Climate and glaciers

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ABSTRACT   The reaction of a glacier to fluctuations of
climatic elements and its adjustment to different
climatic zones is analytically formulated. Changes in
air temperature, precipitation and radiation balance are
found to contribute with equal efficiency and probability
to the shift of the equilibrium line. The vertical mass
budget profile is discussed in terms of the relative
importance of climatic variables. The budget gradient,
the elevation of the equilibrium line above the zero
degree level of the free atmosphere, and precipitation,
are the parameters of the mass and energy budget that are
most suitable for a climatic classification of glaciers.

1   INTRODUCTION

The vast ice masses of the polar regions and the Earth's seasonal
snow cover provide a boundary for the atmosphere which profoundly
influences global circulation. As this boundary reacts itself to
atmospheric mass and energy fluxes it demonstrates the truly
interactive nature of the relationship between climate and the
large scale ice masses.

Glaciers of the mid latitudes and tropics, on the other hand,
have comparatively little effect on the atmosphere, their inter-
action with climatic elements is restricted to their immediate
surroundings. The size of an extrapolar glacier is too small for
the establishment of a true feedback to the atmosphere: the scale
typical for synoptic processes is 10^3 km, compared to the size of
larger alpine glaciers of only 10 km. While, thus, the vertical
fluxes of energy and mass from a glacier can contribute only
negligibly to the total exchange between the atmosphere and the
Earth's surface, the storage capacity of glaciers for these
quantities is much larger than that of the atmosphere. A glacier
can store a 100 m column of water compared to 1-2 cm of
precipitable water present in the atmosphere in mid latitudes.
10^7 J will heat a 1-m^2 column of our entire atmosphere or only
about 5 m of ice by 1°C or will melt 0.03 m of ice.

It is understandable that this high storage capacity for mass
and energy will make the glacier respond slowly to the fast
changing state of the atmosphere. A glacier acts as an integra-
tor of meteorological processes in a variety of time scales: the
surface snow cover and the adjustment of the equilibrium line has
a yearly cycle. The time for flow adjustment and size changes
has a scale of 1-10 years (or 100 for larger glaciers), and
finally, 100-1000 years is the order of magnitude of the average
residence time of ice in the glacier.

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These three time scales can be applied to three climatological problems:

(a) the immediate reaction of the glacier surface to climatic variables,

(b) the dynamic adjustment of the glacier to climatic trends or several years' fluctuations,

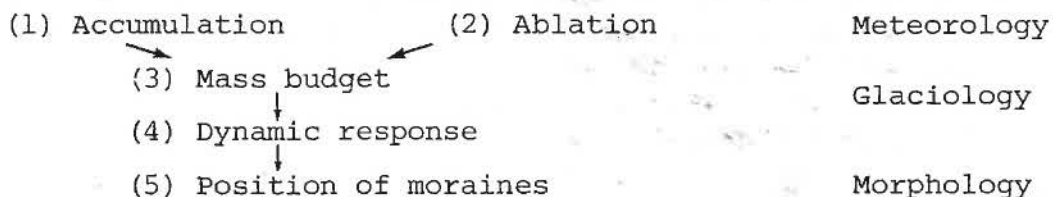(c) the archive of secular changes in atmospheric composition contained in the ice and its air inclusions.

It is the intent of this paper to formulate the first of these problems, the present day response of temperate glaciers to small variations in climatic parameters. In many respects this amounts to using a glacier as a device for measuring atmospheric conditions as integrated over a season or a year.

It is a basic requirement of any measurements that the measuring device does not interfere with the quantity being measured and in this respect we may argue that a glacier is a very suitable tool for the measurement of atmospheric conditions, and that from a study of present day coupling between glaciers and their environment, we should be able to retrieve the information about past climates that is stored in a glacier and in the morphology of its surroundings.

This optimism, however, is damped by the fact that the signal from the immediate vicinity may be of a similar magnitude to that from the atmosphere. In extreme cases, the system will undergo irreversible changes since the glacier's response is faster than that of morphological or botanical adjustments. For instance, a glacier may advance through grassland, but when it retreats its terminus will be covered and surrounded by debris.

The study of the climate-glacier relation is relatively young: 50 years ago the Scandinavian school applied micrometeorology to the glacier surface while the first theories of ice flow were being tested. The development of glacial meteorology has been reviewed by Hoinkes (1964, 1970) and Radok (1978), and a comprehensive list of energy and mass budget studies up to 1966 has been compiled by Paterson (1969).

Fifteen years ago Meier (1965) summarized the state of the art by a chain of processes which combined the climatic and the dynamic responses that lead to the morphological evidence of past variations. From this chain (modified from Meier, 1965):

(1) Accumulation   (2) Ablation   Meteorology

    (3) Mass budget   Glaciology

    (4) Dynamic response

    (5) Position of moraines   Morphology

we can easily identify the fundamental difficulties inherent in the reconstruction of past climates from glacial morphology. Although the dynamic behaviour of a glacier is basically understood (Nye, 1961; Paterson, 1969; Budd & Jenssen, 1975), it is difficult to assess the response for an unmeasured glacier (Budd & Allison, 1975). This problem can be overcome in practice by analogue and by empirical parametrization of glacier flow.

Another fundamental problem is the branch in the chain from

mass budget to either accumulation or ablation: it is strictly impossible to decide without further information, whether a small variation in the net mass budget stems from a small variation in accumulation or in ablation.

The fundamental problem is then, that broad band frequency information flows from (at least) two sources through a low pass filter to be stored in the glacier. The inversion of this flow cannot tell from which source it came, nor can it restore the high frequency part of the information content.

This explains why some shortcuts like the one from glacier length variation to variations in atmospheric temperature and precipitation, or the one from changes in equilibrium line position to changes in atmospheric temperature, must be classified as fundamental mistakes.

