Cavity optomechanics with ultracold atoms
Theory review

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Introduction

▶ Reviews
[Ritsch, Domokos, Brennecke, Esslinger, Rev. Mod. Phys. (2013)]

▶ micro ⇔ cavity QED → macro ⇔ atomic ensemble in cavity
– monitoring and cooling motion ✓
– cavity enhanced diffusion ✓
– feedback cooling ✓
Rempe et al., Kimble et al 1997–
[Horak, Ritsch, Phys. Rev. Lett. (1997)]

▶ selection of few modes of the ensemble → cavity optomechanics

1. quantum regime of optomechanics: initially ground state
2. tunability of system parameters and interactions (radiation pressure coupling, Dicke-model, etc)
3. granular regime: single excitation influences the other subsystem
Possible applications: low intensity light manipulation

- Photon Blockade Effect in Optomechanical Systems  

- Single-Photon Optomechanics, non-Gaussian steady states  

- All-optical transistor based on a cavity optomechanical system with Bose-Einstein condensates  


Cavity QED basic scheme and dispersive interaction

\[ H = \hbar \omega_c a a^\dagger + \frac{p^2}{2m} + V_{\text{trap}}(x) + \hbar U_0 \cos^2(kx) a a^\dagger \]

Detection, dissipation and noise \((k_B T = 0)\)

\[
\frac{d}{dt} \hat{\rho} = -\frac{i}{\hbar} [H, \hat{\rho}] + \kappa \left( 2 a \hat{\rho} a^\dagger - a^\dagger a \hat{\rho} - \hat{\rho} a^\dagger a \right)
\]

Detunings

\[ \Delta_c = \omega - \omega_c \]

Polarizability

\[ U_0 = -\frac{\omega_c}{V} \chi' \approx \frac{g^2}{\Delta_A} \]

\[ \Gamma_0 = -\frac{\omega_c}{V} \chi'' \approx \gamma \frac{g^2}{\Delta_A^2} \approx 0 \]

Trapped atoms in a cavity

\[ H = \hbar \omega_c a^\dagger a + H_{\text{mech}} + \hbar U_0 \sum_{j=1}^{N} \cos^2 \left( kx_j \right) a^\dagger a \]

Lamb-Dicke regime
\[ x_j = x_{\text{trap}}^{(j)} + \delta x_j \quad k|\delta x_j| \ll 1 \]

\[ H \approx \hbar \omega_c + U_0 \sum_{j=1}^{N} \cos^2 \left( kx_{\text{trap}}^{(j)} \right) a^\dagger a \]

\[ + \sum_j \frac{p_j^2}{2m} + \frac{m}{2} \omega_M^2 \delta x_j^2 \]

\[ + \hbar U_0 a^\dagger a \sum_j \sin \left( 2kx_{\text{trap}}^{(j)} \right) k \delta x_j \]

Bose-Einstein condensate in an optical resonator

Density wave excitations
\[ \omega_R = \frac{\hbar k^2}{2m} \quad \text{recoil frequency} \]
\[ \kappa \quad \text{cavity resonance} \]

\[ \omega_R \ll \kappa \quad \text{(Zürich)} \]
\[ \omega_R \geq \kappa \quad \text{(Hamburg)} \]

Quantized atom field in a single-mode resonator

One-dimensional toy model for coupled matter and light fields

\[ H = \omega_C \hat{a}^\dagger \hat{a} + i \eta (\hat{a}^\dagger e^{-i \omega t} - \hat{a} e^{i \omega t}) + \int \hat{\Psi}^\dagger (x) \left[ -\frac{\hbar}{2m} \frac{d^2}{dx^2} + g_c \hat{\Psi}^\dagger (x) \hat{\Psi} (x) \\
+ U_0 \hat{a}^\dagger \hat{a} \cos^2 (kx) + i \eta_t \cos kx (\hat{a}^\dagger e^{-i \omega t} - \hat{a} e^{i \omega t}) \right] \hat{\Psi} (x) dx, \]

