# Two models of reliability by imprecise parameters of lifetime distributions

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## 1 Introduction

By analyzing the reliability of a system, it is very often assumed that all probabilities are precise, that is, that every probability involved is perfectly determinable. However, the information about reliability of components may be supplied by experts and it is difficult to expect that all experts provide precise and true reliability assessments. One of the promising tools for dealing with such assessments and for computing the system reliability on their base is the *imprecise probability theory* (also called the theory of lower previsions [19], the theory of interval statistical models [14], the theory of interval probabilities [21, 22]), whose general framework is provided by upper and lower previsions. Some examples of the successful application of imprecise probabilities to reliability analysis can be found in [13, 16, 17], where it is assumed that probability distributions of the component times to failure are unknown and there exists only some partial information about the component reliability behavior in the form of interval probabilities, interval moments, etc. At the same time, there are cases when a type of probability distributions of the component times to failure is known, for example, from physical nature of components, but their parameters or a part of parameters are defined by experts. If experts provide possible intervals of parameters and these experts are absolutely reliable, i.e., they provide always true assessments, then the problem of computing the system reliability is reduced to the well known interval analysis [1]. However, in reality there is some degree of our belief to each expert's judgement whose value is determined by experience and competence of the expert. Therefore, it is necessary to take into account the available information about experts to obtain more credible assessments of the system reliability.

The uncertainty in parameters can be considered in a framework of *hierarchical uncertainty models* which are rather common in uncertainty theory. The different application examples of these models can be found in [6, 8]. A comprehensive review of hierarchical models is given in [5] where it is argued that the most common hierarchical model is the Bayesian one [2, 3, 10, 11, 12, 15, 24].

At the same time, the Bayesian hierarchical model is unrealistic in problems where there is available only partial information about the system behavior. In order to describe the partial information in hierarchical models de Cooman and Walley [4, 6, 9, 20] proposed to use the possibility measure [7, 23]. However, this approach can not be used in cases when experts assess the distribution parameters because the assessed intervals of parameters may be arbitrary and do not satisfy the condition of their nesting.

Therefore, the models studied in the presented paper can be regarded as an extension of the Bayesian hierarchical model on the case of imprecise parameters of probability distributions. Two approaches for computing reliability measures are considered and analyzed from their practical application point of view. Moreover, updating the beliefs to experts after observing a failure is investigated. Numerical examples illustrate the proposed model.

## 2 Preliminary definitions

Suppose there is a continuous random variable $X(x)$ defined on the sample space $\Omega$ and information about this variable is represented as a set of $m$ interval-valued expectations of functions $f_1(X),...,f_m(X)$. Denote these lower and upper expectations $\underline{a}_i = \underline{\mathbb{E}}f_i$ and $\overline{a}_i = \overline{\mathbb{E}}f_i$, $i = 1,...,m$. In terms of the theory of imprecise probabilities the corresponding functions $f_i(X)$ and interval-valued expectations $\underline{\mathbb{E}}f_i$ and $\overline{\mathbb{E}}f_i$ are called *gambles* and *lower and upper previsions*, respectively. Various types of information can be modelled by means of lower and upper previsions. For example, if $f_i$ is the indicator function of an event $A$, then previsions $\underline{\mathbb{E}}f_i$ and $\overline{\mathbb{E}}f_i$ can be regarded as lower and upper probabilities of the event $A$. If $f_i(X) = X$, then $\underline{\mathbb{E}}f_i$ and $\overline{\mathbb{E}}f_i$ are bounds for a mean value of the corresponding random variable. The lower and upper previsions $\underline{\mathbb{E}}f_i$ and $\overline{\mathbb{E}}f_i$ can also be interpreted as bounds for an unknown precise prevision $\mathbb{E}f_i$ which will be called a *linear prevision*.