2. THE BASIC EQUATIONS LINKING ATMOSPHERIC AND GLACIAL MASS AND ENERGY FLUXES

The principle of continuity requires that vertical fluxes from the atmosphere to the ice surface and from the surface into the ice be exactly equal. Quantities thus transferred are

Energy
Mass
Momentum

The balance of energy fluxes requires no further explanation. The mass flux through the interface incorporates dry air, precipitation, and water vapour. Since snow or ice become impermeable little below the surface, only precipitation and water vapour are of interest, and of the latter, again only that part changing phase at or below the interface. Consequently one of the basic equations describing the atmosphere, viz. the equation of state of water vapour, is reduced to one parameter, the saturation vapour pressure at surface conditions, which appears in the energy and mass balances. Atmospheric momentum is dissipated completely at the interface, at a rate of less than 1 W m^{-2} which need not be considered. While the atmospheric energy balance is coupled to the momentum balance by the temperature advection $\vec{v} \cdot \nabla T$, this effect is negligible compared to vertical exchange in the energy balance of an extrapolar glacier. The glacier's momentum balance can thus be discarded and we are required only to match three fluxes, those of energy, precipitation and water vapour.

The interface energy balance

net radiation + turbulent transfer =   (Atmosphere)
= melting + heating (cooling)   (Glacier)   (1)

In this balance melting means strictly runoff, since that part of meltwater which percolates and refreezes contributes to heating.

The general water balance

precipitation (P) = runoff + evaporation + storage   (2)

For glaciers: storage = mass budget = accumulation - ablation.

Accumulation (c)

$c = P_{solid} + P_{stored} + condensation + drift^{+} + avalanches^{+}$ (3)

Ablation (a)

$a = melting + evaporation + drift^{-} + avalanches^{-} + calving$ (4)

And the mass budget (b) follows from subtraction of equation (4) from (3):

$b = c - a = P_{solid} + P_{stored} - melting + condensation -$

$evaporation + drift + avalanches - calving$ (5)

Let us now consider the simplifications applicable to a temperate glacier with a melting surface. In equation (1) heating becomes zero and that equation reduces to

net radiation (R) + turbulent transfer of latent (V) and
of sensible heat (H) = melting (6)

$R + H + V = \Sigma Q$

where ΣQ is the heat flux available for melting, having dimensions $W\ m^{-2}$.

In equations (3) and (4) the solid precipitation and that part of the liquid precipitation which is stored in the snow cover (by refreezing or capillarity) are approximated by total precipitation P, the error introduced decreasing with altitude. Evaporation and condensation are negligible if their seasonal sum is compared to melting or precipitation. For short periods and in dry climates, evaporation and sublimation can be considerable, as will be discussed later. The net drift (D) is counted positive when it contributes to accumulation. The net accumulation due to avalanches, although of major importance in some mountains, is considered negligible here, and so is calving. Equation (5) then reduces to

$c = P + D$ and $a = M$ (7)

or

$M = P + D - (c - a) = P + D - b$

for a temperate glacier.

The combination (P + D)/P was called snow concentration by Kotlyakov & Krenke (1979). Values of over 2 (Hoinkes, 1957; Kotlyakov, 1973) show that D cannot be neglected on alpine glaciers.

If the heat flow ΣQ acts on the ice for the duration of melting τ, then

$M = \frac{\tau}{L} \Sigma Q$ $(kg\ m^{-2})$ (8)

where L is the latent heat of fusion, 334 kJ kg^{-1}.

The energy balance (equation (6)) and the mass budget (equation (7)) can now be combined with the help of equation (8):

$P + D - (c - a) = \frac{\tau}{L} \Sigma Q$ $(kg\ m^{-2})$ (9)

Accepting the simplifications mentioned above the balance (equation (9)) is valid for every point on the glacier. At the

equilibrium line of a glacier, by definition $c - a = 0$ and equation (9) further reduces to

$$P + D = \frac{\tau}{L} \Sigma Q = c = a \tag{10}$$

The equilibrium line thus seems to be the place for which the atmosphere-ice interaction is easiest to formulate.

From equation (5) it is clear that the left-hand side of equation (10) is total accumulation and the right-hand side total ablation. The heat required for melting is furnished by the radiation balance R and the turbulent flux of sensible heat which can be parameterized as a function α of the difference between the temperature T_a that would prevail at level z in the free atmosphere outside the thermal influence of the glacier, and the surface temperature T_s at z. In the simplest case, which shall be considered here, $T_s = 0°C$ and α is a constant (Kuhn, 1979). At any height z on the glacier equation (9) becomes

$$P(z) + D(z) = c(z) = \frac{\tau}{L} \left[R(z) + \alpha(T_a(z) - T_s) \right] + b(z) \tag{11}$$

and at the height h_o of the equilibrium line in the steady state

$$c(h_o) = \frac{\tau}{L} \left[R(h_o) + \alpha(T_a(h_o) - T_s) \right] \tag{12}$$

While h denotes the height of the equilibrium line under arbitrary conditions, let the subscript o refer to the steady state, or equilibrium state ($b = 0$) of the glacier. We shall next investigate what can be the reasons for the equilibrium line to move from h_o in the steady state to h in a state disturbed by small changes in climatic variables.

We treat here the quantities P, D, R, and T_a of equation (11) as independent variables and note that keeping τ, α, and T_s constant is a simplification that is accepted for the sake of clarity of the present derivation. In the global atmosphere, only the shortwave component of R is an independent variable, it is the forcing function to which the other variables respond. In the time and space scales of a glacier this does not apply rigorously, e.g. the precipitation does not depend on the air temperature above the glacier, but the reader will find examples for the interdependence of R and T_a or R and c at the end of the next section. Again for the sake of clarity these connections shall first be neglected and c, R and T_a will be treated as independent.

