\theta=0} x_r \) and \( v_r \) with mode at \( v_r \) (see the triangular distribution in Figure 5 with the solid hypotenuse).

Our explanation for this specific shape of the rating distribution is as follows. Those innovators that publish a rating of \( v_r \) have a taste of \( \tau = 0 \). Thus, the number of ratings of \( v_r \) equals the indifferent risk premium for these innovators \( \tilde{\theta}_1^{\tau=0} \). For decreasing ratings, the indifferent risk premium and, therefore, the number of ratings decreases. The lower bound of ratings for products that do not fail is the rating of the indifferent imitator with a risk premium of zero, \( v_r - \tilde{\tau}_1^{\theta=0} x_r \). Thus, the mode of the triangular distribution must be at \( v_r \) and the number of ratings strictly decreases with increasing taste down to a rating of \( v_r - \tilde{\tau}_1^{\theta=0} x_r \). A decreasing number of consumer ratings
from high ratings to low ratings can also be observed empirically and is explained by the so called 
acquisition bias (Hu et al. 2017) meaning that “consumers with a favorable predisposition toward 
a product are more likely to purchase a product” (Hu et al. 2017, p. 450).

Considering the under-reporting bias (cf., Assumption 3), a mass of innovators $b \in [0, 1 - f_r]$ with 
mediocre experiences may not publish a rating (see the area between the solid hypotenuse and the 
dashed line in Figure 5). However, all innovators with extreme experiences publish a rating. In our 
model, extreme experiences are represented by a perfectly matched taste (i.e., a rating of $v_r$), a 
perfectly mismatched taste (i.e., a rating of $v_r - \tilde{\tau}^{\theta_1=0}x_r$) or product failure (i.e., a rating of zero). 
Even in this case, imitators can observe the corners of the triangular distribution and resolve the 
under-reporting bias (Hu et al. 2017).

The resulting rating distribution has the typical J-shape which has been found for almost all 
products sold on Amazon.com (Hu et al. 2017). This empirical consistency provides evidence that 
helps justify our assumptions.

In contrast to consistent quality goods, the enjoyment of inconsistent quality goods depends not 
only on two, but on three product characteristics. Thus, it is not sufficient to consider only the 
average and the variance of ratings to derive the relevant product characteristics from the rating 
distribution. For example, solely based on the average and the variance of the rating distribution, 
imitators cannot distinguish if a mediocre average rating and a positive variance is caused by taste 
differences, quality differences, or a combination of these attributes. However, by decomposing the
total variance into variance caused by taste differences and variance caused by quality differences, imitators and the retailer can distinguish between these cases. Variance caused by taste differences, denoted as $V_t$, can be derived by disregarding all negative ratings which are caused by product failure and computing the variance of the triangular distribution on the right in Figure 5. As we only have two sources of variance, the variance caused by quality differences, denoted as $V_q$, must be equal to the difference between the total variance and the variance caused by taste differences. $M$, $V_t$, and $V_q$ can be computed, respectively, as:

$$M = (v_r - \frac{\tau^{\theta=0}_1 x_r}{3})(1 - f_r), V_t = (\frac{\tau^{\theta=0}_1 x_r^2}{18})(1 - f_r), \text{ and } V_q = \frac{(1 - f_r) f_r (3v_r - \tau^{\theta=0}_1 x_r)^2}{9}. \quad (8)$$

**Second Period:** Based on $M$, $V_t$, and $V_q$ imitators learn about the product. A product with a rating distribution with large $M$, large $V_t$, and small $V_q$ suggests that the product has a high matched quality and substantial mismatch costs but only a small failure rate. A product with large $M$, small $V_t$, and large $V_q$ has a high matched quality with a substantial failure rate but little mismatch costs. Imitators derive $v_r$, $x_r$, and $f_r$ for inconsistent quality goods by rearranging (8):

$$v_r = M + \frac{V_q + \sqrt{2V_t(M^2 + V_q)}}{M}, x_r = \frac{3\sqrt{2V_t(M^2 + V_q)}}{M\tau^{\theta=0}_1}, \text{ and } f_r = \frac{V_q}{M^2 + V_q}. \quad (9)$$

After deriving $v_r$, $x_r$, and $f_r$ imitators have no remaining uncertainty about the product characteristics. They know the exact matched quality and mismatch costs of the product and, therefore, how well the product matches their tastes. Even if imitators know the exact failure rate of the product, they do not know whether their individual product will fail. Thus, the expected net utility for imitators is

$$u_2 = (v_r - x_r \tau)(1 - f_r) - p_2 - f_r z \theta,$$

where the term $f_r z \theta$ captures the risk associated with product failure. Based on the net utility, the retailer can derive second-period demand. Compared to the first period, second-period demand $D_2$ is scaled by $k$ and equals $0.5k \hat{\tau}^{\theta=0}_2 \hat{\theta}_2 = 0$. In terms of $v_r$, $x_r$, and $f_r$, second-period demand can be written as:
\[ D_2 = \frac{k(v_r(1 - f_r) - p_2)^2}{2f_rzx_r(1 - f_r)}. \]  

(10)

Based on this demand the retailer again maximizes profits by choosing second-period price: \( \max_{p_2} p_2 D_2 \). This results in optimal second-period price and demand:

\[ p_2^* = \frac{v_r(1 - f_r)}{3}, \quad \text{and} \quad D_2^* = \frac{2kv_r^2(1 - f_r)}{9f_rzx_r}. \]  

(11)

Using (9), optimal price and demand can be rewritten as functions of \( M, V_t, \) and \( V_q \):

\[ p_2^* = \frac{M}{3} + \frac{M\sqrt{2V_t(M^2 + V_q)}}{3(M^2 + V_q)} \quad \text{and} \quad D_2^* = \frac{Mk\tau_1^\theta = 0 \sqrt{2(M^2 + V_q)(\sqrt{2V_t + \sqrt{M^2 + V_q}})^2}}{27V_q\sqrt{V_t}}. \]  

(12)

Based on these representations of \( p_2^* \) and \( D_2^* \), we derive the effects of the average rating, variance caused by taste differences, and variance caused by quality differences on optimal price and demand in the next three propositions.

