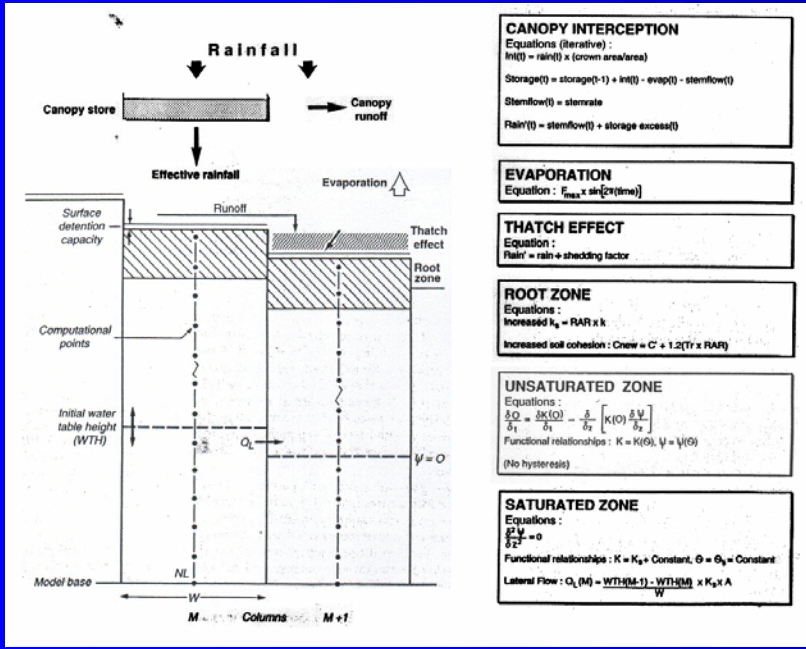
A classification approach to uncertainty analysis in a hydrology/stability model using statistical and imprecise information

Jim Hall
University of Bristol
Department of Civil Engineering

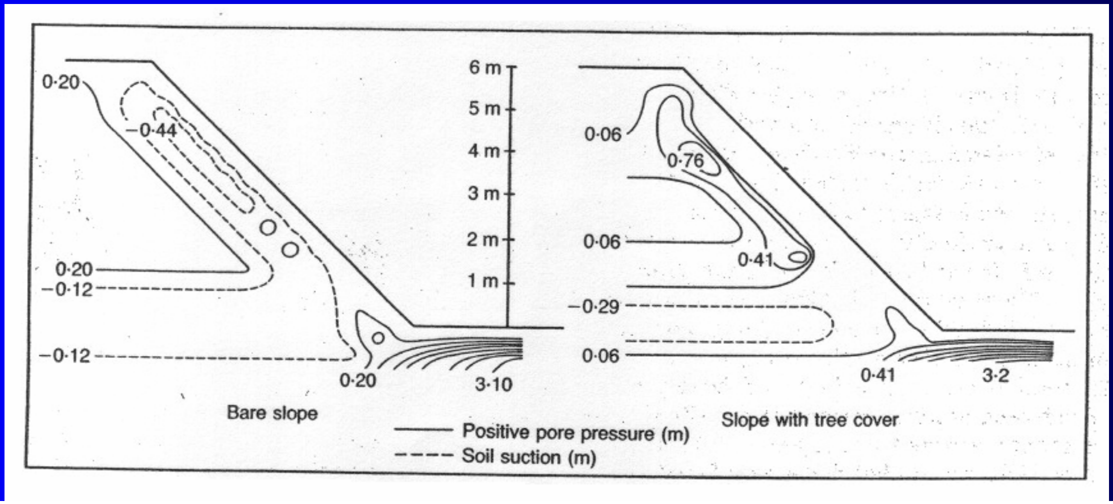
Contents

- Background to the CHASM slope hydrology-stability model
- A classification approach for avoiding inappropriate assumptions about joint distributions
- Dealing with statistical and interval-based information
- Combining information from different sources
- Extending the uncertain information through CHASM

