

Assignment 4

Problem 1:

Find VC $\text{Cov}[\hat{\beta}, \hat{\beta} - b]$

$$\hat{\beta}_{OLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} y$$

$$\hat{b}_{OLS} = (X' X)^{-1} X' y$$

$$y = X\beta + \epsilon$$

$$\hat{\beta}_{OLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} X \beta + (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \epsilon$$

$$= \beta + (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \epsilon$$

$$\hat{b}_{OLS} = (X' X)^{-1} X' X \beta + (X' X)^{-1} X' \epsilon$$

$$= \beta + (X' X)^{-1} X' \epsilon$$

$$\hat{\beta}_{OLS} - \hat{b}_{OLS} = \beta + (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \epsilon - \beta - (X' X)^{-1} X' \epsilon$$

$$= [(X' \Omega^{-1} X)^{-1} X' \Omega^{-1} - (X' X)^{-1} X'] \epsilon$$

$$E(\hat{\beta}_{OLS}) = \beta \quad \left. \begin{array}{l} \text{both unbiased} \\ \text{as } E[\epsilon | X] = 0 \end{array} \right\}$$

$$E(\hat{b}_{OLS}) = \beta$$

$$E(\hat{\beta}_{OLS} - \hat{b}_{OLS}) = \beta - \beta = 0$$

$$\text{Cov}(\hat{\beta}_{OLS}, \hat{\beta}_{OLS} - b) = E[(\hat{\beta}_{OLS} - \beta)(\hat{\beta}_{OLS} - \beta)']$$

$$= E[(X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \epsilon \epsilon' (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} X - (X' X)^{-1} X']$$

$$= (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \sigma^2 \Omega^{-1} X - (X' X)^{-1} X'$$

$$= (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \sigma^2 \Omega^{-1} X (X' \Omega^{-1} X)^{-1} - X (X' X)^{-1}$$

$$= \sigma^2 (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} \Omega^{-1} X (X' \Omega^{-1} X)^{-1} -$$

$$- \sigma^2 (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} X (X' X)^{-1} =$$

$$- \sigma^2 (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} X (X' \Omega^{-1} X)^{-1} -$$

$$- \sigma^2 (X' \Omega^{-1} X)^{-1} X' X (X' X)^{-1} =$$

$$= \sigma^2 (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} X (X' \Omega^{-1} X)^{-1} - \sigma^2 (X' \Omega^{-1} X)^{-1} X' X (X' X)^{-1} = 0$$

$(AB)' = B'A'$
 $(A+B)' = A'+B'$
 $\Omega = \text{symmetrisch}$