**Proposition 2.** For inconsistent quality goods, price and demand both increase with the average rating.

The intuition for Proposition 2 is similar to the intuition underlying the first part of Proposition 1 for consistent quality goods. A high average rating acts as a credible signal of a high product quality (i.e., high matched quality and low failure rate; see (9)). Therefore, price and demand both increase with the average rating (see (11)).

**Proposition 3.** For inconsistent quality goods, price increases and demand decreases with the variance caused by taste differences.

A high variance of ratings caused by taste differences indicates a product with high mismatch costs. Again, this means that an imitator with a perfectly matched taste enjoys such a product more (i.e., the product has a higher matched quality) than a product with low variance caused by taste differences (see (9)). Thus, the retailer charges a higher price to skim the higher willingness to
pay of imitators with tastes that closely match the product. This higher price deters some imitators with tastes that do not closely match the product and, therefore, results in a lower demand (see (11)). Figure 6 illustrates the relationship between price and demand, and the variance caused by taste differences.

![Figure 6](image)

**Figure 6** Optimal Price and Demand for Inconsistent Quality Goods - Variance caused by Taste Differences

**Proposition 4.** For inconsistent quality goods: (a) Price decreases with variance caused by quality differences; (b) If the variance caused by quality differences is sufficiently low, then demand decreases with variance caused by quality differences; (c) If the variance caused by quality differences is sufficiently high and the variance caused by taste differences is sufficiently low, then demand increases with variance caused by quality differences.

A high variance of ratings caused by quality differences indicates a high failure rate (see (9)). A high failure rate is a signal of quality issues with the product and consequently the retailer sets a lower price (see (11)). A high failure rate also has a direct negative effect on demand as imitators do not like products with potential quality issues. If this direct negative effect of failure rate on demand outweighs the indirect positive effect through a lower price (i.e., if \( V_q < 2M^2 \), see proof of proposition 4 in the appendix), then demand decreases with the variance caused by quality differences. Figure 7 illustrates the relationship between optimal price and demand, and variance caused by quality differences for a product with \( V_q < 2M^2 \).
If the indirect positive effect of failure rate through a lower price outweighs the direct negative effect (i.e., if $V_q > 2M^2$ and $V_i < \frac{(M^2 + V_q)(-2M^2 + V_q)^2}{2(2M^2 + V_q)^2}$, see proof of proposition 4 in the appendix), then demand increases in the variance caused by quality differences. However, in a typical five star rating system with one indicating the lowest and five indicating the highest rating, the condition $V_q > 2M^2$ is not valid.

Comparing prices across periods, our analysis for inconsistent quality goods further shows that a discounted second-period price in response to the average rating and the two parts of the variance of ratings results from an overestimation of matched quality, $v_e > v_r$, and/or an underestimation of the failure rate, $f_e < f_r$ (see (7) and (11)), in the first period.

In our analyses so far, we have investigated the effects from increasing one part of the variance (e.g., variance caused by taste differences) while the other part of the variance (e.g., variance caused by quality differences) stays constant. To further analyze different decompositions of a constant total variance, we substitute $V_i$ by $V - V_q$ in (12). This means that an increase of variance caused by taste differences goes along with a decrease of variance caused by quality differences, but the total variance stays constant. With this substitution, optimal price and demand can be written as

$$p_2^* = \frac{M}{3} + \frac{M \sqrt{2(V - V_q)(M^2 + V_q)}}{3(M^2 + V_q)}$$

and

$$D_2^* = \frac{Mk\tilde{\theta}=0 \sqrt{2(M^2 + V_q)} (\sqrt{2(V - V_q)} + \sqrt{M^2 + V_q})^2}{27zV_q \sqrt{V - V_q}}.$$  

(13)
Based on these representations of $p_2^*$ and $D_2^*$, we derive the effects of different shares of variance caused by taste differences (quality differences) on optimal price and demand in the next proposition.

**Proposition 5.** For inconsistent quality goods and a constant total variance: (a) Price increases (decreases) with an increasing relative share of variance caused by taste differences (quality differences); (b) If the total variance is sufficiently low, then demand increases (decreases) with an increasing share of variance caused by taste differences (quality differences); (c) If the total variance is sufficiently high, then demand decreases (increases) with an increasing share of variance caused by taste differences (quality differences).

The intuition for Proposition 5 is as follows. An increasing relative share of variance caused by taste differences is necessarily associated with a decreasing relative share of variance caused by quality differences. Again, a higher variance of ratings caused by taste differences indicates a product with higher mismatch costs. Similar to Proposition 3, an imitator with a perfectly matched taste enjoys such a product more than a product with lower variance caused by taste differences. Thus, the retailer charges a higher price to skim the higher willingness to pay of imitators with tastes that closely match the product. At the same time, a lower variance caused by quality differences indicates a lower failure rate. This leads to a further increase of the price as the retailer associates less quality issues with the product.

Holding the average rating constant, a lower failure rate makes the product more attractive to risk averse consumers. If the total variance is lower than a threshold, $V < \overline{V}$ where $\overline{V}$ is defined in the proof of Proposition 5 (see appendix), then the direct positive effect of the lower failure rate on demand is greater than the indirect negative effect of a higher price on demand. Thus, in this case, both price and demand increase in the share of variance caused by taste differences. Alternatively, if the total variance is higher than a threshold, $V > \overline{V}$ where $\overline{V}$ is also defined in the proof of Proposition 5, then the direct positive effect of the lower failure rate on demand is smaller than the indirect negative effect through price. In this case, the total effect of an increasing share of variance caused by taste differences on demand is negative.
Figure 8 illustrates the response of optimal price and demand to changes in the decomposition of the variance of consumer ratings for $V < \overline{V}$ in the left-hand side graph and $V > \overline{V}$ in the right-hand side graph.

Through the mechanism described in Proposition 5b, price and demand can increase with total variance of consumer ratings which is illustrated in the following numerical example.