Dissipation and noise \((k_B T = 0)\)

\[ \frac{d}{dt} \hat{\rho} = -\frac{i}{\hbar} [H, \hat{\rho}] \\
+ \kappa \left( 2 \hat{a} \hat{\rho} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \hat{\rho} - \hat{\rho} \hat{a}^\dagger \hat{a} \right) \]

Time scales

\[ \omega_R = \hbar k^2 / 2m \quad \text{recoil frequency} \]
\[ \kappa, \Delta_C (\equiv \omega - \omega_C) \quad \text{cavity resonance} \]
\[ N U_0, N \eta_t \quad \text{tunable interaction} \]

Collective spin models

Fourier-expansion of the matter wave

\[ g_c = 0 \implies \hat{\Psi}(x) = \frac{1}{\sqrt{L}} \hat{c}_0 + \sqrt{\frac{2}{L}} \hat{c}_1 \cos kx \]
\[ + \sqrt{\frac{2}{L}} \hat{c}_2 \cos 2kx + \ldots \]

\[ \left[ \hat{c}_i, \hat{c}_j^\dagger \right] = 1 \quad i = 0, 1, 2, \ldots \]

Cavity pump

\[ \eta \neq 0, \eta_t = 0 \]
\[ \cos 2kx \text{ excitation mode} \]
\[ \text{[Wolke, Klinner, Kessler, Hemmerich, Science (2012)]} \]

\[ H/\hbar = -\delta_C \hat{a}^\dagger \hat{a} + i \eta (\hat{a}^\dagger - \hat{a}) + 4 \omega_R \hat{S}_z \\
+ \frac{U_0}{\sqrt{2}} \hat{a}^\dagger \hat{a} \hat{S}_x \]

Transverse pump

\[ \eta = 0, \eta_t \neq 0 \]
\[ \cos kx \text{ excitation mode} \]
\[ \text{[Baumann, Guerlin, Brennecke, Esslinger, Nature (2010)]; [Nagy, Kónya, Szirmai, Domokos, PRL (2010)]} \]

\[ H/\hbar = -\delta_C \hat{a}^\dagger \hat{a} + 4 \omega_R \hat{S}_z \\
+ y (\hat{a}^\dagger + \hat{a}) \hat{S}_x \]

Two-mode approx

\[ N = c_0^\dagger c_0 + c_i^\dagger c_i = \text{const} \]

\[ \hat{S}_x = \frac{1}{2} (c_i^\dagger c_0 - c_0^\dagger c_i) \]
\[ \hat{S}_y = \frac{1}{2i} (c_i^\dagger c_0 - c_0^\dagger c_i) \]
\[ \hat{S}_z = \frac{1}{2} (c_i^\dagger c_i - c_0^\dagger c_0) \]

Optomechanics

Dicke-model (cf. Rafael Mottl’s talk)
Optomechanical bistability

**undepleted BEC approximation**

\[ \hat{S}_x \rightarrow \sqrt{\frac{N}{2}} \hat{\chi} \]

**Radiation pressure coupling**

\[ H_{\text{OM}} = \frac{\omega_M}{2} (\hat{\chi}^2 + \hat{\chi}^2) - (\delta_C - G \hat{\chi}) \hat{a}^\dagger \hat{a} - i\eta (\hat{a}^\dagger - \hat{a}) \]

\[ \omega_M = 4\omega_R \quad G = \sqrt{N} U_0 / 2 \]

[Brennecke, Ritter, Donner, Esslinger, Science (2008)]

**nonlinear oscillations → effective potential**

\[ H_{\text{eff}} / \hbar = \frac{\omega_M}{2} (X^2 + Y^2) + \frac{G^2}{\kappa} \arctan \left( \frac{G X - \delta_C}{\kappa} \right) \]

