Hydrological Aspects of Alpine and High Mountain Areas (Proceedings of the Exeter Symposium, July 1982). IAHS Publ. no. 138.

A glacier discharge model based on results from field studies of energy balance, water storage and flow

D. BAKER, H. ESCHER-VETTER, H. MOSER, H. OERTER
GSF-Institut für Radiohydrometrie, Ingolstädter Landstrasse 1, D-8042, Neuherberg, Germany
O. REINWARTH
Kommission für Glaziologie der Bayerischen Akademie der Wissenschaften, Marstallplatz 8, D-8000 München 22, Germany

ABSTRACT In this paper, a discharge model is described which calculates the runoff of the glacier Vernagtferner (9.3 km^2, Oetztal Alps, Austria), using the meltwater production on its surface as input data. The runoff from Vernagtferner is recorded at the gauging station "Pegelstation Vernagtbach" (2640 m a.m.s.l.), where meteorological measurements are also made. Based on these measurements, the meltwater production for any point of the glacier surface is calculated with the aid of an energy balance model. This serves as input for the discharge model, which consists of three parallel linear reservoirs corresponding to runoff from three different areas of the glacier. A small constant fourth component is added to this to allow for groundwater. For the ablation period of 1979, the hourly mean values of the model output are in good agreement with the recorded runoff.

INTRODUCTION

Computations of glacier discharge require an understanding of two separate processes: meltwater production on the glacier surface and the drainage process within the glacier. One possibility for computing meltwater production is the so-called degree-day method based on temperature measurements. This method yields rather good results for periods of several days or weeks, but cannot compute hourly runoff values, the goal of the model to be described here. For this reason another method was chosen, namely computation of the energy balance on the glacier surface.

Several attempts have been made in the past to describe the meltwater flow within a glacier. At the symposium on Hydrology of glaciers at Cambridge in 1969 for example Campbell & Rasmussen (1973) and Derikx (1973) lectured on models describing the whole glacier as a porous medium, and Lang (1973) tried a regression analysis between discharge and meteorological data, obtaining regression coefficients for further calculations. Later, Nilsson & Sundblad (1975) proposed a model consisting of a system of reservoirs. They tried to determine the model parameters as well as the meltwater input by statistical methods. In this paper we also present a reservoir model, but the model parameters and in particular the meltwater input will be determined by a direct analysis of the drainage and melting processes within and on the glacier.

In 1973 the gauging station "Pegelstation Vernagtbach" (altitude 2640 m a.m.s.l.) in the Oetztal Alps, Austria, was constructed at the glacial runoff stream from the glacier Vernagtferner (lat. 46° 52'N, long. 10° 49'E) (Bergmann & Reinwarth, 1976). At a distance of about 1200 m from the glacier snout, the runoff is recorded for an 11.44 km^2 drainage basin, which is 81% glacierized by Vernagtferner (9.30 km^2). The glacier lies between the elevations 2800 m and 3628 m a.m.s.l. Its mean altitude is 3125 m. The recording runs continuously from May until October every year. The discharge data for the ablation periods in the years 1974-1980 are compiled by Oerter (1981a). The mean runoff for the ablation periods (May to September) for this time is 0.970 m^3 s^{-1}. The mean discharge for the later-discussed ablation period of 1979 was 1.16 m^3 s^{-1} (101 l s^{-1} km^{-2}), higher than this average. At the same site there is also a meteorological station which records the data necessary for the calculation of the energy balance and thus of the meltwater production on Vernagtferner. These data serve as input for a deterministic discharge model, which is a mathematical analogue of the storage and discharge behaviour within the glacier. The output of the model is then compared with the measured runoff from the gauging station.

CALCULATION OF MELTWATER PRODUCTION

The meteorological data measured at the Vernagtbach gauging station are solar radiation, longwave radiation, air temperature, relative humidity of the air, wind direction and velocity, and precipitation. Another important piece of data is the albedo of the glacier surface. As it is difficult to record this quantity continuously during the ablation period, another method was applied in this study: once each day, a large part of the glacier was photographed by an automatic camera, thus recording the distribution of newly fallen snow and of firn and ice. With typical albedo values - snow 80%, firn 60%, ice 40% - the shortwave radiation balance is calculated for each point of the glacier surface, which is represented by a digital terrain model. In doing this, the solar radiation measurements at the Vernagtbach gauging station are adapted to the glacier surface with the aid of a radiation distribution model. Wind velocity, air temperature, and relative humidity, which are needed to calculate the sensible and latent heat fluxes are also calculated for the altitude of each grid point of the terrain model. The energy balance model, developed by Escher-Vetter (1980), then makes it possible to calculate the whole energy budget of any point of the glacier surface and thus the meltwater production.

