Quantum Annealing – Basics and …

Hidetoshi Nishimori

Tokyo Institute of Technology

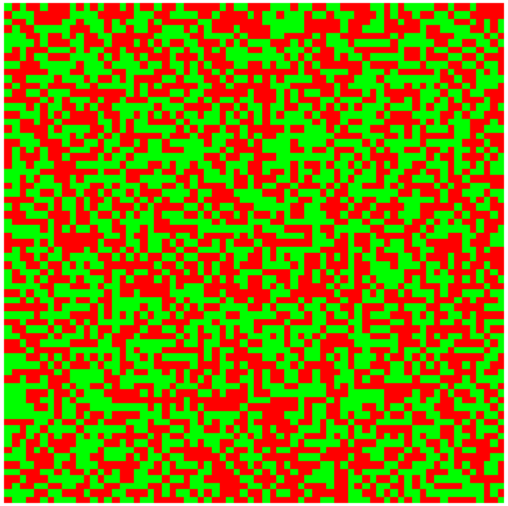
Motivation

Combinatorial optimization

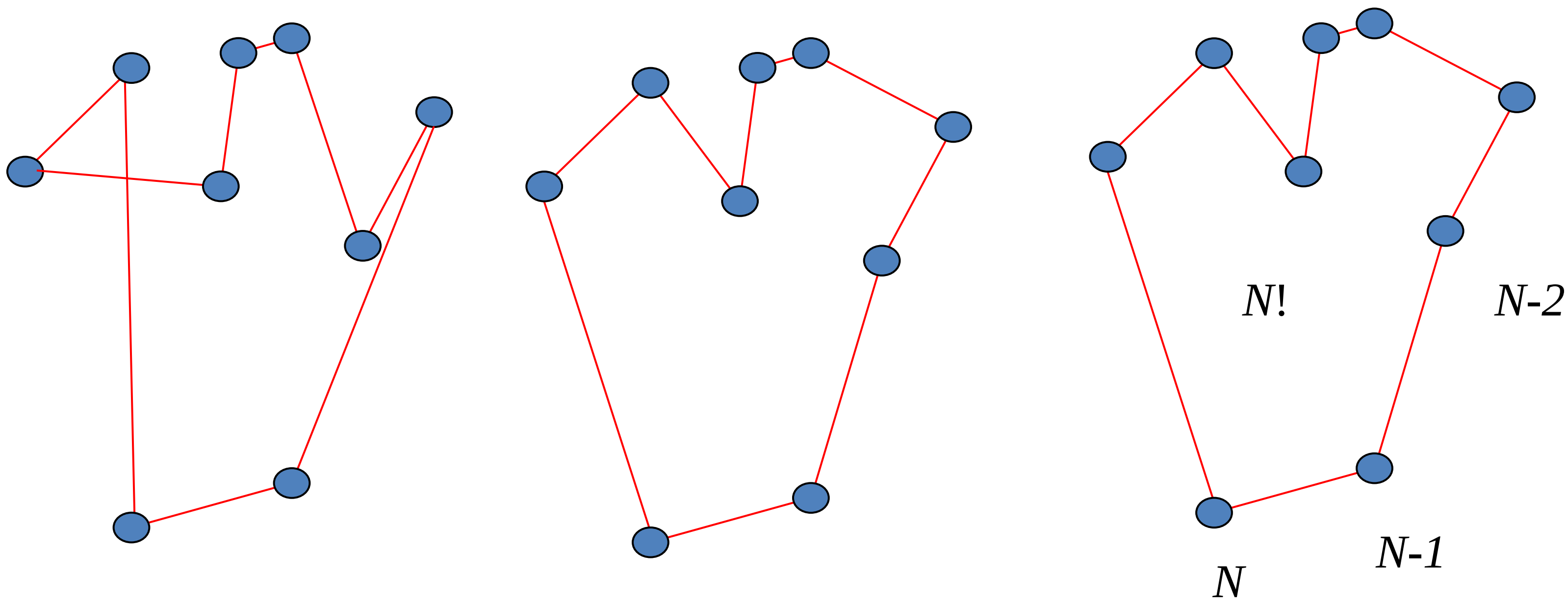
Minimization of cost function :
Multivariable (discrete) & Single-valued

