Assignment 5

Problem 1:
Consider the model $y_t = x_t'\beta + \epsilon_t$ with $\epsilon_t = \rho\epsilon_{t-1} + u_t$ where ϵ_t and u_t are independent of each other. Also note that $E(u_t) = 0$, $Var(u_t) = \sigma_u^2$ and $Cov(u_t, u_s) = 0$ for $t \neq s$. Additionally the process is stationary meaning that $E(\epsilon_t) = E(\epsilon_{t-1})$ and $Var(\epsilon_t) = Var(\epsilon_{t-1})$.

Write down the variance-covariance matrix of ϵ for 3 time lags ($Cov(\epsilon_t, \epsilon_{t-1})$, $Cov(\epsilon_t, \epsilon_{t-2})$...) . What happens to the OLS estimator in this case when we have autocorrelated disturbances, is it still BLUE (Check if there is a bias and calculate the variance of the estimator b).

(Hint: When you calculate the covariances e.g. $Cov(\epsilon_t, \epsilon_{t-1}) = E(\epsilon_t\epsilon_{t-1})$ substitute $\epsilon_t = \rho\epsilon_{t-1} + u_t$ only for ϵ_t to get quadratic terms of ϵ_{t-1} like $...E(\rho\epsilon_{t-1}^2 + \epsilon_t u_t)$ which you then can solve. Do such substitutions also for the higher order lags.)

Problem 2:
Show for each of the following processes if it is a stationary process. Additionally calculate the autocorrelations ρ. a, b and c are constants.
$\epsilon_t \sim i.i.d.N(0, \sigma^2)$

1. $y_t = a + b\epsilon_t + c\epsilon_{t-1}$
2. $y_t = \epsilon_{t-2} + 2\epsilon_t$
3. $y_t = \epsilon_t\epsilon_{t-1}$
4. $y_t = bt + \epsilon_t$

Problem 3:
Use the dataset LakeHuron which is data on the water level in feet from 1875-1972 of Lake Huron and do the following tasks:

1. Plot the series and estimate a linear trend $y_t = \alpha + \beta t + u_t$. Plot the residuals against time and the autocorrelation and partial autocorrelation, what do you conclude?
2. Estimate an AR(1) process $y_t = \alpha + \rho_1 y_{t-1} + u_t$. What is the value of ρ? Again plot the residuals and the corresponding autocorrelation/partial autocorrelation, what do you conclude now?
3. Estimate an AR(2) process $y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + u_t$? What are the estimated coefficients for ρ_1 and ρ_2? Plot the residuals and the autocorrelation and partial autocorrelation, what are your conclusions?

Some useful STATA commands:

- tsset *timevar* defines that this is a time series dataset. Use this at the beginning of your analysis and define the time variable.
- ac *varname* plots the autocorrelation
- pac *varname* plots the partial autocorrelation
- L.*varname* use this in a regression to specify one lag of the variable
- L(1/2).*varname* in this regression STATA uses lag 1 and 2