

## Assignment 4

### Problem 1:

Use the data in Rental.dta and estimate the following model

$$\log(\text{rent}_{it}) = \beta_0 + \delta_0 y90_t + \beta_1 \log(\text{pop}_{it}) + \beta_2 \log(\text{avginc}_{it}) + \beta_3 \text{pctstu}_{it} + a_i + u_{it}$$

where rent is the average monthly rent paid,  $y90$  is a dummy variable for the year 1990,  $\text{pop}$  is the total city population,  $\text{avginc}$  the average city income and  $\text{pctstu}$  the student population as a percent of the total population.  $a_i$  denotes an unobserved effect.

1. Estimate the equation by pooled OLS and report the results. Interpret the coefficient on the dummy variable  $y90$ .
2. Generate the first differences of the variables and estimate the coefficients by OLS. (Hint for Stata users: to compute the first differences define the dataset as a panel first by "xtset city year" and calculate the first differences for each variable used with e.g. "by city: gen dlrent=lrent-lrent[\_n-1]")
3. Estimate the model by the fixed effects estimator provided by your software package and compare it to your estimates in 3.
4. Compare your estimates of point 1 and 4. What is the difference between these two models regarding the unobserved effect? Which model would you prefer? Perform an adequate test. State the null hypothesis of this test! (Hint: Use the following test statistics:  $\frac{(R^2 - R_r^2)/q}{(1 - R^2)/(nT - n - k)} \sim F_{(q, nT - n - k)}$  with  $R_r^2$  as the  $R^2$  of the restricted model and  $q$  as the number of restrictions)

### Problem 2:

Fixed Effects.

Show that in the model  $y_{it} = \alpha_i + x'_{it}\beta + \epsilon_{it}$  the within group estimator coincides with the estimator obtained by using first differences when  $T=2$ .

**Problem 3:**

Fixed Effects.

Consider the fixed effects model  $y_{it} = x'_{it}\beta + \alpha_i + \epsilon_{it}$ . The LSDV approach yields the same estimate of  $\beta$  as by transforming the data and estimating  $(y_{it} - \bar{y}_i) = (x_{it} - \bar{x}_i)'\beta + \epsilon_{it} - \bar{\epsilon}_i$ . Write down the matrices  $M_D y$  and  $M_D X$  for the LSDV approach for  $i = 1, 2$  and  $t = 1, 2$  and show that you get the same transformed model as with the within transformation.