3 THE VARIATION OF THE EQUILIBRIUM LINE IN RESPONSE TO CLIMATIC DISTURBANCES

For small variations in the position of the equilibrium line, $\Delta h = h - h_o$, we can express the variables by expanding their steady state values

$$c(h) = c(h_o) + (\partial c/\partial z)\Delta h \tag{13}$$

$$R(h) = R(h_o) + (\partial R/\partial z)\Delta h \tag{14}$$

$$T_a(h) = T_a(h_o) + (\partial T_a/\partial z)\Delta h \tag{15}$$

Let us now consider a small perturbation in these terms, $(\delta c, \delta R, \delta T_a)$ which is a function only of time and not of height. The values at height h now change to

$$c'(h) = c(h) + \delta c \qquad (16)$$

$$R'(h) = R(h) + \delta R \qquad (17)$$

$$T_a'(h) = T_a(h) + \delta T_a \qquad (18)$$

Reacting to this perturbation the equilibrium line will change to an altitude $h = h_o + \Delta h$ where again

$$c'(h) = \frac{\tau}{L} \left[R'(h) + \alpha(T_a'(h) - T_s) \right] \qquad (19)$$

Subtracting equation (12) from (19) with the help of equations (13-18) we find

$$c'(h) - c(h_o) = (\partial c/\partial z) \Delta h + \delta c$$
$$= \frac{\tau}{L} \left[\frac{\partial R}{\partial z} \Delta h + \delta R + \alpha(\frac{\partial T_a}{\partial z} \Delta h + \delta T_a) \right] \qquad (20)$$

If we interpret these perturbations δ as climatic fluctuations then equation (20) shows how the latter can be compensated by altitudinal adjustments of the mass balance terms. In order to appreciate the efficiency of the altitudinal adjustment and the likelihood of changes in the variables, a numerical example is given for the terms in equation (20). The following values which are approximately valid for central alpine conditions are used in the example:

$$\frac{\partial c}{\partial z} = \frac{1 \text{ kg m}^{-2}}{\text{m}} \quad (\hat{=} 1 \text{ mm m}^{-1})$$

$$\tau = 100 \text{ days}$$

$$L = 335 \text{ kJ kg}^{-1}$$

Since the albedo pattern is largely fixed with respect to the equilibrium line rather than to absolute height, and vertical variation of downward short and longwave are small and opposite,

$$\frac{\partial R}{\partial z} = 0$$

$$\alpha = 1.7 \text{ MJ m}^{-2} \text{ day}^{-1} \text{ }^{\circ}C^{-1}$$

$$\frac{\partial T_a}{\partial z} = -0.006 \text{ }^{\circ}C \text{ m}^{-1}$$

Inserting these figures and $\Delta h = 100$ m we find

$$100 + \delta c = (0.3 \times 10^{-3})\delta R - (0.3 \times 10^{-3})(1.7 \times 10^6)(0.6) +$$
$$(0.3 \times 10^{-3})(1.7 \times 10^6)\delta T_a \qquad (21)$$

$$400 = (0.3 \times 10^{-3})\delta R + 500\delta T_a - \delta c \qquad (\text{kg m}^{-2}) \qquad (22)$$

Thus the equilibrium line rises by 100 m if either

$\delta c = -400$ kg m^{-2}: the annual accumulation decreases by
400 kg m^{-2} (23)

$400 = (0.3 \times 10^{-3})\delta R$: the radiation balance increases
by 1.330 MJ m^{-2} day^{-1}, or (24)

$400 = 500\delta T_a$: the free air temperature increases by
0.8°C (25)

Note that the δT_a required to raise the equilibrium line by 100 m

is not equal to the temperature difference corresponding to the adiabatic lapse rate but $\delta T_a > 100 \ (\partial T_a / \partial z)_{adiab}$ since it was specified that accumulation increases with height.

Fluctuations in accumulation of ∓ 400 kg m^{-2}, in radiation balance of ± 1.33 MJ m^{-2} day^{-1}, or in free air temperature of $\pm 0.8°C$ thus have an equivalent effect on the variation of the equilibrium line by ± 100 m. Of these values, δc refers to the accumulation period, while δR and δT_a are averages during the ablation period.

The probability of the occurrence of such fluctuations can be assessed by comparing their magnitude to the long term variance of seasonal means. Such values can conveniently be determined for $\bar{T}$ of the ablation period and for c. Substituting the variance of the precipitation of the accumulation period for the variance of the accumulation may give the wrong picture since the contribution of drift snow to c is of equal magnitude as precipitation and highly variable. Due to the lack of data, an objective assessment of σ_R cannot be attempted here.

The following examples are taken from the vicinity of Hintereisferner (Kuhn et al., 1979):

Station Hintereis 3030 m, 1 May–30 September, 1969–1978:

$$\bar{T} = 0.4°C \qquad \sigma_T = \pm 0.8°C$$

Accumulation-pit Teufelsegg (Hintereisferner, 3070 m),
1 October–31 May, 1966/1967 – 1977/1978 with the exception of
1969/1970:

$$\bar{c} = 1620 \text{ kg m}^{-2} \qquad \sigma_c = 540 \text{ kg m}^{-2} = 38\%$$

Precipitation on Hintereisferner, 2970 m, same period as
accumulation:

$$\bar{P} = 816 \text{ kg m}^{-2} \qquad \sigma_P = 213 \text{ kg m}^{-2} = 26\%$$

Comparing the δ values that are required for a 100 m shift of the equilibrium line to the respective deviations

$$\delta c / \sigma_c = 400/540 = 0.74 \qquad \delta T_a / \sigma_T = 0.8/0.8 = 1.0$$

we find that the corresponding fluctuation is slightly more likely to occur in accumulation than in temperature.

