Assignment 3

Problem 1:
Interpretation of regression coefficients.

1. Show for the model $ln(y) = \beta_0 + \beta_1 ln(x)$ that the coefficient β_1 can be interpreted as an elasticity meaning that a 1% change in the variable x causes a β percent change in y. The elasticity between x and y is defined by $E_{xy} = \frac{dy}{dx}\frac{x}{y}$.(Hint: Differentiate both sides of the above equation with respect to x)

2. Differentiating a function with respect to a variable x means that we examine the change in that function for infinitesimal changes of x. In the case of a logarithmic function we get e.g. $\frac{dln(x)}{dx} = \frac{1}{x} \Leftrightarrow dln(x) = \frac{dx}{x}$ which means that the change in a logarithmic function equals the relative change of that variable.

   For discrete changes we can use Taylor approximation to get $ln(x + \Delta x) - ln(x) \approx \frac{\Delta x}{x}$ which is again the relative change of x. This approximation is only true for small changes and small β 's due to the nonlinearity of the logarithm.

   Show the effect of a discrete change of Δx on y. (Hint: Start with building differences for a change of Δx: $ln(y + \Delta y) - ln(y) = \beta_1 ln(x + \Delta x) - \beta_1 ln(x)$ and manipulate this expression until you get the percentage change in y i.e. $\frac{\Delta y}{y} \cdot 100$)

Problem 2:
In Problem 2 of Assignment 2 you estimated the regression model

$$log(price) = \beta_0 + \beta_1 log(lotsize) + \beta_2 log(sqrft) + \beta_3 bdrms + \epsilon$$

In this model the house price is related to certain characteristics. *price* is the house price in 1000's, *lotsize* is the size of the lot in square feet and *bdrms* is the number of bedrooms.

Give now an interpretation of the estimated coefficients in the log specification. Calculate effects of a discrete change of Δx on y. The formula for the effect of a discrete change of Δx of a log-linear specification like for the variable *bdrms* is $\frac{\Delta y}{y} = (e^{b\Delta x} - 1) \cdot 100$

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. reg lprice llotsize lsqrft bdrms

      Source |       SS       df       MS              Number of obs =      88
-------------+------------------------------           F(  3,    84) =   50.42
       Model |  5.15504028     3  1.71834676           Prob > F      =  0.0000
    Residual |  2.86256324    84  .034078134           R-squared     =  0.6430
-------------+------------------------------           Adj R-squared =  0.6302
       Total |  8.01760352    87  .092156362           Root MSE      =   .1846

------------------------------------------------------------------------------
      lprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    llotsize |   .1679667   .0382812     4.39   0.000     .0918404     .244093
      lsqrft |   .7002324   .0928652     7.54   0.000     .5155597    .8849051
       bdrms |   .0369584   .0275313     1.34   0.183    -.0177906    .0917074
       _cons |  -1.297042   .6512836    -1.99   0.050    -2.592191    -.001893
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Problem 3:
Heteroscedasticity.
Given a sample of observations on y_i and x_i we want to estimate the model including only a constant with $y_i = \mu + \epsilon_i$. $E(\epsilon_i|x_i) = 0$ and $Var(\epsilon_i|x_i) = \sigma^2 \frac{x_i}{2}$.

1. Calculate the standard OLS estimator for μ.
2. Transform the model (in this case $P = \Omega^{-1/2}$) to account for the heteroscedastic error terms and perform OLS estimation on the transformed model. Show that the error term in this transformed model is

now homoscedastic.
(Hint: Transform the variables and write down the matrices to calculate $b = (X'X)^{-1}X'y$)

3. Use $b_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y$ to calculate the estimator (write down the matrices) and compare it to the OLS estimators of 1 and 2.