Ground state of Ising SG

$$H = -\sum J_{ij}\sigma_i\sigma_j$$



Travelling salesman problem



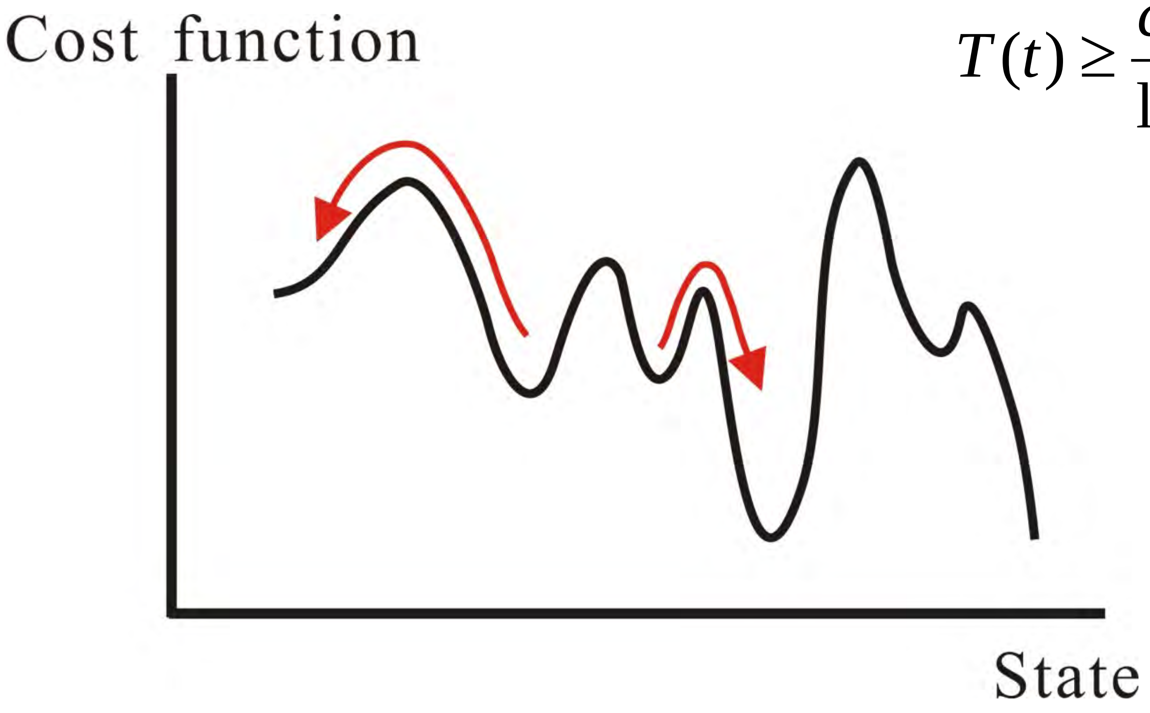
Configuration 1 Configuration 2

Minimize the cost function (=*tour length*)

Simulated Annealing (SA)

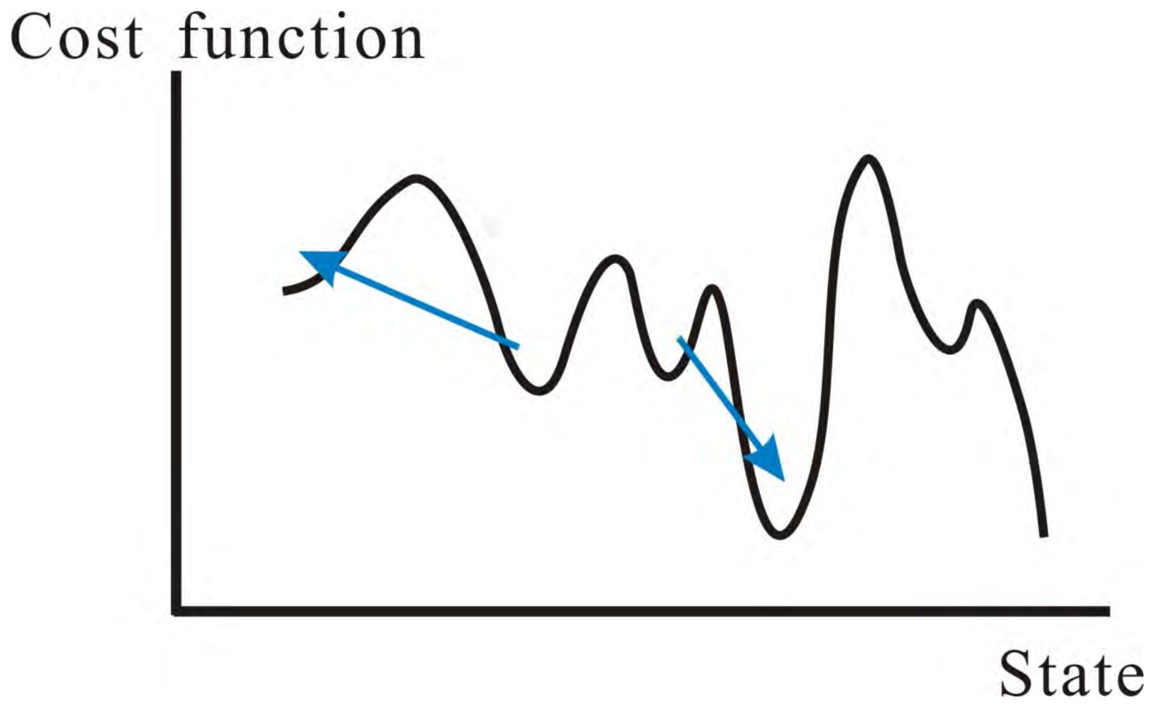
- *Generic, approximate* algorithm
- Phase-space search by thermal fluctuations