The variation $\delta R = \pm 1.33$ MJ m^{-2} day^{-1} cannot directly be compared to the above values since σ_R is not known. According to Wagner (1979, 1980) the mean values of the radiation fluxes at the equilibrium line of Hintereisferner (2960 m) from 6 July to 30 September 1971 were:

global irradiance, G	20 MJ m^{-2} day^{-1}
shortwave balance, $(1-a)$ G	10 MJ m^{-2} day^{-1}
longwave downward flux, A	23 MJ m^{-2} day^{-1}
longwave upward flux, E	27 MJ m^{-2} day^{-1}
longwave balance, A-E	-4 MJ m^{-2} day^{-1}
overall radiation balance, R	6 MJ m^{-2} day^{-1}

where a denotes albedo in this context. When considering the likelihood of a change $\delta R = \pm 1.3$ MJ day^{-1} m^{-2}, we have to keep in mind that it can be caused by any of the components of the radiation balance

$$R = (1-a)\ G + A - E \tag{26}$$

$$\delta R = (1-a)\ \delta G - G\delta a + \delta A - \delta E \tag{27}$$

With $a = 0.5$, δR becomes 1.3 MJ m^{-2} day^{-1} if either

$\delta G = 2.6$ MJ m^{-2} day^{-1}

$\delta a = -0.07$

$\delta A = +1.3$ MJ m^{-2} day^{-1}, or

$\delta E = -1.3$ MJ m^{-2} day^{-1}

Although to deduce the probability of the occurrence of these fluctuations is still a matter of personal judgement it seems safe to summarize the discussion by the conclusion that $\delta c/\sigma_c$ and $\delta T_a/\sigma_T$ are approximately equal and that $\delta R/\sigma_R$ is of the same order of magnitude. By no means can changes in accumulation and radiation be ignored when palaeotemperatures are derived from past snow lines or moraines.

In the above considerations, c, T_a, and R have been treated as independent variables. In reality, δc, δT_a, δR (δG, δa, δA, δE) will act simultaneously and are mutually dependent.

For instance, increased precipitation can increase the surface albedo as well as accumulation. This effect is difficult to describe quantitatively since the temporal distribution of the snowfalls is of prime importance, but it can be put qualitatively that, because of continuously high albedo, a low, steady rate of snowfall contributes more to the mass balance than the same mass deposited at one time.

Increased free air temperature does not only raise the turbulent heat transfer but also atmospheric downward radiation, A. By introducing an effective emissivity ε^*, A can be linked to the free air temperature in a rough approximation (Kuhn, 1979)

$$A = \varepsilon^* \sigma T_a^4 \tag{28}$$

where $\sigma = 5.67 \times 10^{-8}$ W m^{-2} °C^{-4}. The dependence of A(T_a) can be linearized

$$\frac{\partial A}{\partial T_a} = \varepsilon^*\ 4\ \sigma\ T_a^3 = \varepsilon^*\ 0.42 \text{ MJ m}^{-2} \text{ day}^{-1} \text{ °C}^{-1} \text{ at } T_a = 0°C \tag{29}$$

Using an effective emissivity of 0.7, which seems to be representative for alpine conditions, we thus find that a warming $\delta T_a = 1°C$ causes increased atmospheric downward radiation $\delta A = 0.3$ MJ m^{-2} day^{-1} and increased turbulent heat transfer to a melting glacier $\delta H = \delta(\alpha T_a) = 1.7$ MJ m^{-2} day^{-1}.

Going back to equation (22) with $\delta R = \delta A = 0.3 \times 10^6\ \delta T_a$

$$400 = 0.3 \times 10^{-3}\ (0.3 \times 10^6\ \delta T_a) + 500\ \delta T_a \rightarrow \delta T_a = 0.7°C$$

we find that only 0.7°C warming is required to raise the equilibrium line over 100 m by δT_a alone.

4   LOCAL CHANGES OF THE BUDGET COMPONENTS

Of the individual components of the mass budget, D is mainly influenced by topography and prevailing wind, also by T_a since it diminishes with wet snowfalls; R is determined by aspect and slope as well as albedo and T_a or cloudiness; and little is known

about the change of the turbulent transfer with topography. The marginal regions of a glacier gain additional energy from the snow free surroundings by longwave radiation and heat advection.

These effects combined cause the high spatial variability of the specific net budget b on a glacier (see for instance Hoinkes, 1970, Fig. 3). It certainly does not help our physical understanding, but it is customary and in many respects necessary to average b over areal increments dS bounded by two isohypses, and to ignore differences in the length of the budget year of individual regions on one glacier.

This yields a budget profile $b(z)$ (kg m^{-2}) whose gradient reflects the predominance of certain budget components, since P, D, R and H have characteristic profiles of their own.

Table 1 and Fig. 1 show averages of budget profiles grouped in three classes, representing negative (subscript n), nearly balanced (o) and positive (p) mean specific budgets. It is obvious from Fig. 1 that the individual profiles have a similar shape and appear shifted along the abscissa. The amount of this shift

$$b_i(z) = b(z) - b_o(z) \tag{30}$$

Table 1 Mean altitudinal profiles of specific mass budget, standard deviations, and budget gradients at Hintereisferner, 1952/1953–1958/1959 and 1961/1962–1977/1978

ΔZ (m)	Class n: $\bar{b} < -250$ kgm^{-2}, 13 cases, $\bar{b}_n = -646$ kgm^{-2}			Class o: $-250 < \bar{b} < 250$ kgm^{-2}, 6 cases, $\bar{b}_o = -2$ kgm^{-2}			Class p: $\bar{b} > 250$ kgm^{-2}, 5 cases, $\bar{b}_p = 555$ kgm^{-2}		
	b (kgm^{-2})	σ_b (kgm^{-2})	F (kgm^{-2}m^{-1})	b (kgm^{-2})	σ_b (kgm^{-2})	F (kgm^{-2}m^{-1})	b (kgm^{-2})	σ_b (kgm^{-2})	F (kgm^{-2}m^{-1})
3600–3700	+150	150	−0.5	+370	350	−1.1	+840	510	−2.6
3500–3600	+200	130	−1.0	+490	300	−2.4	+1100	450	−2.7
3400–3500	+300	200	−2.2	+730	150	−2.6	+1270	440	−3.0
3300–3400	+520	210	1.3	+990	170	1.4	+1570	370	1.1
3200–3300	+390	210	0.7	+850	200	0.5	+1460	340	0.4
3100–3200	+320	240	2.5	+800	180	2.6	+1420	330	2.8
3000–3100	+70	300	5.3	+540	110	3.6	+1140	300	5.3
2900–3000	−460	450	7.1	+180	120	4.3	+610	210	4.7
2800–2900	−1170	530	10.0	−250	140	9.8	+140	200	7.7
2700–2800	−2170	500	9.2	−1230	180	9.7	−630	250	12.4
2600–2700	−3090	490	10.1	−2210	300	11.0	−1870	400	12.8
2500–2600	−4100	550	12.9	−3300	300	12.2	−3050	440	12.0
2400–2500	−5400	640		−4520	240		−4250	620	