During May 1979, most of the melting took place in the region between the glacier snout and the gauging station, making it difficult to separate glacier discharge from the total amount of water recorded at the gauging station. For this reason the comparison between meltwater production and runoff starts only when this region is mostly free of snow. This was approximately the case at the end of May; therefore 1 June was used as the starting point. From this time it can also be assumed that, on the whole, the glacier is at the melting point, an important assumption for the energy balance calculations.

The four months from June to September started with melting conditions at the beginning and the end of June, interrupted by a longer period without melting. From 9 July to 11 August there was a long, undisturbed period, followed by several snowfalls during August which inhibited any melting. The summer ended with another set of days with melting conditions from 30 August to 21 September. During the entire ablation period the energy balance was calculated for the ice region and six additional sections of the glacier surface between distinct altitude contour lines. In this way it was possible to determine, for any section of the glacier, how much meltwater was produced. This is a basic requirement for the modelling of the glacier discharge from several reservoirs, where the meltwater of the corresponding regions are taken as input (see the following section).

Before discussing the model output, we consider some typical values of energy balance terms. For 13-26 July 1979, the daily averages of those terms, as well as the resulting melting energy S (in W m^{-2}) and the calculated meltwater production S' (in m^3 s^{-1}) are plotted in Fig.1. Except for 14 and 24 July, melting took place on the whole glacier. In most cases it is the whole glacier where melting occurs. This is true for 70% of all days in the entire ablation period 1979. For the remaining days, only parts of the glacier lie in the region where melting takes place.

Melting is only possible when the sum of the following energy balance terms is positive (see Fig.1): the shortwave radiation

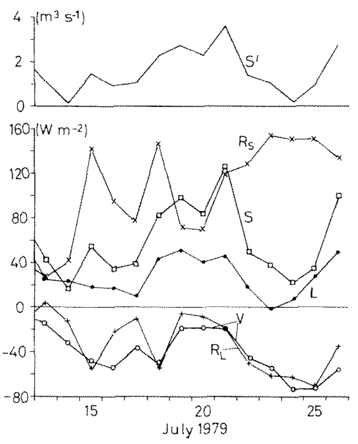

FIG.1 Daily mean values of the components of the energy balance (shortwave radiation balance R_S, longwave radiation balance R_L, sensible and latent heat fluxes L and V), the resulting melting energy S and the meltwater production S' in the time 13-26 July 1979. On 14 and 24 July melting occurs on the glacier only at elevations below 3100 m a.m.s.l., on the other days melting occurs on the whole glacier.

balance (R_S), the longwave radiation balance (R_L), the sensible (L) and the latent (V) heat fluxes. R_S is nearly always the highest positive value of all the terms, just as R_L is with one exception always negative. The latent heat flux V is always negative, which means that energy is consumed by evaporation. On the other hand, the sensible heat flux is generally a heat source during this period. It is only negative on 23 July. If one considers the resulting melting energy of 21 July, for example, it can be seen, that a large R_S and a high value of L are diminished by relatively small quantities of R_L and V, resulting in the highest melting value S of this period. On the other hand, the day with the highest value of R_S, 23 July, gives only a relatively small melting energy, as it is balanced by negative values of all the other terms.

The topmost curve in Fig.1 shows the daily averages of meltwater production S' in $m^3\ s^{-1}$. This can easily be calculated out of the energy balance and the average area. Due to space problems it is not possible to show the hourly values of the energy balance terms for the whole season. However, Fig.2 shows the hourly fluctuations of the energy balance terms for two days: 29 July shows the highest meltwater production in 1979, whereas on 19 August there was no melting at all. As already remarked in the discussion of Fig.1, one sees that the shortwave radiation balance is the largest positive contribution to the energy budget.