For computing new previsions $\underline{\mathbb{E}}g$ and $\overline{\mathbb{E}}g$ of a gamble $g(X)$ from the available information, *natural extension* can be used. This is a general mathematical procedure for calculating new previsions from initial judgements. It produces a coherent overall model from a certain collection of imprecise probability judgements and may be seen as the basic constructive step in interval-valued statistical reasoning. In fact, the natural extension can be written as the following optimization problems:

$$\underline{\mathbb{E}}g = \min_{\rho} \int_{\Omega} g(x)\rho(x)\mathrm{d}x, \overline{\mathbb{E}}g = \max_{\rho} \int_{\Omega} g(x)\rho(x)\mathrm{d}x, \tag{1}$$

subject to

$$\int_{\Omega} \rho(x)\mathrm{d}x = 1,\ \rho(x) \geq 0, \tag{2}$$

$$\underline{a}_i \leq \int_{\Omega} f_i(x)\rho(x)\mathrm{d}x \leq \overline{a}_i,\ i \leq m. \tag{3}$$

Here the minimum and maximum are taken over a set of all possible probability density functions $\{\rho(x)\}$ satisfying conditions (3).

It should be noted that problems (1)-(3) are linear and the dual optimization problems can be written as follows [13, 14]:

$$\overline{\mathbb{E}}g = \min_{c_0, c_i, d_i} \left( c_0 + \sum_{i=1}^{m} (c_i \overline{a}_i - d_i \underline{a}_i) \right), \ \underline{\mathbb{E}}g = -\overline{\mathbb{E}}(-g), \tag{4}$$

subject to $c_i$, $d_i \in \mathbb{R}_+$, $c_0 \in \mathbb{R}$, $i = 1, ..., m$, and $\forall x \in \Omega$,

$$c_0 + \sum_{i=1}^{m} (c_i - d_i) f_i(x) \geq g(x). \tag{5}$$

In order to indicate that expectations are found in accordance with the density $\rho$, we will note it below by $\mathbb{E}_\rho$.

# 3 The problem statement

Suppose that reliability of a system is described by a probability density function $\pi(x|\Theta)$, where $\Theta = (\theta_1, ..., \theta_k)$ is a vector of parameters of the known probability distribution $F$.

It is assumed that information about parameters is represented in the following form:

$$\underline{\alpha}_j \leq \mathbb{E}_\rho f_j(\Theta) \leq \overline{\alpha}_j, \ j = 1, ..., m, \tag{6}$$

or

$$\underline{\alpha}_j \leq \int_\Lambda f_j(\Theta)\rho(\Theta)\mathrm{d}\Theta \leq \overline{\alpha}_j, \ j = 1, ..., m.$$

Here $\rho$ is an unknown joint density function of the vector $\Theta$ formed by constraints (6) and $\Lambda$ is a sample space of the vector $\Theta$. This means that we know lower and upper expectations of some functions of parameters $\Theta$, such that $m$ judgements are available about parameters of the distribution corresponding to the system time to failure. In particular, if $\underline{\alpha}_j$ and $\overline{\alpha}_j$ are lower and upper probabilities that the $j$-th parameter is in bounds $[a, b]$, then $f_j(\Theta)$ is the indicator function $I_{[a,b]}(\theta_j)$ such that $I_{[a,b]}(\theta_j) = 1$ if $\theta_j \in [a, b]$ and $I_{[a,b]}(\theta_j) = 0$ if $\theta_j \notin [a, b]$. By formalizing the information about parameters in the form of (6), we assume that parameters are random variables having the density $\rho$. In fact, there is a set of densities satisfying (6) and each density from this set can be regarded as a candidate to further analysis. If parameters are statistically independent as random variables, then the joint density is represented as a product of marginal ones and this condition can be considered as some additional information about parameters.

The system reliability can be described by the following probability

$$R(t) = \Pr\{X \leq t\} = \mathbb{E}_\pi I_{[0,t]}(X) = \int_{\mathbb{R}_+} I_{[0,t]}(x)\pi(x|\Theta)\mathrm{d}x.$$

Generally, different measures of the system reliability can be obtained as expectations of some functions $g(x)$. For example, the mean time to system failure can be computed as $\mathbb{E}_{\pi}g(X) = \mathbb{E}_{\pi}X$.