Numerical Example: To get realistic values for the average rating and variance of ratings, we scale the risk premium of innovators by $z_1 = 250$. To represent the higher risk aversion of imitators, we scale their risk premium by $z_2 = 500$. Further, we take the number of imitators as four times higher than the number of innovators by setting $k = 4$. The shaded area in Figure 9 illustrates optimal demand for products with an average rating of 4, a total variance of ratings between 1 and 1.5, and varying shares of variance caused by taste differences. For these values, $V < \overline{V}$ holds and consequently an increasing relative share of variance caused by taste differences leads to an increase in demand (cf. Proposition 5b). Thus, the lower bound of the shaded area represents demand for products with the lowest possible relative share of $V_1$, and the upper bound represents demand for products with the highest possible relative share of $V_1$. 

Figure 8  Optimal Price and Demand for Inconsistent Quality Goods - Changes in the Composition of the Variance
The point marked with A represents a product with expected product characteristics of $v_e = 5.50$, $x_e = 4.15$, and $f_e = 0.020$. The realized product characteristics of product A are $v_r = 5.30$, $x_r = 4.10$, and $f_r = 0.023$. The resulting total variance is 1.1, which is composed of approximately 64% of variance caused by taste differences and 36% of variance caused by quality differences. This results in an optimal price of 1.73 and a demand of 0.5. The solid black line in Figure 9 represents products with the same optimal price as product A ($p_2^* = 1.73$). As optimal price increases in the relative share of variance caused by taste differences (cf., Proposition 5(a)), all products above the solid black line have higher prices compared to product A. Thus, holding the average rating constant and increasing the total variance of ratings, we find higher optimal prices and higher demand for products in the top right-hand quadrant from point A. Comparing the worst possible variance composition marked with B ($D_2^* = 0.43$, $p_2^* = 1.66$) and the best possible variance composition marked with C ($D_2^* = 1.34$, $p_2^* = 1.88$) illustrates that product C with 50% higher total variance has a 13% higher price, and a more than three times higher demand compared to product B. This comparison demonstrates that the source of variance of consumer ratings substantially influences
product prices and sales, and that risk averse consumers may prefer products with a higher price and a higher total variance.

4.3. Model Extension: Inconsistent Quality Goods with Overlapping Innovators

In our main model, we assume that innovators who do not purchase in the first period exit the market. In this model extension, we relax this assumption and allow non-purchasing innovators from the first period to reconsider purchasing the same product in the second period (i.e., spillover) and extend the mass of imitators in the second period. We take these overlapping innovators as innovators in the second period. Thus, overlapping innovators remain unaffected by consumer ratings and, even though consumer ratings are available in the second period, continue to decide based on their expectations of product characteristics. Allowing overlapping innovators to become imitators in the second period would contradict Bass (1969) and Rogers (1962) that both take innovators and imitators as mutually exclusive consumer groups with different characteristics in several dimensions, including elements that drive taste and risk preferences evidenced in the risk premium. For there to be overlapping innovators in the second period requires that the retailer reduces the price from first to second period ($p_2^* < p_1^*$). Overlapping innovators may only purchase the product in the second period if they can take advantage of a lower second-period price. Consequently, the demand function for overlapping innovators is given by

$$D_{spill} = \max \left[ 0, \frac{(v_e(1 - f_e) - p_2)^2}{2f_e x_e (1 - f_e)} - \frac{(v_e(1 - f_e) - p_1^*)^2}{2f_e x_e (1 - f_e)} \right]. \quad (14)$$

The behavior of innovators is not strategic as they do not know in the first period the direction of a potential price change which depends on the magnitude and direction of the deviation of expected product characteristics from realized product characteristics. To consider overlapping innovators in the second-period demand function, (10) has to be extended by (14), yielding

$$D_2 = \frac{k(v_r(1 - f_r) - p_2)^2}{2f_r x_r (1 - f_r)} + D_{spill}. \quad (15)$$
The demand function in (15) represents a kinked demand curve with the kink at \( p_2 = p^*_2 \). To maximize profits (i.e., \( \max_{p_2} p_2 D_2 \)), the retailer determines an optimal second-period price with no spillover, \( p^*_{2,ns} \), and an optimal second-period price with spillover, \( p^*_{2,ws} \), and chooses the price which results in higher profits. This retailer behavior is represented by the max-function in (14).

The resulting optimal second-period price and demand can be expressed as functions of \( M, V_t, \) and \( V_q \) by using (9). As the resulting equations and derivatives for optimal second-period price and demand with spillover cannot be simplified to yield clear analytical results, we numerically analyze the effects of overlapping innovators on our results from the main model.

Numerical Analysis: Our numerical analysis proceeds in two steps. First, we analyze the circumstances when a spillover takes place. Second, we analyze whether Propositions 2 to 5 of our main model also hold with overlapping innovators.

Step 1: For the numerical analysis we set all realized product characteristics to 0.5 (i.e., \( v_r = x_r = f_r = 0.5 \)), the number of imitators to be four times higher than the number of innovators (i.e., \( k = 4 \)) and the risk tolerance of innovators to be two times higher than the risk tolerance of imitators (i.e., \( z = 2 \)). As the expected product characteristics of innovators define optimal first-period price (see (7)) and the mass of overlapping innovators (see (14)), we vary the expected product characteristics between 0 and 1. The results are illustrated in the graphs in Figure 10 and Figure 11.