ΔZ (m)	Mean budget imbalances for classes n and p			
	$b_i = b_n - b_o$ (kgm^{-2})	$\sigma(b_n)/|b_i(n)|$	$b_i = b_p - b_o$ (kgm^{-2})	$\sigma(b_p)/|b_i(p)|$
3600–3700	−220	0.68	470	1.09
3500–3600	−290	0.45	610	0.74
3400–3500	−430	0.47	540	0.81
3300–3400	−470	0.45	580	0.64
3200–3300	−460	0.46	610	0.56
3100–3200	−480	0.50	620	0.53
3000–3100	−470	0.64	600	0.50
2900–3000	−640	0.70	430	0.49
2800–2900	−920	0.58	390	0.51
2700–2800	−940	0.53	600	0.42
2600–2700	−880	0.56	340	1.18
2500–2600	−800	0.70	250	1.76
2400–2500	−880	0.74	270	2.30
	$\bar{b}_i = -600$		$\bar{b}_i = 490$	

is called the budget imbalance (Meier, 1962). The means of b_i and the corresponding class mean specific budgets $\bar{b}$ are not identical, since the mean specific budget takes into account the area profile of the glacier.

$$\bar{b} = (1/S)\int b\ dS \qquad \text{and} \qquad \bar{b}_O = (1/S)\int b_O dS = 0 \qquad (31)$$

The inspection of the two $|b_i|$ profiles reveals a fact of climatological relevance: in positive years, $|b_i|$ is larger in the accumulation area, and in negative years it is larger in the ablation area. This can be interpreted to imply that there is, under present climatic conditions, a minimum amount of ablation on the tongue of the glacier, and while the first small deviation from $\bar{b}_O$ can be accomplished either by an increase in accumulation or a decrease in ablation, an extremely positive budget can be accomplished only be excessive accumulation. Likewise, there persists a minimum amount of accumulation in the accumulation area and an extremely negative budget is decided by excessive ablation on the tongue.

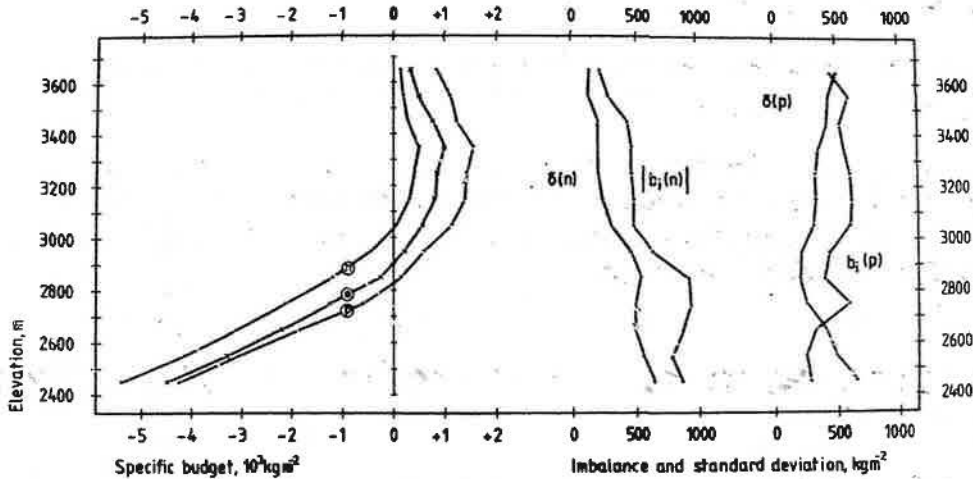In Fig. 1 the standard deviations of b(z) for classes n and p

Fig. 1   Specific budget profiles of negative (n), balanced (o) and positive (p) class means, corresponding standard deviations and specific imbalances for Hintereisferner, 1952/1953-1958/1959 and 1961/1962-1977/1978. The values are listed in Table 1.
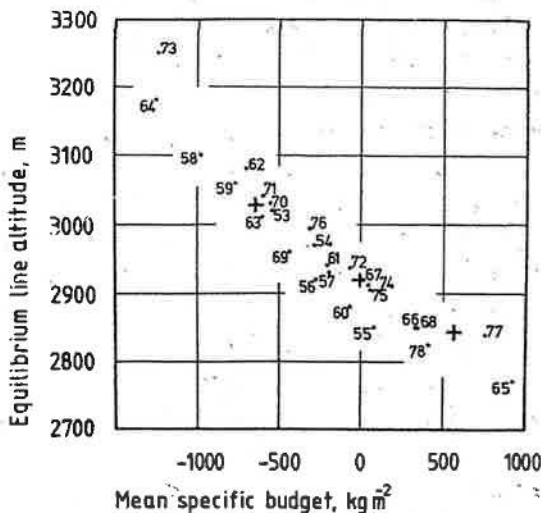

Fig. 2   Mean specific budget, equilibrium line altitude and mean budget gradient at the equilibrium line. The crosses are class mean values for the negative, balanced and positive budgets listed in Table 1.

are entered in the diagram with the budget imbalances. The
behaviour of σ_b agrees with what has been found in $|b_i|$: in a
negative year $\sigma_b(z)$ decreases upward, in a positive year it
increases, and there is a tendency for the ratio $\sigma_b/|b_i|$ to stay
constant over the profile (Table 1). That $\sigma_b/|b_i|$ reaches
maximum values at the tongue in both the p and the n class may
be an indication that the specific budget of the tongue depends
very little on the fate of the rest of the glacier. The area
equally sensitive to both positive and negative deviations from
the steady state budget is thus the region of the equilibrium
line.