Our task is to find the system reliability measures taking into account imprecision of parameters $\Theta$.

In order to give the reader the essence of the subject analyzed and make all the formulas more readable, we will assume for simplicity that $k = 1$, i.e., there is only one imprecise parameter for the distribution of the system time to failure. Furthermore, throughout the paper the obvious constraints for densities $\rho$ (or $\pi$) to the optimization problems such that $\rho(x) \geq 0$, $\int_{\mathbb{R}_+} \rho(x)\mathrm{d}x = 1$ will not be written.

# 4 Model 1 (averaging parameters)

The first approach is to find average bounds $\underline{\mathbb{E}}_{\rho}\theta$ and $\overline{\mathbb{E}}_{\rho}\theta$ for the parameter $\theta$ by using the available partial information about it (the first step) and then compute the lower $\underline{\mathbb{E}}_{\pi}g$ and upper $\overline{\mathbb{E}}_{\pi}g$ system reliability measures by using the known rules of the interval computations (the second step).

The first step can be carried out by using the natural extension in the following form:

$$\underline{\mathbb{E}}_{\rho}\theta\left(\overline{\mathbb{E}}_{\rho}\theta\right) = \min_{\rho}\left(\max_{\rho}\right)\int_{\Lambda}\theta\rho(\theta)\mathrm{d}\theta \tag{7}$$

subject to

$$\underline{\alpha}_j \leq \int_{\Lambda} f_j(\theta)\rho(\theta)\mathrm{d}\theta \leq \overline{\alpha}_j,\ j = 1, ..., m. \tag{8}$$

Here the minimum and maximum are taken over all possible joint densities $\rho$ satisfying the above constraints.

Then the dual optimization problems can be written as follows:

$$\overline{\mathbb{E}}_{\rho}\theta = \min_{c_0, c_i, d_i}\left(c_0 + \sum_{i=1}^{m}\left(c_i\overline{a}_i - d_i\underline{a}_i\right)\right),\ \underline{\mathbb{E}}_{\rho}\theta = -\overline{\mathbb{E}}_{\rho}(-\theta), \tag{9}$$

subject to $c_i$, $d_i \in \mathbb{R}_+$, $c_0 \in \mathbb{R}$, $i = 1, ..., m$, and $\forall\theta \in \Lambda$,

$$c_0 + \sum_{i=1}^{m}\left(c_i - d_i\right)f_i(\theta) \geq \theta. \tag{10}$$

If all functions are indicator ones, then problems (9)-(10) have a finite number of constraints.

The second step is determined as follows:

$$\underline{\mathbb{E}}_{\pi}g = \min_{a \in [\underline{\mathbb{E}}_{\rho}\theta, \overline{\mathbb{E}}_{\rho}\theta]}\int_{\mathbb{R}_+} g(x)\pi(x|a)\mathrm{d}x, \tag{11}$$

$$\overline{\mathbb{E}}_{\pi}g = \max_{a \in [\underline{\mathbb{E}}_{\rho}\theta, \overline{\mathbb{E}}_{\rho}\theta]}\int_{\mathbb{R}_+} g(x)\pi(x|a)\mathrm{d}x. \tag{12}$$

**Example 1** *Suppose that the random time to failure $X$ is governed by the exponential probability distribution with the density*

$$\pi(x|\lambda) = \lambda \exp(-\lambda x).$$

*Suppose that two experts provide the following information about the parameter $\lambda$: the first expert - $\lambda \in [4 \cdot 10^{-5}, 8 \cdot 10^{-5}]$ (failures/hour) and the second expert - $\lambda \in [6 \cdot 10^{-5}, 9 \cdot 10^{-5}]$ (failures/hour). The belief to the first expert is 0.5. This means that the expert provides 50% of true judgements. The belief to the second expert is between 0.3 and 1. This means that the expert provides greater than 30% of true judgements. It is also known that bounds for $\lambda$ are 0 and $25 \cdot 10^{-5}$. Let us find bounds for the system reliability at time 5 hours, i.e., bounds for*