In the graph in Figure 10, \( v_e \) and \( x_e \) are varied between 0 and 1. The area IV represents combinations of \( v_e \) and \( x_e \) that are not defined as we assume \( x \leq v \) in our model. The area VI represents combinations where the indifferent consumers are not defined in the first period (i.e., \( \tilde{\tau}^\theta = 0 \supseteq [0, 1] \) or \( \tilde{\theta}^r = 0 \supseteq [0, 1] \)) and the areas V where this is the case for the second period (i.e., \( \tilde{\tau}_2^\theta = 0 \supseteq [0, 1] \) or \( \tilde{\theta}_2^r = 0 \supseteq [0, 1] \)). The area I represents combinations of \( v_e \) and \( x_e \) which result in an optimal first-period price that is lower than the optimal second-period price (i.e., \( p^*_1 < p^*_2 \)) and, thus, innovators do not spill over (no spillover, case (a)). This is not surprising as in these cases the realized matched quality of \( v_r = 0.5 \) is (much) higher than the expected matched quality and consequently the retailer...
Figure 10  Numerical Analysis for Overlapping Innovators, \( v_e \) and \( x_e \) varied

increases the price from first to second period after imitators and the retailer learn about the actual matched quality by observing consumer ratings. The area II represents combinations of \( v_e \) and \( x_e \) where a spillover would generate a candidate solution for optimal price and demand in the lower part of the kinked demand curve (i.e., \( p^*_{2,ws} < p^*_1 \)). Interestingly, in this area the retailer chooses the profit-maximizing price with no spillover (i.e., \( p^*_{2,ns} \geq p^*_1 \)) as this price results in higher profits (no spillover, case (b)). This higher price means that the profits from a higher price charged to imitators is greater than the forgone profits from a lower price yielding higher demand from imitators plus the innovators that spill over.

Finally, the area III represents combinations where it is profit-maximizing for the retailer to choose an optimal second-period price that is lower than the optimal first-period price (i.e., \( p^*_{2,ws} < p^*_1 \)) and generates overlapping innovators (spillover). This means that the area III represents combinations where innovators spill over between periods and purchase the product in the second period. In the upper right corner of the area III this is not surprising as the realized matched quality is (much) lower compared to the expected matched quality. Naturally, optimal price set by the retailer in the first period is higher than in the second period even without considering the potential for innovators to spill over (i.e., \( p^*_1 > p^*_{2,ns} > p^*_{2,ws} \)). In the lower part of the area III, we
have the interesting situation that without considering innovators that spill over between periods, optimal second-period price would be higher than optimal first-period price. However, due to the potential extra demand from innovators that spill over to the second period, the retailer sets a lower second-period price and increases profits from the higher demand from innovators that spill over into the second period (i.e., $p^*_{2,ns} > p^*_{1} > p^*_{2,ws}$).

Figure 11  Numerical Analysis for Overlapping Innovators, $x_e, f_e$ (left panel) and $v_e, f_e$ (right panel) varied

Interpretations of the two graphs in Figure 11 are analogous. In the left graph, $v_e$ and $f_e$ are varied and in the right graph, $x_e$ and $f_e$ are varied. The locations of the different feasible areas I (no spillover, case (a)), II (no spillover, case (b)), and III (spillover) is primarily driven by the expected failure rate in both graphs: If the expected failure rate is high, then there is be no spillover as the retailer sets a higher second-period price due to the comparably lower realized failure rate.

Step 2: For all parameter combinations represented by the areas III in Figure 10 and Figure 11 we analyze if our Propositions 2 to 5 of the main model also hold for the case with overlapping innovators. Therefore, we incrementally increased, ceteris paribus, the average rating, $M$, to test Proposition 2, the variance caused by taste differences, $V_t$, to test Proposition 3, the variance
caused by quality differences, $V_q$, to test Proposition 4, and the relative share of variance caused by taste differences, $V_t/V$, to test Proposition 5. By analyzing the respective effects on optimal second-period price and demand, we find that for all parameter combinations represented by the areas III, Propositions 2, 3, 4a, 4b, 5a, and 5c of the main model also hold for the case with overlapping innovators. In Table 3 we illustrate this procedure using the parameter settings $v_e = v_r = x_e = x_r = f_e = f_r = 0.5$ as an example.

### Table 3  Numerical Results for Overlapping Innovators for $v_e = v_r = x_e = x_r = f_e = f_r = 0.5$

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<th>$V_t$</th>
<th>$V_q$</th>
<th>$V_t/V$</th>
<th>$V_{q/V}$</th>
<th>$p_1$</th>
<th>$D_1^*$</th>
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<td>$V_r + 1%$</td>
<td>0.19444</td>
<td>0.00319</td>
<td>0.03819</td>
<td>0.04127</td>
<td>0.07478</td>
<td>0.91606</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.08329</td>
<td>0.22143</td>
<td>0.06645</td>
<td>0.29120</td>
</tr>
<tr>
<td>$V_r + 5%$</td>
<td>0.19444</td>
<td>0.00324</td>
<td>0.03970</td>
<td>0.04279</td>
<td>0.07214</td>
<td>0.88568</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.08311</td>
<td>0.21843</td>
<td>0.06622</td>
<td>0.28908</td>
</tr>
<tr>
<td>$V_r + 10%$</td>
<td>0.19444</td>
<td>0.00340</td>
<td>0.04159</td>
<td>0.04686</td>
<td>0.06908</td>
<td>0.86629</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.08289</td>
<td>0.21504</td>
<td>0.06594</td>
<td>0.28565</td>
</tr>
<tr>
<td>$V_r + 1%$</td>
<td>0.19444</td>
<td>0.00350</td>
<td>0.03740</td>
<td>0.04090</td>
<td>0.08547</td>
<td>0.94533</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.08458</td>
<td>0.21568</td>
<td>0.06685</td>
<td>0.28630</td>
</tr>
<tr>
<td>$V_r + 5%$</td>
<td>0.19444</td>
<td>0.00376</td>
<td>0.03576</td>
<td>0.04090</td>
<td>0.12547</td>
<td>0.87453</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.08902</td>
<td>0.19955</td>
<td>0.06822</td>
<td>0.26998</td>
</tr>
<tr>
<td>$V_r + 10%$</td>
<td>0.19444</td>
<td>0.00718</td>
<td>0.03372</td>
<td>0.04090</td>
<td>0.17547</td>
<td>0.82453</td>
<td>0.08333</td>
<td>0.11111</td>
<td>0.09385</td>
<td>0.19067</td>
<td>0.06993</td>
<td>0.26993</td>
</tr>
</tbody>
</table>

For our numerical analysis above, we have chosen a product where Proposition 5c holds in the main model and we found that Proposition 5c also holds in the model extension with overlapping innovators. As our counterintuitive result that risk averse consumers may prefer a higher priced product with a higher total variance results from Proposition 5b, we further analyze a product where the condition for Proposition 5b (i.e., $V < V_q$) holds in the main model. This is the case for product A in our numerical example of the main model (cf., Figure 9) which we now extend by allowing innovators to spill over to the second period.