The mean height of the equilibrium line in practice is deter-
mined graphically from the budget profile. In Fig. 1 the
profile crosses the zero-line at 2830, 2910 and 3030 m, respect-
ively. From this figure we can establish a simple relationship
of $\bar{b}$ with h, provided we accept the empirical identity $b_i(h) = \bar{b}_i$
which was found to hold to a good approximation (Meier, 1962;
p. 259; Hoinkes, 1970; p. 67).

It follows from equations (30) and (31) that

$$\bar{b} = (1/S)\int b_i \, dS \qquad \text{or} \quad \bar{b} = \bar{b}_i \approx b_i(h) \tag{32}$$

and from the geometry of Fig. 1

$$\bar{b} \approx b_i(h) = -(\partial b/\partial z)_h \cdot \Delta h \tag{33}$$

the budget gradient at the equilibrium line being the factor of
proportionality between mean specific budget and the height
change Δh of the equilibrium line. Figure 2 shows this relation-
ship for Hintereisferner.

5 THE VERTICAL BUDGET GRADIENT (ACTIVITY INDEX) IN VARIOUS CLIMATIC ZONES

The budget gradient $F = \partial b/\partial z$ was early recognized as an
indicator of climatic conditions. Shumskii called it energy of
glaciation in 1947 (Shumskii, 1964; p. 442). Meier (1961)
referred to it as activity index which comes more to the point
since a glacier needs more activity to transport high net
accumulation from the firn basin to the ablation area while
maintaining a steady state surface profile, and obviously for a
given glacier size the absolute values of c and a increase with
$\partial b/\partial z$, provided an equilibrium line exists on the glacier.

Haefeli (1962) found a typical value of 10 kg m^{-2} m^{-1} per year
for what he called ablation gradient in the temperate zones,
values decreasing to 3 kg m^{-2} m^{-1} per year in polar zones. While
Haefeli's postulation of $F > 10$ kg m^{-2} m^{-1} a^{-1} for tropical
regions was later confirmed (e.g. Allison: 25 kg m^{-2} m^{-1} a^{-1} on
Meren Glacier, New Guinea; 1975, 1976), his concept of a mere
latitude effect was oversimplifying, as can be seen from a
compilation by Budd & Allison (1975).

From the principles of Section 2 we can determine the relative
contributions of the budget terms to $\partial b/\partial z$ and we can conseque-
ntly predict its values for different climatic regions provided
we are in possession of climatic records of that region. In

order to do so we first eliminate the restrictive assumption of a
constant duration τ of the ablation season by taking the time
derivative of equation (11) and inserting $\dot{b} = db/dt$ and $\dot{c} = dc/dt$

$$\dot{b} = \dot{c} - \frac{R}{L} - \frac{\alpha}{L} (T_a - T_s) \qquad (\text{kg m}^{-2} \text{ day}^{-1}) \qquad\qquad (34)$$

the height derivative of which is the budget gradient per unit
time (per day):

$$\partial\dot{b}/\partial z = \partial\dot{c}/\partial z - (1/L)\, \partial R/\partial z - (\alpha/L)\, \partial(T_a - T_s)/\partial z \qquad (35)$$

Equation (35) can now be discussed in terms of climatic
variables. The first effect that becomes obvious is

$$F = \partial b/\partial z = \int_{t=0}^{t=\tau} \partial\dot{b}/\partial z \; dt \qquad (36)$$

i.e. the budget gradient increases with time. This is one of
the reasons why F is so large in a tropical climate where
ablation and accumulation are active on every day of the year,
while τ at the equilibrium line is about 100 days in the Alps and
decreases with increasing latitude.

Since in general the free air temperature decrease is less
than dry adiabatic $(\partial T_a/\partial z > -0.01°C \text{ m}^{-1})$ there is an upper limit
for the contribution of turbulent transfer to F. Again assuming
$T_s = $ constant $= 0°C$ during ablation we find this limit

$$-(\alpha/L)(\partial T_a/\partial z) \leq 0.051 \text{ kg m}^{-2} \text{ m}^{-1} \text{ day}^{-1} = 18.6 \text{ kg m}^{-2} \text{ a}^{-1} \qquad (37)$$

or 14 kg m^{-2} m^{-1} a^{-1} with a lapse rate of 0.007°C m^{-1}. These
values are exceeded by glaciers with large accumulation and long
ablation periods, like tropical or very maritime glaciers:
on New Guinea, Meren Glacier has $\tau = 365$ days and $F = 25$ kg m^{-2}
m^{-1} a^{-1} (Allison, 1975, 1976) and in the Southern Ocean, Glacier
Ampère (49°30'S) and Tasman Glacier (43°S) both have balance
gradients of 20 kg m^{-2} m^{-1} a^{-1} (Vallon, 1978).

The yearly accumulation changes only slowly with height in a
region with appreciable seasonal march of air temperature. If a
glacier extends through a height interval large enough that rain
falls on its lower parts while precipitation is solid in the
upper parts, the accumulation gradient becomes larger than the
precipitation gradient. This situation is observed in the Alps
in summer: on Hintereisferner a crude estimate for the ablation
period June–September is 500 kg m^{-2} which falls as rain on the
terminus, 2500 m, and as snow in the névé, 3500 m, thus yielding
an accumulation gradient of 0.5 kg m^{-2} m^{-1} in 120 days or 0.004
kg m^{-2} m^{-1} day^{-1} which is one order of magnitude less than the
gradient due to turbulent transfer.