$$Q(5) = 1 - R(5) = 1 - \Pr\{X \leq 5\}.$$

*First, we compute average bounds for $\lambda$ from (7)-(8) or from (9)-(10) as follows:*

$$\underline{\mathbb{E}}_{\rho}\lambda \left(\overline{\mathbb{E}}_{\rho}\lambda\right) = \min_{\rho} \left(\max_{\rho}\right) \int_{\Lambda} \lambda \rho(\lambda) \mathrm{d}\lambda$$

*subject to*

$$0.5 \leq \int_0^{25 \cdot 10^{-5}} I_{[4 \cdot 10^{-5}, 8 \cdot 10^{-5}]}(\lambda) \rho(\lambda) \mathrm{d}\lambda \leq 0.5,$$

$$0.3 \leq \int_0^{25 \cdot 10^{-5}} I_{[6 \cdot 10^{-5}, 9 \cdot 10^{-5}]}(\lambda) \rho(\lambda) \mathrm{d}\lambda \leq 1.$$

*The solutions to optimization problems are $\underline{\mathbb{E}}_{\rho}\lambda = 2.6 \cdot 10^{-5}$ and $\overline{\mathbb{E}}_{\rho}\lambda = 16.47 \cdot 10^{-5}$. Hence*

$$\underline{Q}(5) = \min_{a \in \{\underline{\mathbb{E}}_{\rho}\lambda, \overline{\mathbb{E}}_{\rho}\lambda\}} \exp(-a \cdot 5) = \exp(-16.47 \cdot 10^{-5} \cdot 5) = 0.99918,$$

$$\overline{Q}(5) = \max_{a \in \{\underline{\mathbb{E}}_{\rho}\lambda, \overline{\mathbb{E}}_{\rho}\lambda\}} \exp(-a \cdot 5) = \exp(-2.6 \cdot 10^{-5} \cdot 5) = 0.99987.$$

# 5 Model 2 (averaging the system reliability)

For computing $\underline{R}$ and $\overline{R}$, the following optimization problems should be solved:

$$\underline{R}(\overline{R}) = \min_{\rho}(\max_{\rho}) \int_{\Lambda} \left( \int_{\mathbb{R}_+} g(x) \pi(x|\theta) \mathrm{d}x \right) \rho(\theta) \mathrm{d}\theta,$$

subject to (6). The variable of optimization is the density $\rho$.

Now we have the optimization problem which can be reformulated in terms of the imprecise probability theory. It is necessary to find lower and upper previsions of the gamble

$$G(\theta) = \int_{\mathbb{R}_+} g(x) \pi(x|\theta) \mathrm{d}x \tag{13}$$

by given some partial information about the parameter $\theta$. So, we can write the following optimization problems for computing $\underline{R}$ and $\overline{R}$:

$$\underline{R}(\overline{R}) = \min_{\rho} \left( \max_{\rho} \right) \int_{\Lambda} G(\theta)\rho(\theta)\mathrm{d}\theta, \tag{14}$$

subject to (6).

The corresponding dual optimization problems are of the form:

$$\underline{R} = \max_{c_0, c_i, d_i} \left( c_0 + \sum_{i=1}^{m} \left( c_i \underline{a}_i - d_i \overline{a}_i \right) \right), \tag{15}$$

subject to $c_i$, $d_i \in \mathbb{R}_+$, $c_0 \in \mathbb{R}$, $i = 1, ..., m$, and $\forall \theta \in \Lambda$,

$$c_0 + \sum_{i=1}^{m} \left( c_i - d_i \right) f_i(\theta) \leq G(\theta), \tag{16}$$

and

$$\overline{R} = \min_{c_0, c_i, d_i} \left( c_0 + \sum_{i=1}^{m} \left( c_i \overline{a}_i - d_i \underline{a}_i \right) \right), \tag{17}$$

subject to $c_i$, $d_i \in \mathbb{R}_+$, $c_0 \in \mathbb{R}$, $i = 1, ..., m$, and $\forall \theta \in \Lambda$,

$$c_0 + \sum_{i=1}^{m} \left( c_i - d_i \right) f_i(\theta) \geq G(\theta). \tag{18}$$