$$T(t) \geq \frac{cN}{\ln t}$$



Quantum Annealing (QA)

- *Generic, approximate* algorithm
- Phase-space search by quantum fluctuations



Problems

- *Quantum vs thermal fluctuations:*

  Is QA useful for optimization purposes?

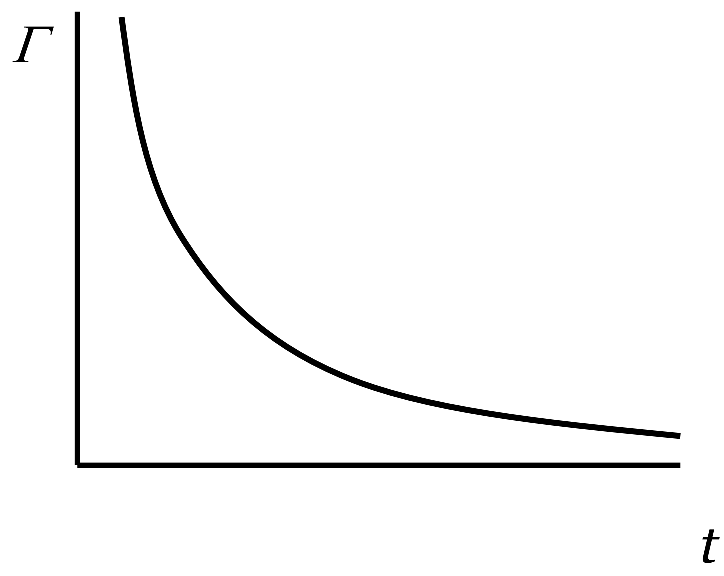
  Yes, but be careful…

- *Related with quantum computation?*

  Yes, equivalent. *Aharonov et al (2007)*

Implementation

$$H(t) = H_{\text{classical}} + H_{\text{quantum}} = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum \sigma_i^x$$



Experiments

$$H = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x$$

Magnetic material

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Brooke, Bitko, Rosenbaum, Aeppli (1999)

Numerical evidence

$$H = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x$$

T vs Γ: Hopfield model

$$H = -\sum J_{ij}\sigma_i\sigma_j \quad (\text{Finite } T) \qquad J_{ij} = \sum_{\mu=1}^{p} \xi_i^{\mu}\xi_j^{\mu}$$

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Amit, Gutfreunt, Sompolinsky (1985)

T vs *Γ*: Hopfield model

$$H = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x \quad (T=0)$$

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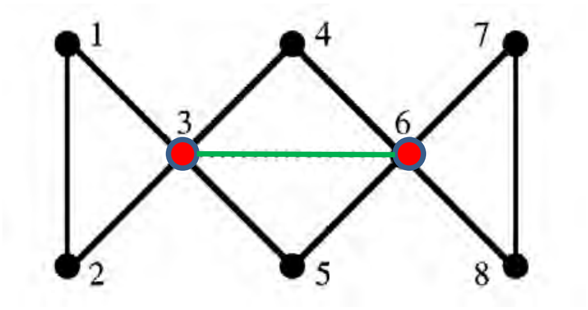
Nishimori & Nonomura (1996)