**mean cavity field intensity**

\[ \hat{G}^4 \hat{I} + 2\delta_C \frac{\hat{G}^2}{\omega_M} \hat{I} + (\delta_C^2 + \kappa^2) I - \eta^2 = 0 \]

**critical pump strength**

\[ \eta^2 \geq \frac{8}{3 \sqrt{3}} \frac{\omega_M \kappa^3}{G^2} \]

**Bogoliubov description including collisions**

\[ H = H_{\text{OM}} + G X^2 \]

**canonical transformation**

\[ \begin{align*}
X &\rightarrow \chi X \\
Y &\rightarrow Y / \chi
\end{align*} \]

\[ \chi = 4 \left( \frac{\omega_M + 2G}{\omega_M} \right) \]

**renormalization of parameters**

\[ \bar{\omega}_M = \sqrt{\omega_M (\omega_M + 2G)} \approx \omega_M + G \]

\[ \bar{G} = G / \chi \approx \left( 1 - \frac{G}{2\omega_M} \right) G \]

**parameters**

\[ \begin{align*}
N &= 6 \times 10^4 \\
U_0 &= 0.96 \omega_R \\
\eta &= 549.5 \omega_R \\
\kappa &= 363.9 \omega_R \\
G &= \left\{ \begin{array}{ll}
1 & \text{green} \\
2 & \text{blue}
\end{array} \right.
\end{align*} \]
Trapped BEC in the Thomas-Fermi limit

Condensate wave function
(i) $g_c \neq 0$ & (ii) size $\sim 10 \lambda$

Effect on the mean field

Dynamics in optomechanical bistability

Spectrum of excitations — Real part

(brute force, 1400 grid point)

[Nagy, Szirmai, Domokos, EPJD (2013)]

Finite size effect is weak!
Optomechanical heating

Open system quantum dynamics $\kappa \gg G$

\[ \dot{\rho} = -i [H_{\text{eff}}, \rho] - [d(X), [d(X), \rho]] - \frac{i}{2} [g(X), [Y, \rho]] \]

“double well” potential
\[ H_{\text{eff}} = \frac{\omega M}{2} (X^2 + Y^2) + \frac{\eta^2}{\kappa} \arctan \left( \frac{G \cdot \delta C}{\kappa} \right) \]

heating rate
\[ D(X) = 2 \left( \frac{\partial d(X)}{\partial X} \right)^2 = G^2 \frac{2\kappa\eta^2}{(G \cdot \delta C)^2 + \kappa^2} \]

friction
\[ \Gamma(X) = \frac{\partial g(X)}{\partial X} = -16\omega_R G^2 \kappa \eta^2 \frac{\delta C - G \cdot X}{((G \cdot \delta C)^2 + \kappa^2)^2} \]

[Noj, Domos, Vukics, Ritsch, EPJD (2009)]

Quantum depletion

\[ \eta = 80.06 \omega_R \]
\[ \eta = 283.8 \omega_R \]
\[ \eta = 549.5 \omega_R \]

linearization outside the bistable region
[Noj, Noos, Zwerger, Kippenberg, PRL (2007)]
[Marquardt, Chen, Clerk, Girvin, PRL (2007)]
[Genes, Vitali, Tombesi, Gigan, Aspelmeyer, PRA (2008)]

\[ \tilde{\eta}(X_0) = \frac{D(X_0)}{2\Gamma(X_0)} \approx \frac{\delta C^2 + \kappa^2}{-16\omega_R \delta C} \sim \frac{\kappa}{\omega_R} \]
Summary

Quantum limit of the granular regime of cavity optomechanics is reached

Significant heating effects

[Guerra, Moore, Gupta, Stamper-Kurn, Nat. Phys. (2008)]

Nonlinear dynamics

[Guerra, Moore, Gupta, Stamper-Kurn, PRL (2007)]

[Brennecke, Ritter, Donner, Esslinger, Science (2008)]