**Example 2** *Let us compute bounds for the reliability function by using the second method and data described in Example 1. The optimization problems are*

$$\underline{Q}(5) \left( \overline{Q}(5) \right) = \min_{\rho} \left( \max_{\rho} \right) \int_{0}^{25 \cdot 10^{-5}} G(\lambda)\rho(\lambda)\mathrm{d}\lambda,$$

*subject to*

$$0.5 \leq \int_{0}^{25 \cdot 10^{-5}} I_{[4 \cdot 10^{-5}, 8 \cdot 10^{-5}]}(\lambda)\rho(\lambda)\mathrm{d}\lambda \leq 0.5,$$
$$0.3 \leq \int_{0}^{25 \cdot 10^{-5}} I_{[6 \cdot 10^{-5}, 9 \cdot 10^{-5}]}(\lambda)\rho(\lambda)\mathrm{d}\lambda \leq 1.$$

*Here*

$$G(\lambda) = \int_{\mathbb{R}_+} I_{[5,\infty)}(x)\lambda \exp(-\lambda x)\mathrm{d}x = \exp(-5 \cdot \lambda).$$

*A dual optimization problem for computing the upper bound is of the form:*

$$\overline{Q}(5) = \min_{c_0, c_i, d_i} \left( c_0 + 0.5c_1 - 0.5d_1 + 1c_2 - 0.3d_2 \right)$$

*subject to* $c_i$, $d_i \in \mathbb{R}_+$, $c_0 \in \mathbb{R}$, $i = 1, 2$, *and* $\forall \lambda \in [0,\ 25 \cdot 10^{-5}]$,

$$c_0 + (c_1 - d_1) I_{[4\cdot 10^{-5}, 8\cdot 10^{-5}]}(\lambda) + (c_2 - d_2) I_{[6\cdot 10^{-5}, 9\cdot 10^{-5}]}(\lambda) \geq \exp(-5 \cdot \lambda).$$

*By substituting different values of* $\lambda$ *from the interval* $[0,\ 25 \cdot 10^{-5}]$ *into the above constraints, we can rewrite these constraints as follows:*

$$\begin{aligned} c_0 &\geq \exp(-5 \cdot 0), \\ c_0 + (c_1 - d_1) &\geq \exp(-5 \cdot 4 \cdot 10^{-5}), \\ c_0 + (c_1 - d_1) + (c_2 - d_2) &\geq \exp(-5 \cdot 6 \cdot 10^{-5}), \\ c_0 + (c_2 - d_2) &\geq \exp(-5 \cdot 8 \cdot 10^{-5}). \end{aligned}$$

*By solving this simple linear programming problem, we obtain* $\overline{Q}(5) = 0.99987$. *Similarly, the lower bound can be computed* $\underline{Q}(5) = 0.99918$.

# 6 Comparison of models

Examples 1 and 2 show that both considered models may give the same results. However, it can be not valid in general. Let us denote lower and upper reliability measures obtained by means of the first and second models $\underline{M}_1$, $\overline{M}_1$, $\underline{M}_2$, $\overline{M}_2$, respectively.

**Theorem 1** *If the function* $G$ *in (13) is arbitrary, then* $\underline{M}_1 \leq \underline{M}_2$, $\overline{M}_1 \geq \overline{M}_2$. *If the function* $G$ *is monotone on* $\Lambda$, *then* $\underline{M}_1 = \underline{M}_2$, $\overline{M}_1 = \overline{M}_2$.

It follows from Theorem 1 that some part of the available information is lost if the first model is used for computing lower and upper bounds for reliability characteristics. Therefore, in spite of possible computational difficulties the second models is more preferable than first one. Moreover, it is stronger from the mathematical point of view.