Frustrated system

Ferro interaction

Antiferro interaction



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Master eqn vs. Schrödinger eqn

Spin glass (SK model) with 8 spins

$$\Gamma(t)=\frac{3}{\sqrt{t}}$$

Schrödinger

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$$T(t)=\frac{3}{\sqrt{t}}$$

Master /Equilibrium

Kadowaki & Nishimori (1998)

Monte Carlo for TSP

$$H(t) = \frac{t}{\tau} H_{\text{classical}} + \left(1 - \frac{t}{\tau}\right) H_{\text{quantum}}$$

$$H(0) = H_{\text{quantum}} \Rightarrow H(\tau) = H_{\text{classical}}$$

cf: classical SA $T(0) = \text{large} \Rightarrow T(\tau) = 0$

Residual energy : $H(\tau) - E_{\text{true}}$

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Martonak, Santoro &Tosatti (2004)

Monte Carlo for 3SAT

QA

QA

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SA

$$C_1 = x_1 \vee \bar{x}_2 \vee x_3$$

$$C_2 = \bar{x}_4 \vee x_1 \vee x_2$$

$$(x_i = \text{true or false})$$

$$F = C_1 \wedge C_2$$

Can F be true?

Battaglia, Santoro, Tosatti (2005)

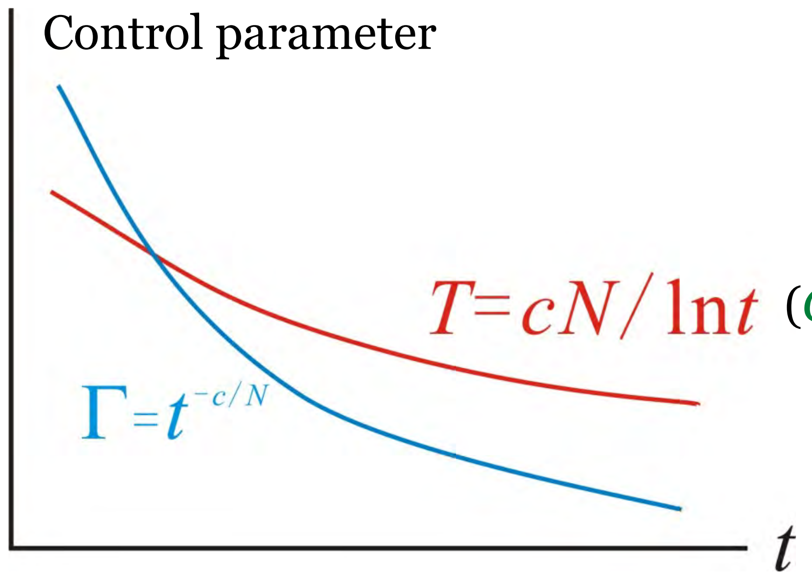
Theoretical foundation

$$H = -\sum J_{ij} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x$$

Convergence theorem

$$H = H_{\text{classical}} + H_{\text{quantum}} = -\sum J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum \sigma_i^x$$

Convergence condition $\Gamma(t) = t^{-c/N}$ *Morita & Nishimori*



(*Geman-Geman* for SA)

Computational complexity

$$\Gamma(t) = t^{-c/N} = \delta \quad \Rightarrow \quad t = \exp(aN|\ln\delta|)$$

$$T(t) = \frac{cN}{\ln t} = \delta \quad \Rightarrow \quad t = \exp(bN \ln|\ln\delta|)$$

Classical-quantum mapping

Classical equilibrium state ⟺ Quantum ground state

$$\langle A \rangle_T = \frac{\sum A(\sigma) e^{-\beta H(\sigma)}}{Z}$$

$$\langle A \rangle = \langle \psi(T) | A | \psi(T) \rangle$$

$$|\psi(T)\rangle = \frac{e^{-\beta H(\sigma_z)/2}}{\sqrt{Z}} \sum_\sigma |\sigma\rangle$$

$$H_q |\psi(T)\rangle = 0$$

TFIM $$H_q = 1 - e^{\beta H/2} M(T) e^{-\beta H/2}$$

(Quasi) equilibrium ⟺ Adiabatic condition

$$T(t) = \frac{cN}{\ln t}$$

$$\Gamma(t) = t^{-c/N}$$

Somma, Batista & Ortiz (2007)

