25.) $A = (1,-1,2),\ B = (5,3,-6)$

a) $AB = B - A = (5,3,-6) - (1,-1,2) = (4,4,-8)$

$\vec{X} = A + \lambda \cdot AB = (1,-1,2) + \lambda(4,4,-8)$

b.)

$$\left.\begin{array}{ll} \text{I} & x = 1 + 4\lambda \\ \text{II} & y = -1 + 4\lambda \\ \text{III} & z = 2 - 8\lambda \end{array}\right\} \quad \begin{array}{l} \text{I}-\text{II}: E_1: x - y = 2 \\ 2\,\text{II}+\text{III}: E_2\ 2y + z = 0 \end{array}$$

26.)

$$a_n = \frac{n+7}{n^3+n+1} = \frac{\frac{1}{n^2}+\frac{7}{n^3}}{1+\frac{1}{n^2}+\frac{1}{n^3}} \xrightarrow[n\to\infty]{} 0 \quad \left(= \frac{0}{1}\right)$$

$$b_n = \frac{1-3n^2}{7n+5} = \frac{\frac{1}{n}-3\cdot n}{7+\frac{5}{n}} \xrightarrow[n\to\infty]{} \text{"}-\infty\text{"} \left(= \text{"}\frac{0-3\cdot\infty}{7+0}\text{"}\right) \text{ also divergent}$$

$$c_n = n - \frac{n^2+3n+1}{n} = n - n - 3 - \frac{1}{n} = -3-\frac{1}{n} \xrightarrow[n\to\infty]{} -3$$

$$d_n = 1 + (-1)^n = \begin{cases} 2 & n \text{ gerade} \\ 0 & n \text{ ungerade} \end{cases} \Rightarrow \text{nicht konvergent}$$

$$e_n = -n + \frac{1}{n} \xrightarrow[n\to\infty]{} \text{"}-\infty\text{"} \text{ nicht konvergent}$$

Einschließung

$$f_n = \left(-\frac{1}{n}\right)^n \quad \forall n: \underbrace{-\frac{1}{n^n}}_{\downarrow n\to\infty\ 0} \le f_n \le \underbrace{\frac{1}{n^n}}_{\downarrow n\to\infty\ 0} \Rightarrow \lim_{n\to\infty} f_n = 0$$

27.)

$$-\ \sum_{k=1}^{\infty} \underbrace{\frac{1}{k}\cdot\left(\frac{1}{2}\right)^k}_{0 \le} \le \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k - \left(\frac{1}{2}\right)^0 = \frac{1}{1-\frac{1}{2}} - 1 = \frac{1}{\frac{1}{2}} - 1 = 2-1 = \underline{\underline{1}}$$

$$\Rightarrow \sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{1}{2}\right)^k \le 1 \text{ beschränkt} \Rightarrow \sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{1}{2}\right)^k \text{ konvergiert}$$

$$-\ \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \sum_{k=1}^{\infty}\left(\frac{1}{4}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k - \left(\frac{1}{4}\right)^0 = \frac{1}{1-\frac{1}{4}} - 1 = \frac{1}{\frac{3}{4}} - 1 = \frac{4}{3} - 1 = \underline{\underline{\frac{1}{3}}}$$

$$\Rightarrow \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} = \frac{1}{3} \Rightarrow \sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{2k} \text{ konvergiert}$$

$$-\ \sum_{k=1}^{\infty}\frac{2}{k!} = 2\cdot\sum_{k=1}^{\infty}\frac{1}{k!} = 2\cdot\left(\sum_{k=0}^{\infty}\frac{1}{k!} - \frac{1}{0!}\right) = 2\cdot(e^1 - 1) \Rightarrow \sum_{k=1}^{\infty}\frac{2}{k!} \text{ konvergiert}$$

28.) $\sum_{k=0}^{\infty} \frac{2^k}{k!}$ konvergent ?