On Meren Glacier these conditions prevail throughout the year.
From Allison & Bennett (1976) we could expect an annual precipi-
tation of 3000 kg m^{-2} to fall mostly as sleet or wet snow in the
accumulation area (4700 m) and mostly as rain in the ablation
area (4400 m) which means an accumulation gradient of 10 kg m^{-2}
m^{-1} a^{-1}, almost equal to the ablation gradient due to turbulent
transfer.

Apart from the conditions just described, the vertical
variation of snow drift must be taken into account on glaciers of
all climatic zones. Since it is mostly determined by topography

it is difficult to describe analytically and is not further discussed. It seems, however, worthwhile to remember, that snow drift is responsible for the shape of the uppermost part of the budget profile.

We now turn to the remaining budget term in equation (35), the gradient due to radiation. For its evaluation, the height derivative of equation (26) is taken:

$$\frac{\partial R}{\partial z} = (1-a)\ \frac{\partial G}{\partial z} - G\ \frac{\partial a}{\partial z} + \frac{\partial A}{\partial z} - \frac{\partial E}{\partial z} \tag{38}$$

Of the right-hand terms, $\partial E/\partial z = 0$ for a melting surface, $\partial A/\partial z$ depends partly on temperature and has values of -2 kJ m^{-2} m^{-1} day^{-1} under alpine conditions, while $\partial G/\partial z$ reaches $+2.5$ kJ m^{-2} m^{-1} day^{-1} with an average global radiation of 20 MJ m^{-2} day^{-1} (Kuhn, 1979). The albedo gradient is concentrated to the region below the equilibrium line where it may reach values of about 10^{-3} m^{-1} at an average a of 0.5. Inserting these examples in equation (38) and dividing both sides by $-L$, we find

$$-(1/L)(\partial R/\partial z) = -0.004 + 0.060 + 0.006 \quad (\text{kg m}^{-2}\text{m}^{-1}\text{day}^{-1}) \tag{39}$$

where the dominance of the term containing the albedo gradient is obvious. In a region with rapid change of albedo, the contribution of the radiation term to the budget gradient may thus exceed that of the turbulent transfer term.

Of the four factors affecting the annual budget gradient, (a) length of ablation period, (b) amount of precipitation fallen around 0°C, (c) vertical temperature gradient and (d) rapid albedo changes, none can explicitly be related to latitude in the sense that global radiation depends on latitude.

However, it so happens that a polar, continental glacier (a) has a short ablation period, as is typical for the annual march of temperature in polar regions, (b) has a short or missing period during which rain may fall, and frequently, but not necessarily, (c) has little debris cover and therefore only minor albedo changes in the ablation area.

Meier & Post (1962, p.69) clearly state that in general, the activity index and the equilibrium line altitude reflect the continentality of the climate. They present a convincing example which is contained in Table 2 and show that the influence of continentality on the glacier regime deserves further attention.

6 MARITIME AND CONTINENTAL GLACIERS

If we concentrate on annual precipitation and cloudiness as criteria of the distinction between maritime and continental climate we can apply the reasoning given in the third section with due regard to the complete physical separation of the two glaciers. When in Section 3 the equilibrium line moved from height h_0 to h the proportional contributions of the budget terms at h were related to those at h_0 by the vertical gradients of the individual components (equations (13)-(15)).

For large horizontal distances and large values of Δh the simple linearization is no longer appropriate except, with caution for T_a, and consequently the terms containing $\partial c/\partial z$ and

Table 2 Comparison of a maritime and a moderately continental glacier in the northwestern US. Values from Meier & Post (1962) and Court (1974)

	South Cascade Washington, 48°N		Dinwoody Wyoming, 43°N
h	2000 m	$\Delta h = 1690$ m	3690 m
$\partial b/\partial z$	17 kg^{-2} m^{-1}		8 kg^{-2} m^{-1}
Estimate of c	3500 kg m^{-2}	$\delta c = -2500$ kg m^{-2}	1000 kg m^{-2}
Nearest climatic station	Seattle		Landers
elevation	4 m		1690 m
$\overline{T}$ June-September			
at actual height	17.4°C		18.4°C
at 1690 m (extrapolated with			
−6.5°C km^{-1})	6.4°C	$\delta T_a = 12.0$°C	18.4°C
Global radiation June-September	2.2 GJ m^{-2}		3.0 GJ m^{-2}
$\delta R = (1 - 0.65)\,\delta G$		$\delta R = 280$ MJ m^{-2}	

$\partial R/\partial z$ in equation (20) are dropped with δc and δR now referring to the respective equilibrium lines, whereas δT_a is the horizontal difference. For our purpose of elaborating the peculiarities of continental glaciers we need to remember that one of the simplifications of Section 2 was the disregard of evaporation. As evaporation may be of considerable importance in continental climate we shall introduce the term δV which represents the heat flux due to evaporation but also incorporates errors of computation. Equation (20) is now modified:

$$\delta c = (\tau/L)\left[\delta R + \alpha(\partial T_a/\partial z)\,\Delta h + \alpha\delta T_a + \delta V\right] \tag{40}$$

Since this equation can be solved for only one variable we shall choose δV for the present purpose and use additional information from Table 2 for the other variables.

The solution of equation (40) then yields $V = -10.7$ MJ m^{-2} day^{-1} or -1300 MJ m^{-2} which is equivalent to a mass loss due to sublimation and evaporation of 3.8 kg m^{-2} day^{-1} or 460 kg m^{-2} in the period June-September.

This energy loss is nearly equal to the gain from the short-wave radiation balance (assuming an albedo of 0.65) and is larger than the gain from turbulent transfer of sensible heat H. The relationship of the magnitudes of the budget terms is typical for continental climates, with high solar radiation and dry air.

Given a certain surplus of the energy budget (R+H), of the remaining terms V has priority over M. The vapour pressure gradient $\partial e/\partial z < 0$ will determine the demand, V, of sublimation and evaporation and only if additional energy is available will the ice be heated or melted. In other words, R and H are forcing functions to which V responds, and only if V has reached a steady state, can any remaining available heat be used for melting. In the special case when this is fulfilled only during the hours around noon, and only for surfaces nearly perpendicular to the incident solar rays, ice and snow penitentes will form (Kotlyakov & Lebedeva, 1975). It is well to be aware that this condition is independent of latitude and can be met even in polar regions.