Remark: *Type of the dynamics*

Convergence condition (*methods of proof*)

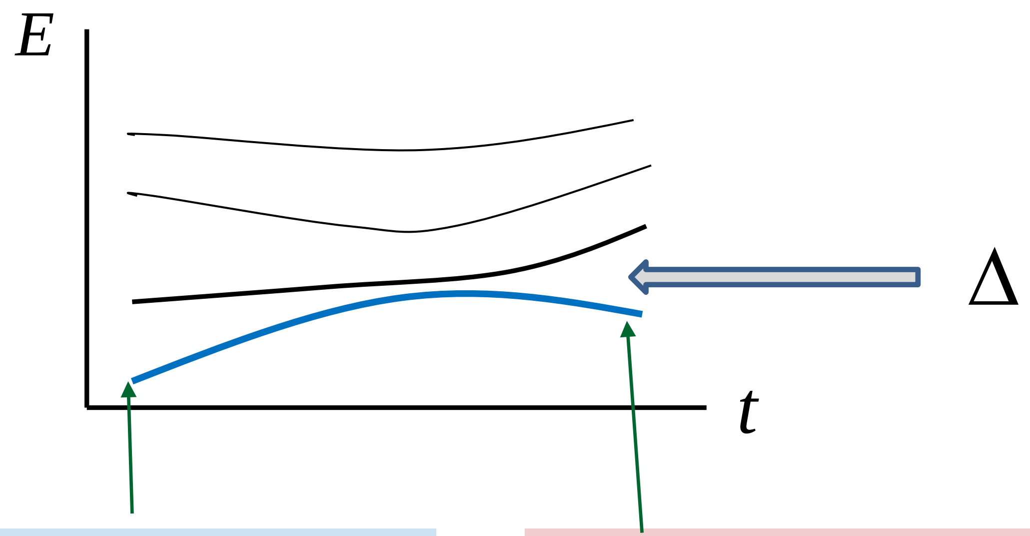
$$\Gamma(t) = t^{-c/N}$$

- ✓Real-time Schrödinger (*adiabatic condition*)
- ✓Imaginary-time Schrödinger (*adiabatic condition*)
- ✓Quantum Monte Carlo (*reduction to SA by Suzuki-Trotter*)

Adiabatic evolution

Quantum adiabatic evolution

$$\frac{1}{\tau}\left|\frac{\left\langle \psi_m(t)\left|\frac{\partial H}{\partial t}\right|\psi_0(t)\right\rangle}{\Delta(t)^2}\right| = \varepsilon$$



Trivial initial state

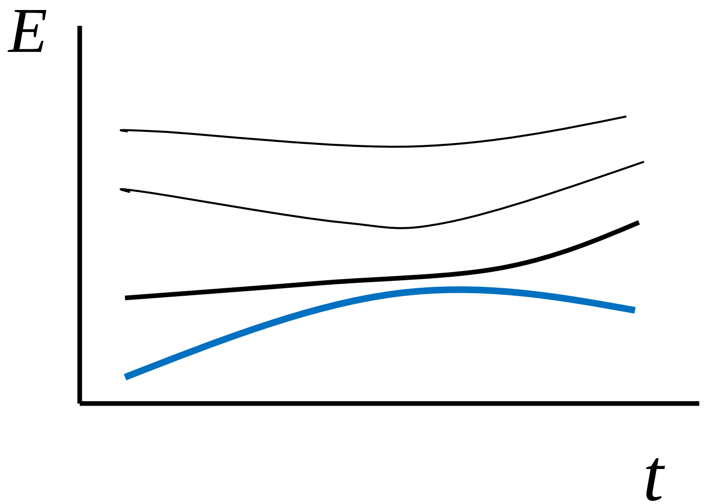
Non-trivial final state

$$H(t) = -\left(1-\frac{t}{\tau}\right)\sum \sigma_i^x - \frac{t}{\tau}\sum J_{ij}\sigma_i^z\sigma_j^z$$

Computational complexity

Finite-size analysis

Adiabatic theorem $\tau \propto \Delta^{-2}$



Gap scaling $\Delta \propto \begin{cases} e^{-aN} \\ N^{-b} \end{cases}$

Complexity $\tau \propto \begin{cases} e^{2aN} & \text{(hard)} \\ N^{2b} & \text{(easy)} \end{cases}$

View from quantum computation

Any problem <u>*hard*</u> classically
but <u>*easy*</u> quantum mechanically?