**Numerical Example Extension**: To recapitulate, product A has an average rating of $M = 4$ and a total variance of $V = 1.1$ which is composed of 64% variance caused by taste differences and 36% variance caused by quality differences. These numbers, the resulting optimal first-period price
and demand, and the optimal second-period price and demand with and without spillover are illustrated in the row Initial Setting of Table 4.

### Table 4  Numerical Example Extension Results for Overlapping Innovators

<table>
<thead>
<tr>
<th>What is changed</th>
<th>M</th>
<th>V_i</th>
<th>V_r</th>
<th>V</th>
<th>V/V</th>
<th>V/V</th>
<th>( p_t )</th>
<th>( D_t^a )</th>
<th>( D_t^b )</th>
<th>( \Delta p_t )</th>
<th>( \Delta D_t^a )</th>
<th>( \Delta D_t^b )</th>
<th>Result</th>
<th>Does proposition hold also with spillover</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Setting</td>
<td>4.0000</td>
<td>0.7118</td>
<td>0.3883</td>
<td>1.1000</td>
<td>0.6470</td>
<td>0.3530</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7263</td>
<td>0.5028</td>
<td>1.3476</td>
<td>0.7034</td>
<td>P2 holds</td>
<td></td>
</tr>
<tr>
<td>M + 1%</td>
<td>4.0400</td>
<td>0.7118</td>
<td>0.3883</td>
<td>1.1000</td>
<td>0.6470</td>
<td>0.3530</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7397</td>
<td>0.5295</td>
<td>1.3615</td>
<td>0.7213</td>
<td>P2 holds</td>
<td></td>
</tr>
<tr>
<td>M + 5%</td>
<td>4.2000</td>
<td>0.7118</td>
<td>0.3883</td>
<td>1.1000</td>
<td>0.6470</td>
<td>0.3530</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7934</td>
<td>0.5962</td>
<td>1.4183</td>
<td>0.7976</td>
<td>P3 holds</td>
<td></td>
</tr>
<tr>
<td>M + 10%</td>
<td>4.4000</td>
<td>0.7118</td>
<td>0.3883</td>
<td>1.1000</td>
<td>0.6470</td>
<td>0.3530</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.8604</td>
<td>0.7022</td>
<td>1.4914</td>
<td>0.9046</td>
<td>P5 holds</td>
<td></td>
</tr>
<tr>
<td>( V_t ) + 1%</td>
<td>4.0000</td>
<td>0.7118</td>
<td>0.3883</td>
<td>1.1072</td>
<td>0.6429</td>
<td>0.3507</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7283</td>
<td>0.5014</td>
<td>1.3480</td>
<td>0.7020</td>
<td>P4a holds</td>
<td></td>
</tr>
<tr>
<td>( V_t ) + 5%</td>
<td>4.0000</td>
<td>0.7473</td>
<td>0.3883</td>
<td>1.1356</td>
<td>0.6267</td>
<td>0.3419</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7360</td>
<td>0.4962</td>
<td>1.3494</td>
<td>0.6976</td>
<td>P4b holds</td>
<td></td>
</tr>
<tr>
<td>( V_t ) + 10%</td>
<td>4.0000</td>
<td>0.7829</td>
<td>0.3883</td>
<td>1.1712</td>
<td>0.6077</td>
<td>0.3215</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7455</td>
<td>0.4901</td>
<td>1.3512</td>
<td>0.6960</td>
<td>P5a holds</td>
<td></td>
</tr>
<tr>
<td>( V_t/V ) + 1pp</td>
<td>4.0000</td>
<td>0.7228</td>
<td>0.3773</td>
<td>1.1000</td>
<td>0.6570</td>
<td>0.3430</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7295</td>
<td>0.5148</td>
<td>1.3542</td>
<td>0.7191</td>
<td>P5b holds</td>
<td></td>
</tr>
<tr>
<td>( V_t/V ) + 5pp</td>
<td>4.0000</td>
<td>0.7668</td>
<td>0.3333</td>
<td>1.1000</td>
<td>0.6970</td>
<td>0.3030</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7419</td>
<td>0.5171</td>
<td>1.3826</td>
<td>0.7704</td>
<td>P5a holds</td>
<td></td>
</tr>
<tr>
<td>( V_t/V ) + 10pp</td>
<td>4.0000</td>
<td>0.8218</td>
<td>0.2783</td>
<td>1.1000</td>
<td>0.7470</td>
<td>0.2530</td>
<td>1.7967</td>
<td>0.3175</td>
<td>1.7570</td>
<td>0.6695</td>
<td>1.4239</td>
<td>0.8717</td>
<td>P5b holds</td>
<td></td>
</tr>
</tbody>
</table>

We again incrementally increase the average rating, the variance caused by taste differences, the variance caused by quality differences, and the relative share of variance caused by taste differences. Reassuringly, we find that the directions of the effects on optimal second-period price and demand are the same with and without spillover for product A. By this extension of our numerical example into the case when innovators spill over to the second period, we find that there are products with a sufficiently low total variance, where optimal second-period price and demand increase with an increasing relative share of variance caused by taste differences (Proposition 5b).

In addition to the numerical analyses above, we analyzed numerous other parameter combinations for multiple products and found that the propositions of our main model hold for almost all products when innovators spill over (details are available from the authors). We found only a few extreme cases where our propositions do not hold across the board. These extreme cases can be categorized broadly into two groups. The first group comprises cases with extreme realized failure rates of close to one; something that can hardly be observed in practice. The second group comprises cases where the realized matched quality and realized mismatch costs are small (e.g.,
$v_r = x_r = 0.1$) and expected matched quality and expected mismatch costs are much higher (e.g., $v_e = x_e = 1$). Again, such cases can rarely be observed in practice. Overall, based on our numerical analyses we find that our results from the main model hold for the model extension allowing a potential spillover of innovators in the second period over a wide range of realistic values.