There is one point that needs to be spelled out clearly: it

is not high altitude of the equilibrium line that indicates
continentality as Meier & Post (1962) claimed but rather the
elevation of the equilibrium line above the zero degree level,
$z = y$, of the free atmosphere. Above y, $\Delta T < 0$ and the snow or
ice surface gives off heat to the atmosphere, at $z < y$, $\Delta T > 0$
and the surface receives sensible heat.

In the case of South Cascade and Dinwoody glaciers, the terms
of equation (40) involving vertical and horizontal temperature
differences were of opposite sign and nearly equal. This is why
on both glaciers the summer (June–September) temperature at the
equilibrium line is equal at about 5°C and Dinwoody Glacier may
be called only moderately continental.

Consider now the case where, all other conditions remaining
unchanged, accumulation c will change by a large $\delta c < 0$, which
would be a sign of increased dryness or continentality. The
equilibrium line will then retreat to a higher altitude, where
$H = \alpha(T_a - T_s)$ decreases to compensate for δc. At first, while
$T_s = 0°C$, $(T_a - T_s)$ will decrease linearly with altitude, but at
some height above y chances are that the mean value of T_s
becomes less than zero and H may become independent of altitude.

This fact has two climatological implications: (a) with the
retreat of the equilibrium line above the zero degree level of
the atmosphere ($h > y$) during the ablation period, exceptional
dryness is indicated, and (b) under these conditions the mean
value of $\partial H/\partial z$ and therefore $\partial b/\partial z$ was associated with low total
mass exchange. This is now confirmed by the connection of $\partial b/\partial z$
with total accumulation.

If we accept the height of the equilibrium line above the zero
degree level of the atmosphere during the ablation season ($\Delta y =$
$h - y$) as a parameter of the turbulent flux of sensible heat H,
and annual precipitation as an indicator of accumulation c we can
qualitatively divide climatic zones by the sign of Δy: with
abundant precipitation, $\Delta y < 0$ and in dry zones $\Delta y > 0$.

There could hardly be any better example than the latitudinal
variation of Δy in the Andes of South America. Figure 3, which
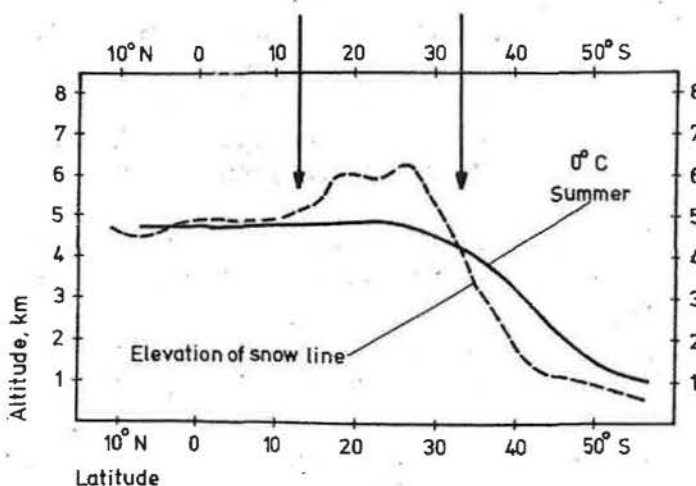

Fig. 3 Elevation of snow line and zero degree level of the free atmosphere in summer
over the Andes. The coincidence of the two lines in Peru indicates moderate precipitation,
the snow line climbs above 0°C in the dry subtropics from 14 to 33°S and falls below
0°C in the humid climate south of 33°S. Simplified from Schwerdtfeger (1976).

was simplified from Schwerdtfeger (1976), shows $\Delta y = 0$ in
the Peruvian Andes. The desert conditions of the Puna de
Atacama in the subtropical high pressure zone are reflected
by $\Delta y > 0$ with values in excess of 1 km. South of 33°S the
trend reverses, and in southern Chile, where annual
precipitation exceeds 3000 kg m^{-2}, Δy becomes negative
and glaciers soon reach down to sea level. This latitudinal
profile was described in detail in an earlier paper (Kuhn,
1980).

7 CONCLUSION

The reaction of a glacier to small climatic variations can be
described by a combination of energy and mass budget. The
behaviour of the equilibrium line is predictable from accumula-
tion, radiation budget and air temperature and their vertical
gradients. A variation of winter accumulation by 400 kg m^{-2}, of
summer free air temperature by 0.8°C, or of mean summer radiation
balance by 1.3 MJ m^{-2} day^{-1} will individually change the equili-
brium line altitude by 100 m under alpine conditions. These
variations are nearly equal to the standard deviation of long
term yearly fluctuations of the respective variables. As they
are not mutually independent it is most likely that a combination
of the three will occur.

The vertical profile of the mass budget shows little variation
from its equilibrium shape when positive or negative budget
situations are inspected. Small as it is, this variation is
systematic and shows the importance of the glacier tongue for
negative budgets while the budget at the tongue does not vary
much from balanced to positive years.

The vertical budget gradient reflects the relative weight of
the individual budget components, as well as the total mass
exchange, and is therefore a very suitable climatic index. It
has maximum values of 25 kg m^{-2} m^{-1} a^{-1} in tropical climate and
decreases proportional to the duration of the ablation season,
reaching minimum values in dry climates, where the mass budget
is in equilibrium high above the zero degree level of the free
atmosphere during the ablation season.

For a given glacier, climatic fluctuations can best be traced
from the observations of the varying equilibrium line altitude.
For a comparison of glacier regimes in different climatic zones,
the elevation of the equilibrium line with respect to the zero
degree level of the free atmosphere, and the budget gradient or
annual accumulation, are the most suitable criteria.

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This paper is dedicated to the memory of my teacher H. C. Hoinkes, 1916-1975.