"Exact cover"

Maybe (*Fahri, Goldstone, Gutman, Lapan, Ludgrenm, Preda, 2001*)

No (*Young, Knysh, Smelyanskiy, 2010*)

“XORSAT”

No *(Jörg, Krzakala, Semerjian, Zamponi, 2010)*

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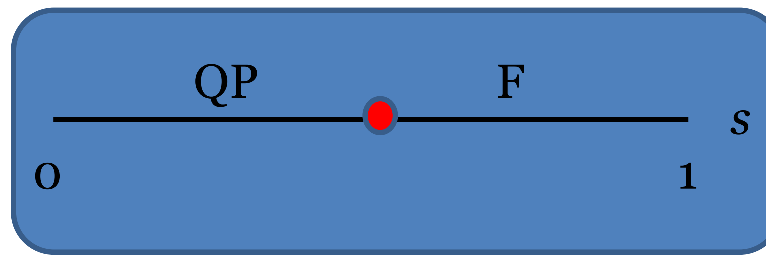
p-spin ferromagnet

$$H(s) = -sN\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p - (1-s)\sum_{i=1}^{N}\sigma_i^x \quad (s = t/\tau)$$



- 1st order transition at finite s
- Exponentially small energy gap. $\Delta \propto e^{-aN}$
- Exponentially large time for adiabatic computation. $\tau \propto e^{bN}$

Jörg, Krzakala, Kurchan, Maggs, Pujos (2010)
"The problem that quantum annealing *cannot* solve"

Seki and Nishimori (2012) "The problem that quantum annealing CAN

An additional quantum term

$$H(\lambda,s) = s\left(H_0 \qquad\qquad \right) + (1-s)H_{\mathrm{TF}}$$

$$H_0 = -N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p, \quad H_{\mathrm{TF}} = -\sum_{i=1}^{N}\sigma_i^x$$

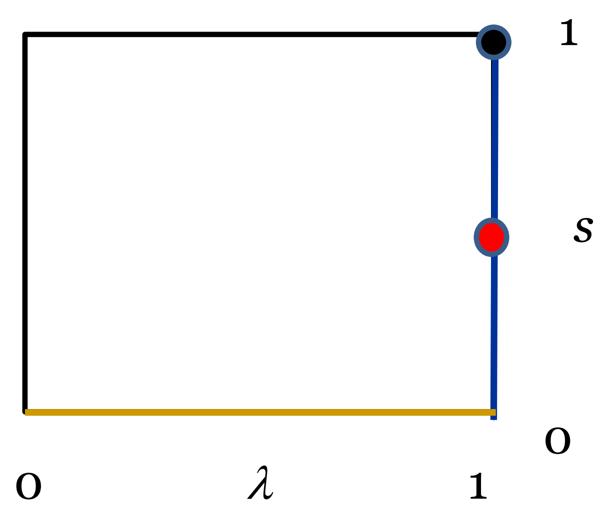
$$H(\lambda,s) = s\left(\lambda H_0 + (1-\lambda)H_{\mathrm{AFF}}\right) + (1-s)H_{\mathrm{TF}}$$

$$H_{\mathrm{AFF}} = N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^x\right)^2$$

