Assignment 1

Problem 1:
Least squares estimation.
The following table shows (randomly chosen) data for two variables y and x.

y_i	x_i
1	1
1	2
2	3
2	4
4	5

1. Show formally that the least squares normal equations for the bivariate regression model $y_i = \alpha + \beta x_i + \epsilon_i$ imply $\sum_i e_i = 0$ and $\sum_i x_i e_i = 0$.
2. Show that the estimator $b = (X'X)^{-1}(X'y)$ of the multiple regression model leads in the bivariate case to $b = \frac{n\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{n\sum_i x_i^2 - (\sum_i x_i)^2}$.
3. Now use one of the formulas in point 2 to calculate the least squares estimator for the data above.
4. Calculate the R^2 (coefficient of determination).
5. Calculate the $Est.Var[b|X] = \frac{\sigma^2}{\sum_i (x_i - \bar{x})^2}$ with $s^2 = \frac{\sum_i e_i^2}{n-k}$ as an unbiased estimator for σ^2.

Problem 2:
Frisch-Waugh-Lovell.
Show for the simple regression model $y_i = \alpha + \beta x_i + \epsilon_i$ that the estimator b for β can be obtained by regressing first the y_i and x_i on the remaining constant and then regressing the residuals from the first regression on the residuals from the second regression.
(Note from above that $b = \frac{n\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{n\sum_i x_i^2 - (\sum_i x_i)^2}$ where you get after some manipulations $b = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$)

Problem 3:
Changing the units of measurement.
Let $y = X\beta + \epsilon$.

1. Assume e.g. that y is the birth weight of children in gramme but you want to express it in kilogramme. This means you have to premultiply y by $\frac{1}{1000}$. Show the effect on the estimator b for the general model above when you multiply y by a scalar c. (Hint: Start with the normal equation $X'y = X'Xb$.)

2. Now suppose that X is multiplied by an invertible matrix P ($Z = XP$). What does this mean?

3. Calculate the estimator b under the conditions in point 2. What effect does this transformation has on R^2?
(Hint: Show the effect of XP on the residuals by using the projection matrix M and $e = My$)