CHASM slope hydrology-stability model



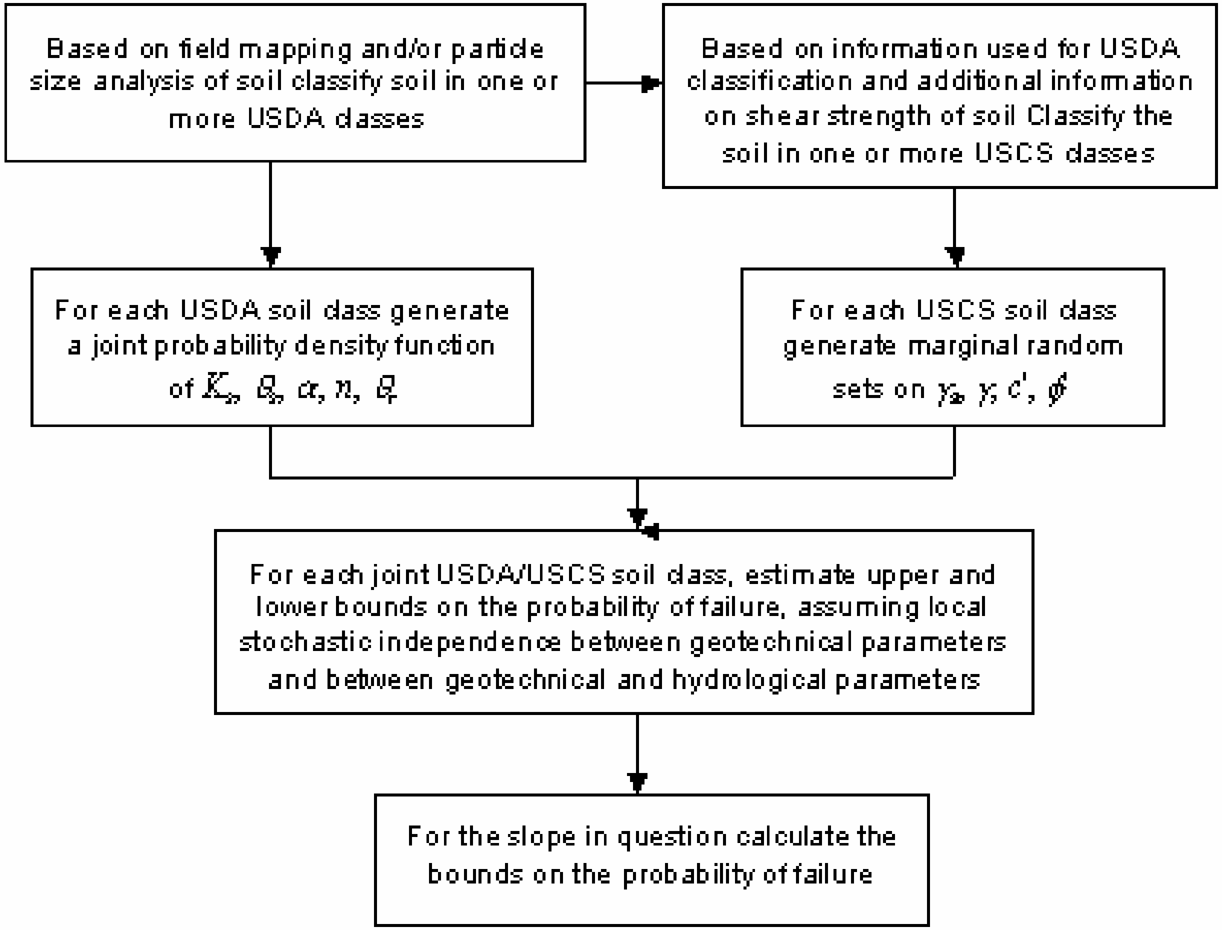
- 2D FD slope hydrology model
- Bishop slope stability analysis
- Driven by a specified rainfall event



Parameters used in CHASM

Parameter	Symbol
Saturated hydraulic conductivity	K_s
Saturated soil moisture content	θ_s
α coefficient: Van Genuchten suction-moisture curve	α
n coefficient: Van Genuchten suction-moisture curve	n
Residual soil moisture content: Van Genuchten suction-moisture curve	θ_r
Saturated bulk density (unit weight)	γ_s
Unsaturated bulk density	γ
Effective cohesion	c'
Effective friction angle	ϕ'

- Means, variances and correlation coefficients of hydrological parameters are published
- For ϕ', c', γ and γ_s only interval values are published
- All values are published according to (inconsistent soil classifications)



Suppose that the lower and upper bounds on the probability of the factor of safety exceeding some value x are written $P^u(FoS > x)$ and $P_l(FoS > x)$, respectively, then the probability of slope failure P_f lies in the interval $[P_l(FoS \le 0), P^u(FoS \le 0)]$.