Hinweis: $k! \geq 4^k$ für $k \geq 9$

Weil $9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 > 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$9 \cdot 7 \cdot \underbrace{6 \cdot 5 \cdot 3}_{90 > 16 = 4 \cdot 4} > 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

$9 \cdot 7 \cdot 5 > 4 \cdot 4 \cdot 4 \cdot 4$

$315 > 16 \cdot 16 = 256$ ✓

$10! = 10 \cdot 9! > 10 \cdot 4^9 > 4 \cdot 4^9 = 4^{10}$

$k! \geq 2^{2k} = 2^k \cdot 2^k$

$\frac{1}{2^k} \geq \frac{2^k}{k!}$

$\underbrace{\sum_{k=0}^{\infty} \frac{2^k}{k!}}_{0 \leq} \leq \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{1-\frac{1}{2}} = \underline{\underline{2}}$ $\Rightarrow \sum_{k=0}^{\infty} \frac{2^k}{k!} \leq 2$ beschränkt $\Rightarrow$ konvergent

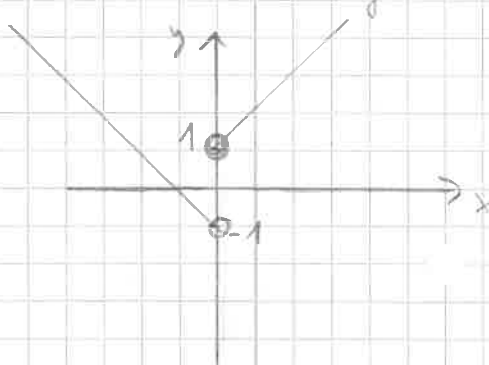
29.)

$D = \mathbb{R} \setminus \{0\}$

- $y = \frac{x + x^2}{|x|} = \frac{x\,(x+1)}{|x|} = \frac{x}{|x|}\,(x+1)$

$\lim_{x \to 0+} \frac{x+x^2}{|x|} = \lim_{x \to 0+} \frac{x+x^2}{x} = \lim_{x \to 0+} 1 + x = 1$

$\lim_{x \to 0-} \frac{x+x^2}{|x|} = \lim_{x \to 0-} \frac{x+x^2}{-x} = \lim_{x \to 0-} -1 - x = -1$

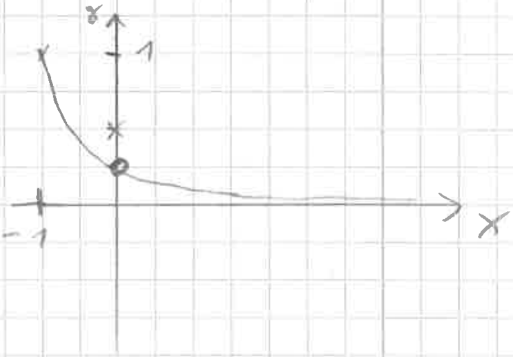


$D = [-1, \infty[\setminus \{0\}$

- $y = \frac{\sqrt{x+1} - 1}{x} = \frac{(\sqrt{x+1} - 1)}{x} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} = \frac{x + 1 - 1}{x(\sqrt{x+1} + 1)} = \frac{x}{x(\sqrt{x+1} + 1)} = \frac{1}{\sqrt{x+1} + 1}$

$\lim_{x \to 0+} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$

$\lim_{x \to 0-} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$



30.)

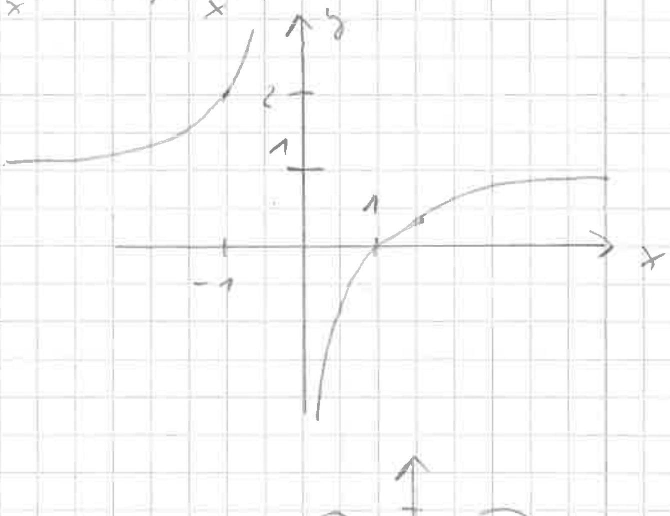
a) $y = \frac{x^2 - 1}{x^2 + x} = \frac{(x+1)(x-1)}{x(x+1)} = \frac{x-1}{x} = 1 - \frac{1}{x}$