5. Conclusion

Online shopping has significantly changed the way people purchase products. Rating systems, which enable consumers to observe the distribution of ratings awarded by other consumers, have contributed to this change. Significant literature has emerged which seeks to understand the effects of different aspects of these rating systems – such as number, average or variance – on product prices and consumer demand. Previous literature that analyzed the role of the variance of consumer ratings concentrated on ratings for products where the variance is caused solely by taste differences on search attributes and experience attributes (Sun 2012). However, a high variance of consumer ratings may also depend on quality differences among instances of the product such as whether a product fails. Our work makes an initial contribution towards understanding how the variance caused by taste differences and quality differences differentially affect product price and demand.

We propose a model where both taste and quality differences may cause variance in consumer ratings. We find that a higher variance caused by taste differences indicates that a product closely matches the tastes of some consumers and less closely matches the tastes of others, resulting in a higher price and lower demand. A higher variance caused by quality differences suggests an unreliable product and is therefore associated with a lower price and lower demand. Our most surprising result is that for products with low variance, holding the average rating as well as the total variance of ratings constant, while increasing the share of the variance caused by taste differences increases optimal price and demand. Thus, counter to intuition, price and demand are capable of increasing concomitantly with a rise in the total variance of consumer ratings. Given the same average rating for two similar products, risk averse consumers may prefer the higher priced product with the higher total variance of ratings. Thus, our results suggest that considering
taste differences and quality differences as separate sources of variance in consumer ratings may be important when empirically analyzing the effects of consumer ratings on product pricing and consumer demand.

Our findings have important managerial implications. First, if retailers were to consider the composition of the variance of consumer’s ratings, then they could improve their sales forecasts and increase profits by adjusting their inventories accordingly to satisfy demand or by charging higher prices for those products for which a relatively larger share of the variance is caused by taste differences. Second, they could implement mechanisms to explicitly communicate information about the decomposition of the variance to allow more consumers to use this important information in their decision making, which would further reduce uncertainty consumers have in e-commerce. There is empirical evidence that strongly suggests that consumers are able to digest information about rating distributions (e.g., Clemons et al. 2006, Sun 2012). Today, consumers can only indirectly infer this information by analyzing specific aspects of the ratings distribution, such as a peak in one-star ratings or by reading through the textual consumer reviews for a specific product. As a first step to making this information directly available, retailers may provide additional information on the percentage of the most negative consumer ratings caused by product failure. Retailers could collect this information by asking each consumer posting a negative rating whether it is based on product failure or on taste mismatch.

As with all research, the current study has limitations that present opportunities for future research. First, in our model the two consumer groups (innovators and imitators) do not exhibit strategic behavior. However, consumers may consider the timing of the purchase and, hence, the timing of the consumers purchase decision could be endogenized (see, e.g., Guo and Villas-Boas 2007, Sun 2012). Sun (2012) allows for strategic behavior in an extension of her baseline model but finds qualitatively the same results compared to not allowing strategic behavior. Whether this also holds for the model proposed in this article remains to be analyzed. Second, pressure from regulators or consumer groups may cause retailers to consider a unique price across both periods
that accounts for how consumers respond to ratings. Such a model would incorporate expectations of consumer ratings, and interesting insights may be gained regarding the balance of demand between innovators and imitators.

Third, our results suggest that consumers and retailers would benefit from information about the decomposition of the variance of consumer ratings, that is, which proportion of the variance is caused by taste differences and which by quality differences, although, this information is sometimes revealed by the textual reviews. However, products sometimes have too many consumer reviews for consumers and retailers to read them all. To solve this issue, researchers could develop text mining approaches or semantic techniques (e.g., as in Archak et al. 2011) that can identify the shares of variance caused by taste and quality differences. Finally, our model generates testable predictions regarding the effect of the variance of consumer ratings on product price and consumer demand. The sign of this effect depends to a large degree on the source of this variance. This provides an interesting direction for further research, especially for field studies and experiments that investigate the effects of the variance of consumer ratings that consider the different sources of variance.
Appendix. Proofs of Propositions

Proof of Proposition 1: Differentiating the optimal price and demand with respect to $M$ and $V$ gives

$$\frac{\partial p^*_2}{\partial M} = \frac{1}{2}, \frac{\partial p^*_2}{\partial V} = \frac{3}{4\sqrt{3V}}, \frac{\partial D^*_2}{\partial M} = \frac{kD^*_1}{4\sqrt{3V}}, \text{ and } \frac{\partial D^*_2}{\partial V} = -\frac{3kD^*_1}{8(3V)^{3/2}}.$$ 

Recall that $M$, $V$, and $D^*_1$ are positive by definition. Thus, we have

$$\frac{\partial p^*_2}{\partial M} > 0, \frac{\partial p^*_2}{\partial V} > 0, \frac{\partial D^*_2}{\partial M} > 0 \text{ and } \frac{\partial D^*_2}{\partial V} < 0.$$ 

Q.E.D.

Proof of Proposition 2: Rearranging (12) and differentiating optimal price and demand with respect to $M$ yields

$$\frac{\partial p^*_2}{\partial M} = \frac{\sqrt{2}Vq}{3(M^2 + Vq)^{3/2}} + \frac{1}{3},$$

and

$$\frac{\partial D^*_2}{\partial M} = \frac{k\tilde{\tau}^{\theta=0}}{27zVq} \left( \sqrt{\frac{2(M^2 + Vq)}{V_i}} + 2 \right) \left( \frac{V_i + 4M^2 + \sqrt{2V_i(M^2 + Vq) + M^2}}{M^2 + Vq} \right).$$

As $M$, $V_i$, $V_q$, $\tilde{\tau}^{\theta=0}$, $k$, and $z$ are positive by definition, we have

$$\frac{\partial p^*_2}{\partial M} > 0, \text{ and } \frac{\partial D^*_2}{\partial M} > 0.$$ 

Q.E.D.

