

Problem Set 3

1. Let $\mathbf{Q}_0 = \mathbf{I}_n \oplus (\mathbf{I}_T - \frac{1}{T}\mathbf{J}_T)$, where \mathbf{J}_T is a $T \times T$ matrix of ones and let $\mathbf{Q}_1 = \mathbf{I}_n \oplus \frac{1}{T}\mathbf{J}_T$. Calculate:

$$\begin{aligned}\mathbf{Q}_0\mathbf{Q}_0 \\ \mathbf{Q}_1\mathbf{Q}_1 \\ \mathbf{Q}_0\mathbf{Q}_1 \\ \mathbf{Q}_0 + \mathbf{Q}_1\end{aligned}$$

2. Let $\Sigma = \sigma_\varepsilon^2\mathbf{I}_T + \sigma_\mu^2\mathbf{J}_T$ and

$$\Sigma^{-1/2} = \frac{1}{\sigma_\varepsilon} \left(\mathbf{I}_T - \frac{\theta}{T}\mathbf{J}_T \right),$$

where

$$\theta = 1 - \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T\sigma_\mu^2}}.$$

Verify that $\Sigma^{-1} = \Sigma^{-1/2}\Sigma^{-1/2}$.