$\lim_{x \to 0+} 1 - \frac{1}{x} = 1 - \infty = -\infty$

$\lim_{x \to 0-} 1 - \frac{1}{x} = 1 + \infty = \infty$

$\lim_{x \to 1} 1 - \frac{1}{x} = 1 - 1 = 0$

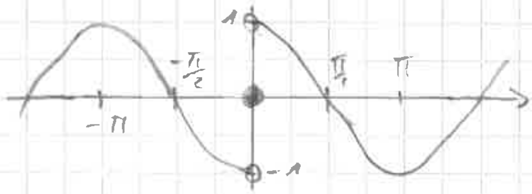
$\lim_{x \to -1} 1 - \frac{1}{x} = 1 + 1 = 2$



b.) $y = \operatorname{sign} x \cdot \sin(x)$

$\lim_{x \to 0} \operatorname{sign} x \cdot \sin(x) = \underline{0} \cdot 0 = \underline{\underline{0}}$

c.) $y = \operatorname{sign}(x) \cdot \cos(x)$



$\lim_{x \to 0+} \operatorname{sign}(x) \cdot \cos(x) = 1$

$\lim_{x \to 0-} \operatorname{sign}(x) \cdot \cos(x) = -1$

$y(0) = 0 \cdot 1 = 0$

31.) $\left| \frac{n^2}{1+n^2} - 1 \right| \overset{!}{<} \varepsilon \qquad \varepsilon > 0$

$\left| \frac{n^2 - 1 - n^2}{1+n^2} \right| < \varepsilon$

$\left| \frac{-1}{1+n^2} \right| < \varepsilon$

$\frac{1}{\varepsilon} < n^2 + 1$

$\frac{1}{\varepsilon} - 1 < n^2$

$\sqrt{\frac{1}{\varepsilon} - 1} < n$

$\underline{z.B.}$ $\varepsilon = \frac{1}{10}$: $\sqrt{\frac{1}{\frac{1}{10}} - 1} = \sqrt{10-1} = \sqrt{9} = 3$

$\Rightarrow N = 4$

32.)
- $f_n(x) = \arctan(n \cdot x)$

$f(x) = \lim_{n \to \infty} f_n(x)$

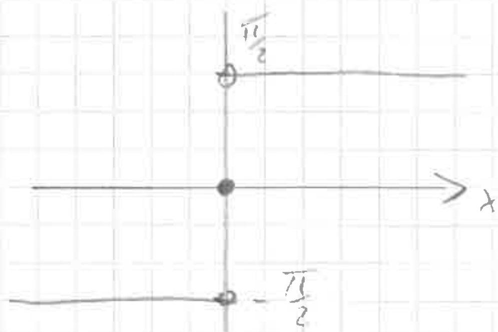
x fix, $x > 0$:

$\lim_{n \to \infty} \arctan(n \cdot x) = \frac{\pi}{2}$

x fix, $x < 0$:

$\lim_{n \to \infty} \arctan(n \cdot x) = -\frac{\pi}{2}$

$x = 0$: $\lim_{n \to \infty} \arctan(n \cdot 0) = 0$



- $g_n(x) = \frac{1}{(1+x^2)^n}$

x fix, $x \neq 0$:

$\lim_{n \to \infty} \frac{1}{\underbrace{(1+x^2)^n}_{>1}} = 0$

$x = 0$: $\lim_{n \to \infty} \frac{1}{1^n} = 1$