# 7 Updating the expert beliefs

Suppose that there is available the set of expert judgements (6) and we observe an event $B(x) = I_{[\underline{b},\overline{b}]}(x)$, for example, "a failure occurs between 10 and 12 hours". It is assumed here that the available information about parameters of distributions is represented by a set of probabilities (beliefs) that the unknown value of the parameter is in intervals $[\underline{\theta}_j, \overline{\theta}_j]$. This implies that all functions $f_j(\theta)$ in (6) are indicator functions $I_{[\underline{\theta}_j, \overline{\theta}_j]}(\theta)$. Now we can update beliefs $\underline{\alpha}_k$ and $\overline{\alpha}_k$ to experts after observing the event $B$, and assuming that bounds for intervals of the parameter provided by experts are not changed, we can construct posterior lower and upper probabilities $\underline{\beta}_k$ and $\overline{\beta}_k$ from prior lower and upper previsions $\underline{\alpha}_k$, $\overline{\alpha}_k$ of the indicator function $I_{[\underline{\theta}_k, \overline{\theta}_k]}(\theta)$ and statistical data $B$ such that

$$\mathbb{E}_\rho(I_{[\underline{\theta}_k, \overline{\theta}_k]}(\theta) | B(x)) \in [\underline{\beta}_k, \overline{\beta}_k].$$

A procedure of updating beliefs for the first model (averaging parameters) is well known [14, 19]. Moreover, the analysis of this model has shown that despite its simplicity from the computational point of view, its application should be limited due to the large imprecision in its results. Therefore, this question is remained outside the paper. Let us consider a procedure of updating which corresponds to the second model.

The linear conditional prevision $\mathbb{E}_{\rho}(I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta)|B(x))$ can be represented as follows [14, 18]:

$$\mathbb{E}_{\rho}(I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta)|B(x)) = \frac{\int_{\Lambda} I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta) G_B(\theta)\rho(\theta)\mathrm{d}\theta}{\int_{\Lambda} G_B(\theta)\rho(\theta)\mathrm{d}\theta},$$

where

$$G_B(\theta) = \int_{\mathbb{R}_+} I_B(x)\pi(x|\theta)\mathrm{d}x.$$

After observing the event $B$, we obtain new previsions $\underline{\beta}_k$ and $\overline{\beta}_k$. This means that the probability that $\mathbb{E}_{\rho} I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta)$ is in the previous interval $[\underline{\alpha}_k, \overline{\alpha}_k]$ is updated and its bounds become $\underline{\beta}_k$ and $\overline{\beta}_k$. Then the following optimization problems can be written for computing $\underline{\beta}_k$ and $\overline{\beta}_k$:

$$\underline{\beta}_k \left(\overline{\beta}_k\right) = \min_{\rho}(\max_{\rho}) \frac{\int_{\Lambda} I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta) G_B(\theta)\rho(\theta)\mathrm{d}\theta}{\int_{\Lambda} G_B(\theta)\rho(\theta)\mathrm{d}\theta},$$

subject to (6).

The above problems can be regarded as linear-fractional ones. Therefore, the more simple dual optimization problems can be constructed. For example, the dual problem for computing the lower bound $\underline{\beta}_k$ is of the form:

$$\underline{\beta}_k = \max d,$$

subject to $d, c \in \mathbb{R}$, $c_i, h_i \in \mathbb{R}_+$, and $\forall \theta \in \Lambda$,

$$c + \sum_{i=1}^{m} (c_i - h_i) I_{[\underline{\theta}_i,\overline{\theta}_i]}(\theta) + dG_B(\theta) \leq I_{[\underline{\theta}_k,\overline{\theta}_k]}(\theta) G_B(\theta),$$

$$c + \sum_{i=1}^{m} \left(\underline{\alpha}_i c_i - \overline{\alpha}_i h_i\right) \geq 0.$$

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