Suppose that there are j possible soil classes and of n samples, m_i samples indicate that the soil should be classified as class C_i, then an estimate of the probability of failure of the slope in question will lie in the interval

$$P_f \in \left[\sum_{i=1}^{j} \frac{m_i}{n} P_l(FoS \le 0 \mid C_i), \sum_{i=1}^{j} \frac{m_i}{n} P^u(FoS \le 0 \mid C_i) \right], \sum_{i=1}^{j} m_i = n \qquad (1)$$

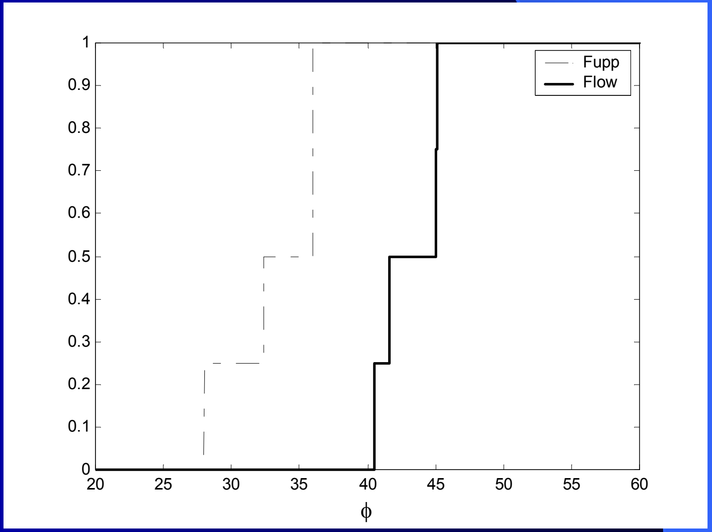
Combination of interval estimates from different sources

- Conjunctive: all of the sources of information are thought of as providing some imprecise yet correct and consistent information abo an unknown quantity or proposition
- Disjunctive: there is at least one correct source of information amongst many, yet it is not known which sources are correct
- Averaging: only one source of information is correct

For each focal element $A \in \mathcal{P}(X)$

$$m(A) = \frac{1}{n}\sum_{i=1}^{n} m_i(A)$$

Reference	Random sets of ϕ (degrees)
7.2	[28.0, 45.0]
8.2	[36.0, 41.6]
8.3	[32.4, 40.5]
8.4	[36.0, 45.1]



Finding the bounds on the system response

If $x_1,\ldots x_n$ are variables on $X =$, the dependency between $x_1,\ldots x_n$ can be expressed as a random relation R, which is a random set $(\mathfrak{R}, \rho)$ on the Cartesian product $X_1\times\ldots\times X_n$. If A_1 and A_2 are sets on X_1 and X_2 respectively and A_1 and A_2 are stochastically independent, then the mass on the random Cartesian product R on $X_1\times X_2$ is given by