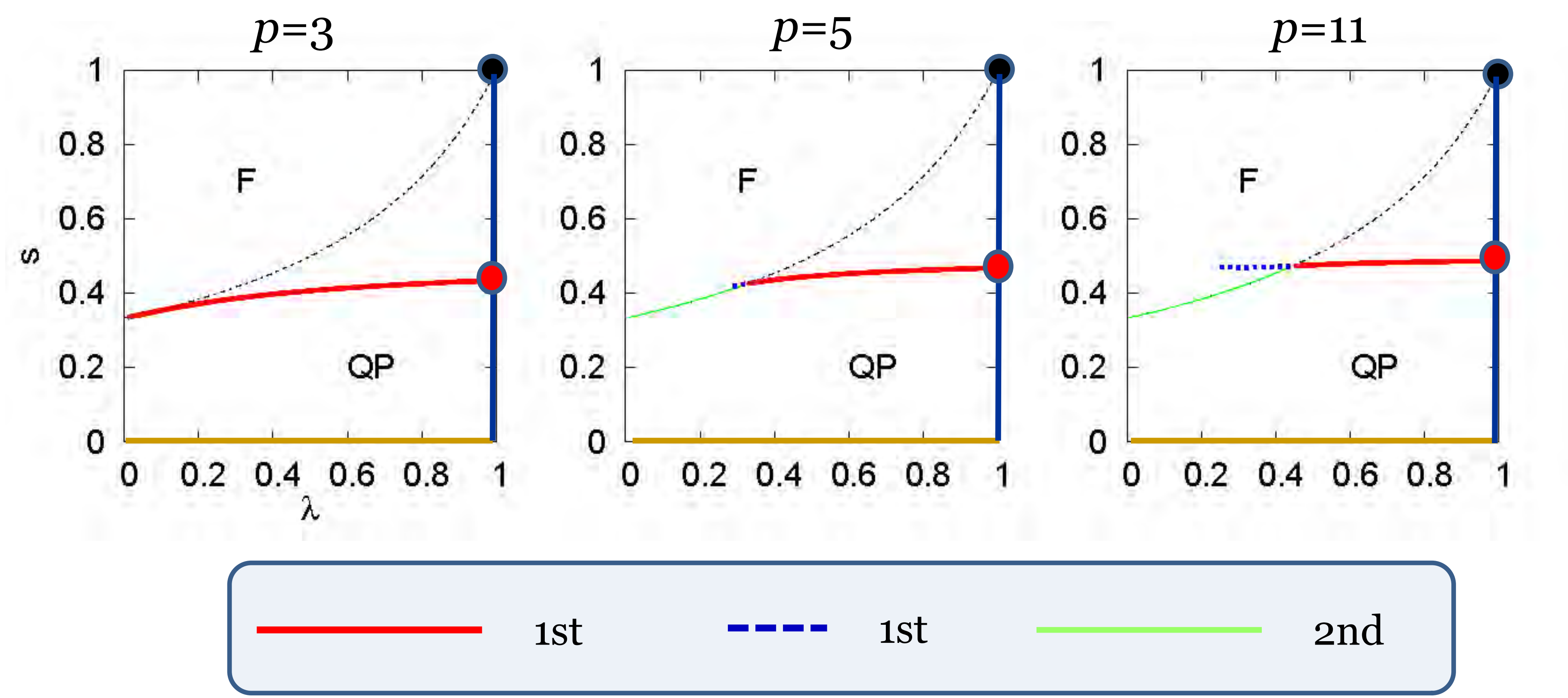
Conventional case: λ=1

Start: s=0, λ=any

Goal: s=1, λ=1 ●



Result



$$H(\lambda, s) = s\big(\lambda H_0 + (1-\lambda) H_{\mathrm{AFF}}\big) + (1-s) H_{\mathrm{TF}}$$

Is $H_{\mathrm{AFF}} = N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^x\right)^2$ essential?

Seoane and Nishimori (2012)

$$H(\lambda, s) = s\left(\lambda H_0 + (1-\lambda) H_{\mathrm{AFF}}^{(k)}\right) + (1-s) H_{\mathrm{TF}}$$

$$H_{\mathrm{AFF}}^{(k)} = N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^x\right)^k$$

It works fine for $k>2$ as well.

Summary

Summary

- ✓QA works fine as a generic, approximate algorithm.
- ✓"Better" than SA.
- ✓Negative evidence for a few difficult problems.
- ✓But there should be ways to avoid 1st order transitions.

Collaborators

- Tadashi Kadowaki
- Helmut G. Katzgraber
- Yoshiki Matsuda
- Satoshi Morita
- Yoshihiko Nonomura
- Masayuki Ohzeki
- Masuo Suzuki
- Sei Suzuki
- Yuya Seki
- Beatriz Seoane

History

✓Apolloni, Carvalho, de Falco (1989)

"QA", *algorithmic*

✓Finnila, Gomez, Sebenik, Stenson, Doll (1994)

Schrödinger, continuous (Lennard-Jones)

✓Tanaka & Horiguchi (1997)

Image restoration, *algorithmic*

✓Kadowaki & Nishimori (1998)

Transverse-field Ising, *Schrödinger*

✓Fahri, Goldstone, Gutmann, Lapan, Lundgren, Preda (2001)

"Adiabatic computation", complexity , not independent of KN