Proof of Proposition 3: Rearranging (12) and differentiating optimal price and demand with respect to $V_i$ yields

$$\frac{\partial p^*_2}{\partial V_i} = \frac{\sqrt{2}M}{6\sqrt{V_i(M^2 + Vq)}}$$

and

$$\frac{\partial D^*_2}{\partial V_i} = \frac{Mk\tilde{\tau}^{\theta=0}\sqrt{2V_i(M^2 + Vq)}(M^2 + Vq - 2V_i)}{54zVqV_i^{2}}.$$ 

As $M$, $V_i$, $V_q$, $k$, and $z$ are positive by definition, we have $\partial p^*_2/\partial V_i > 0$. The sign of $\partial D^*_2/\partial V_i$ solely depends on $(M^2 + V_q - 2V_i)$ which is positive if $V_i < M^2/2 + V_q/2$. From Assumption 2 we have $x \leq v$. Rewriting this inequality in terms of $M$, $V_q$, and $V_i$ by using (9) and simplifying leads to: $V_i < (\tilde{\tau}^{\theta=0})^2(M^2 + V_q)/2(\tilde{\tau}^{\theta=0} - 3)^2$. As $\tilde{\tau}^{\theta=0} \in [0, 1]$, this contradicts $V_i > M^2/2 + V_q/2$. Thus, $\partial D^*_2/\partial V_i < 0$. 

Q.E.D.
Proof of Proposition 4: Rearranging (12) and differentiating optimal price and demand with respect to \( V_q \) yields
\[
\frac{\partial p^*}{\partial V_q} = -\frac{M^2V_t}{3(M^2 + V_q)^2 \sqrt{\frac{2M^2V_t}{M^2 + V_q}}}
\]
and
\[
\frac{\partial D^*}{\partial V_q} = -\frac{k^\theta_1 = 0 \left( 8M^3V_t + \sqrt{\frac{2M^2V_t}{M^2 + V_q}} (2M^4 - V_q^2 + 2V_qV_t + M^2V_q + 4M^2V_t) \right)}{54z^2V_q^2 V_t}.
\] (16)

For (a), as \( M, V_t, \) and \( V_q \) are positive by definition, we have \( \frac{\partial p^*}{\partial V_q} < 0 \).

The sign of \( \frac{\partial D^*}{\partial V_q} \) depends on the sign of the numerator of (16) and especially on the sign of the term in round brackets:
\[
8M^3V_t + \sqrt{\frac{2M^2V_t}{M^2 + V_q}} (2M^4 - V_q^2 + 2V_qV_t + M^2V_q + 4M^2V_t)
\] (17)

For (b), (17) is positive if \( 2M^4 - V_q^2 + 2V_qV_t + M^2V_q + 4M^2V_t > 0 \). Solving this inequality for \( V_q \) we get \( V_q < 2M^2 \) as sufficient condition for (17) > 0 and \( \frac{\partial D^*}{\partial V_q} < 0 \).

For (c), a necessary condition that (17) becomes negative is that \( V_q > 2M^2 \). Taking \( V_q > 2M^2 \) and solving (17) = 0 for \( V_t \) gives
\[
V_t = \frac{(M^2 + V_q) (-2M^2 + V_q)^2}{2 (2M^2 + V_q)^2}.
\]
As (17) is strictly increasing in \( V_t \), we have \( \frac{\partial D^*}{\partial V_q} > 0 \) if
\[
V_q > 2M^2 \text{ and } V_t < \frac{(M^2 + V_q) (-2M^2 + V_q)^2}{2 (2M^2 + V_q)^2}.
\]
Q.E.D.

Proof of Proposition 5: To analyze the effect of the relative share of \( V_q \) (which is the complement of the relative share of \( V_t \)), we differentiate (13) with respect to \( V_q \). Rearranging terms we have
\[
\frac{\partial p^*}{\partial V_q} = -\frac{\sqrt{2} M^2 (M^2 + V)}{6 (M^2 + V_q)^2 \sqrt{\frac{M^2(V - V_q)}{M^2 + V_q}}}
\]
and
\[
\frac{\partial D^*}{\partial V_q} = \Lambda \left( M^2V_q + V_q^2 + \frac{2V_q(V - V_q)^2}{M^2 + V_q} - (V - V_q) (4(V - V_q) + 2M^2 + 4M^2 \sqrt{\frac{V - V_q}{M^2 + V_q} + V_q}) \right),
\] (18)

where
\[
\Lambda = -\frac{\sqrt{2} M k^\theta_1 = 0}{54 z^2 V_q^2 \sqrt{\frac{(V - V_q)^2}{M^2 + V_q}}} > 0.
\]
For (a), as \( V_q \) is by definition always smaller than \( V \), we have \( \partial p_2^*/\partial V_q < 0 \) and, vice versa, \( \partial p_2^*/\partial V_l > 0 \).

For (b) and (c) our approach is to define segments of \( V \) and then address the behavior of (18) within the upper and lower segments. As \( \Lambda > 0 \), the sign of (18) depends only on the sign of the long term in round brackets:

\[
M^2 V_q + V_q^2 + \frac{2V_q (V - V_q)^2}{M^2 + V_q} - (V - V_q) (4 (V - V_q) + 2M^2 + 4M^2 \sqrt{\frac{2V - V_q}{M^2 + V_q}} + V_q).
\]

(19) is strictly increasing in \( V_q \) for \( V_q \in [0, V] \) and strictly decreasing in \( V \).