$$\rho_{12}(A_1\times A_2) = m_1(A_1).m_2(A_2).$$

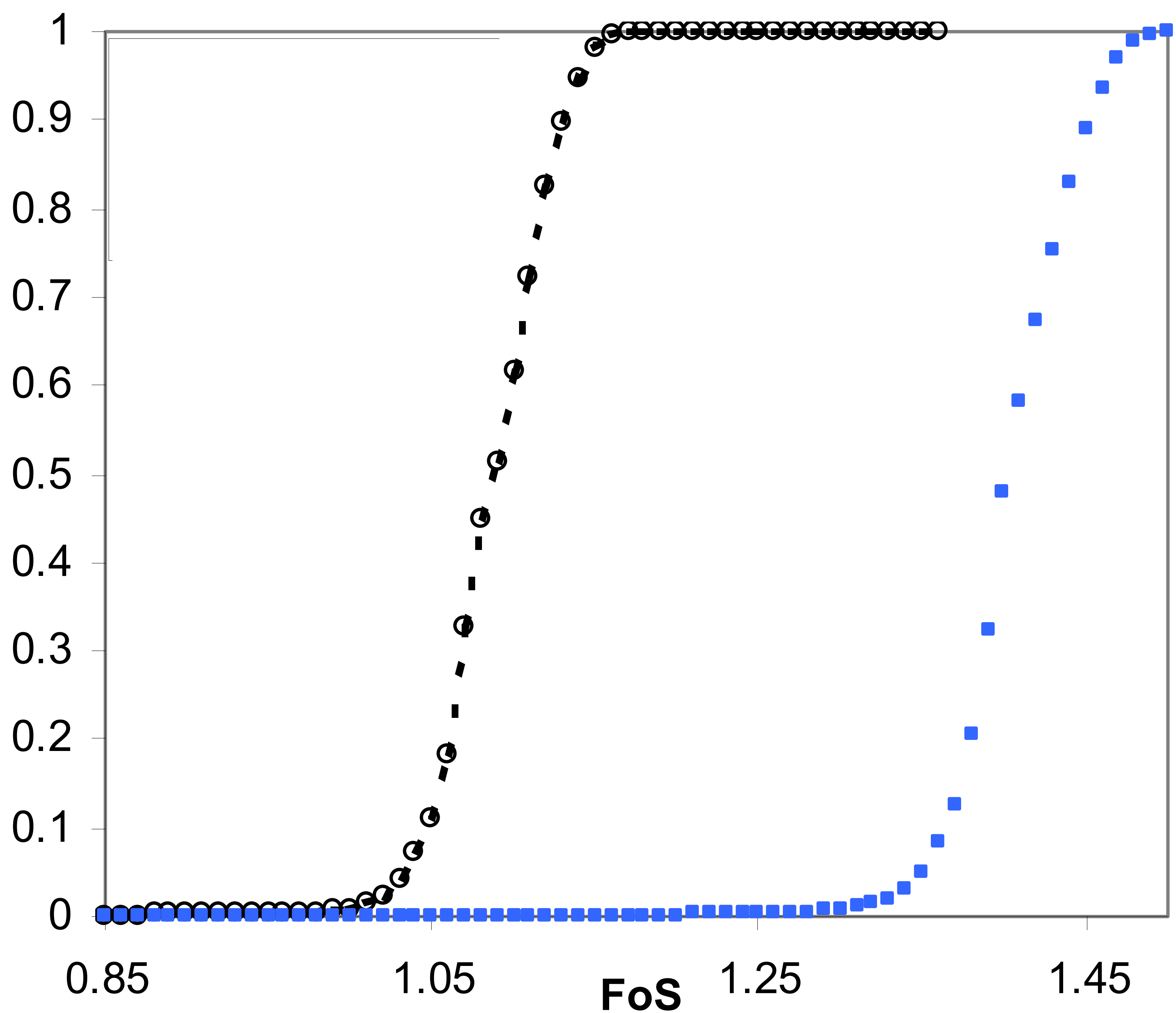
Let f be a mapping $X_1\times\ldots\times X_9 \rightarrow Y$ and $x_1,\ldots x_9$ be variables K_s, θ_s, α, n, θ_r, γ_s, γ, c', ϕ' whose values are expressed as a random relation R on the Cartesian product $X_1\times\ldots\times X_9$, in which case the range of y is the random set $(\mathcal{S}, m)$ such that:

$$\mathcal{S} = \{y(R_i) \mid R_i \in \mathfrak{R}\}, \quad y(R_i) = \{y(\mathbf{x}) \mid \mathbf{x} \in R_i\}$$

$$m(A) = \sum_{A=y(R_i)} \rho(R_i)$$

Finding the bounds on the system response

1. Demonstrate that f is monotonic within the range of geotechnical parameters γ_s, γ, c', ϕ'.
2. For every combination of focal elements in γ_s, γ, c' and ϕ' find the coordinates of the vertices that correspond to the bounding values of f. At these two vertices estimate the cumulative probability distribution on *FoS*. These are the upper and lower bounds on the distribution of *FoS* associated with the given focal elements in the random relation.
3. Generate a combined upper and lower probability distribution by applying equal weight to each of the lower and upper probability distributions generated in (2) above.
4. Repeat the analysis for each relevant soil class. Combine to give an overall estimate of the bounds on the probability of failure.


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.85
1.05
1.25
1.45
FoS

Conclusions

- Demonstration of practical slope hydrology-stability analysis using a combination of probabilistic and imprecise data
- Bayes conditionalisation on soil class is to make use of prior information and localises assumptions of stochastic independence
- Averaging is used to construct random sets from inprecise geotechnical parameters from multiple sources
- Provides an imprecession of the contributions of randomness and imprecision to the probabity of failure
- Could update imprecise priors using site-specific measurements