From Assumption 2 that \( x_r \in [0, v_r] \) and our definition of taste with \( \tilde{\theta}_2^\theta = 0 \in [0, 1] \) we get lower and upper bounds for \( V_q \). To calculate the lower bound for \( V_q \), we take the upper bound of mismatch costs \( x_r = v_r \). For this case \( V_q \) is maximal and consequently \( V_q \) is minimal for a constant total variance. Using the equations for \( v_r \) and \( x_r \) from (9), substituting \( V_q \) by \( V - V_q \), setting \( x_r = v_r \), and solving for \( V_q \), we get the lower bound of \( V_q \). To calculate the upper bound for \( V_q \), we use the taste of the indifferent second-period consumer with zero risk premium \( \tilde{\theta}_2^\theta = 0 = v_r(1 - f_r) - p_2^r \). Using the equations for \( v_r \), \( x_r \), and \( f_r \) from (9) and the equation for \( p_2 \) from (12), substituting \( V_q \) by \( V - V_q \), and differentiating \( \tilde{\theta}_2^\theta = 0 \) with respect to \( V_q \), we find that \( \tilde{\theta}_2^\theta = 0 \) is strictly increasing in \( V_q \). Thus, we take the upper bound of \( \tilde{\theta}_2^\theta = 0 = 1 \) and solve the equation for \( V_q \) to get the upper bound of \( V_q \). The resulting lower and upper bounds for \( V_q \) are given by

\[
V_q^U = V - \frac{(\tilde{\theta}_2^\theta = 0)^2 (M^2 + V)}{3 ((\tilde{\theta}_2^\theta = 0)^2 - 4\tilde{\theta}_2^\theta = 0 + 6)} \leq V_q \leq \frac{(4V - 2M^2) (\tilde{\theta}_2^\theta = 0)^2 - 36V\tilde{\theta}_2^\theta = 0 + 81V}{6(\tilde{\theta}_2^\theta = 0)^2 - 36\tilde{\theta}_2^\theta = 0 + 81} = V_q^L.
\]

Both \( V_q^U \) and \( V_q^L \) are increasing in \( V \).

Suppose (19) and therefore (18) are equal to zero. Inserting \( V_q^U \) and \( V_q^L \), respectively, into (19) and setting (19) = 0, we solve for \( V \) and obtain

\[
V = \frac{2M^2 (\tilde{\theta}_2^\theta = 0)^2 (4\tilde{\theta}_2^\theta = 0 - 9)}{2(\tilde{\theta}_2^\theta = 0 - 9)^2 (4\tilde{\theta}_2^\theta = 0 - 9)} \text{ for the upper bound of } V_q, V_q^U
\]

and

\[
V = \frac{M^2 (\tilde{\theta}_2^\theta = 0)^2 (\tilde{\theta}_2^\theta = 0 - 9/2)}{(2\tilde{\theta}_2^\theta = 0 - 3)(\tilde{\theta}_2^\theta = 0 - 3)} \text{ for the lower bound of } V_q, V_q^L.
\]

To show that \( \nabla \geq 0 \) we subtract \( \nabla \) from \( \nabla \). By rearranging terms we get

\[
M^2(\tilde{\theta}_2^\theta = 0)^2 \left( \frac{\tilde{\theta}_2^\theta = 0 - 4,5}{(\tilde{\theta}_2^\theta = 0 - 3)^2 (2\tilde{\theta}_2^\theta = 0 - 3)} + \frac{54 - 8\tilde{\theta}_2^\theta = 0}{(9 - 2\tilde{\theta}_2^\theta = 0)^2(4\tilde{\theta}_2^\theta = 0 - 9)} \right) \geq 0.
\]

As \( M^2(\tilde{\theta}_2^\theta = 0)^2 \) is positive, the sign of (20) depends only on the two terms in round brackets. The first term is strictly positive and the second term is strictly negative. However, for each and every \( \tilde{\theta}_2^\theta = 0 \in [0, 1] \), the first
term has a greater absolute value compared to the second term. Consequently, we have $V \geq \tilde{V}$ (the equality results from the case where $\hat{\gamma} = 0$).

Thus, we have three segments of the total variance: low, $V < \tilde{V}$; medium, $\tilde{V} \leq V \leq \hat{V}$; and high, $V < \hat{V}$. The delimiters of the segments, $\tilde{V}$ and $\hat{V}$ are each defined by two conditions: $\partial D^*_2 / \partial V_q = 0$ (defined by (18) and (19)), and $V_q = V^l_q$ or $V_q = V^u_q$, respectively.

For (b) when the total variance is low, $V < \tilde{V}$. Suppose $V_q = V^u_q$. Then, because $V^u_q$ is increasing in $V$ the resulting $V^u_q$ is lower, and (19) is increasing in $V_q$. Consequently, (19) and hence (18) are negative at $V_q = V^u_q$.

\[
\frac{\partial D^*_2}{\partial V_q} \bigg|_{V_q = V^u_q} < 0 \implies \frac{\partial D^*_2}{\partial t_q} \bigg|_{V_q = V^u_q} > 0.
\]

As $\partial D^*_2 / \partial V_q$ is increasing in $V_q$ we have

\[
\frac{\partial D^*_2}{\partial V_q} \bigg|_{V_q < V^u_q} < 0 \implies \frac{\partial D^*_2}{\partial t_q} \bigg|_{V_q < V^u_q} > 0.
\]

For (c) when the total variance is high, $V < \hat{V}$. Suppose $V_q = V^l_q$. Then, because $V^l_q$ is increasing in $V$ the resulting $V^l_q$ is higher, and (19) is increasing in $V_q$. Using the same reasoning as in (b) above, (19) and hence (18) are positive at $V_q = V^l_q$.

\[
\frac{\partial D^*_2}{\partial V_q} \bigg|_{V_q = V^l_q} > 0 \implies \frac{\partial D^*_2}{\partial t_q} \bigg|_{V_q = V^l_q} < 0.
\]

And with $\partial D^*_2 / \partial V_q$ is increasing in $V_q$ we have

\[
\frac{\partial D^*_2}{\partial V_q} \bigg|_{V_q > V^l_q} > 0 \implies \frac{\partial D^*_2}{\partial t_q} \bigg|_{V_q > V^l_q} < 0.
\]

Q.E.D.

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