DESCRIPTION OF THE DISCHARGE MODEL

For the discharge model we divide the glacier into three different areas, each of which displays its own characteristic storage and discharge behaviour: the ice region on the tongue of the glacier, the firn area next to the ice, and the snow in the upper region of the glacier. Each area is represented in the model by a so-called "linear reservoir". A linear reservoir can be thought of as a container with water running slowly out of a hole in its bottom. The linearity specifies that, at any time t, the runoff from the bottom of the container, Q(t) (in $m^3\ h^{-1}$), is proportional to the volume of water in the container, V(t) (in m^3):

$$V(t) = k\,Q(t) \tag{1}$$

where the storage constant k has the dimensions of time, being measured in hours and is a measure of how big the hole in the bottom of the container is. If we also allow water to flow into the top of the container at a rate R(t) (in $m^3\ h^{-1}$), then the time rate of change of volume must satisfy

$$\frac{dV}{dt} = R - Q \tag{2}$$

Substituting equation (1) into equation (2) one gets

$$k\frac{dQ}{dt} = R - Q \tag{3}$$

which has as a general solution

$$Q(t) = \int_0^t \frac{R(\tau)}{k} e^{(\tau-t)/k} d\tau + Q(0)e^{-t/k} \tag{4}$$

Thus when the water input function R(t), the storage constant k, and the runoff Q(0) at time t = 0, are given, the runoff Q(t) at any time t is uniquely determined.

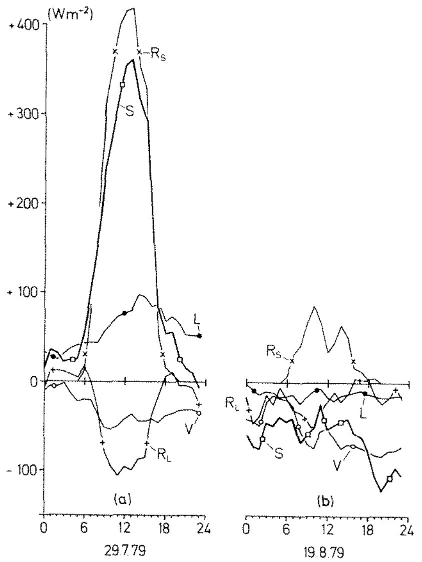

FIG.2 Hourly fluctuations of the energy balance terms for the whole glacier area on two different days: highest meltwater production in 1979 (a), and no meltwater production at all (b).

Finally we remark that, if we have the time-independent equilibrium condition Q = R = constant, then k is the time it takes for water entering the top of the reservoir to flow out of the bottom. In this way we can think of k as an estimate of the time it takes for water to pass through the reservoir (the accuracy of this estimate depends on how close to constant the volume function V(t) is).

This fact can be used to determine the storage constants for the linear reservoirs corresponding to the different areas of the glacier. For Q_1, the model runoff from the ice area, we set k_1 = 4 h. This is an approximation of the time it takes for meltwater from ice to travel from the tongue of the glacier to the gauging station below. Q_2 is runoff from the firn area and we set k_2 = 3 h. Q_3 is runoff from the snow area with k_3 = 430 h. A more detailed description of how these times were determined can be found in Oerter (1981b) and Oerter *et al.* (1981). Finally, a small constant component of groundwater runoff, $Q_4(t) = 0.1\ m^3\ s^{-1}$, was also included.

We now set the total model runoff equal to the sum of its four components:

$$Q_m(t) = Q_1(t) + Q_2(t) + Q_3(t) + Q_4(t) \quad (5)$$

To compute $Q_m(t)$ one still needs the input functions $R_i(t)$ (which correspond to the rate of meltwater production on the parts of the glacier associated with the runoff functions $Q_i(t)$), and the initial values $Q_i(0)$. The functions $R_i(t)$ are the meltwater production values given by the energy balance model (see the previous section). The values $Q_i(0)$ are determined from the following considerations. Because the first reservoir Q_1 drains so quickly, it will usually empty during the night when no melting takes place. Thus we start the modelling process at midnight and set $Q_1(0) = 0$. The modelling in fact starts at midnight on 1 June 1979. At this time there had still been very little melting from the higher areas of the glacier represented by Q_3. Thus we set $Q_3(0) = 0$ as well. $Q_4(t) = Q_4(0) = 0.1\ m^3\ s^{-1}$, so it follows from equation (5) that $Q_2(0) = Q_m(0) - Q_4(0)$. The value for $Q_m(0)$ was taken to be the measured runoff at the gauging station at time $t = 0$. With this information the functions $Q_i(t)$ can be numerically evaluated on a computer using equation (4), and $Q_m(t)$ is then determined from equation (5).

DISCUSSION OF THE RESULTS

Due to space problems it is impossible to show here an hourly comparison of model output and the measured runoff for the whole season. However, Fig.3 does this for the daily averages of model and runoff values. The hourly values (Fig.4) for model and

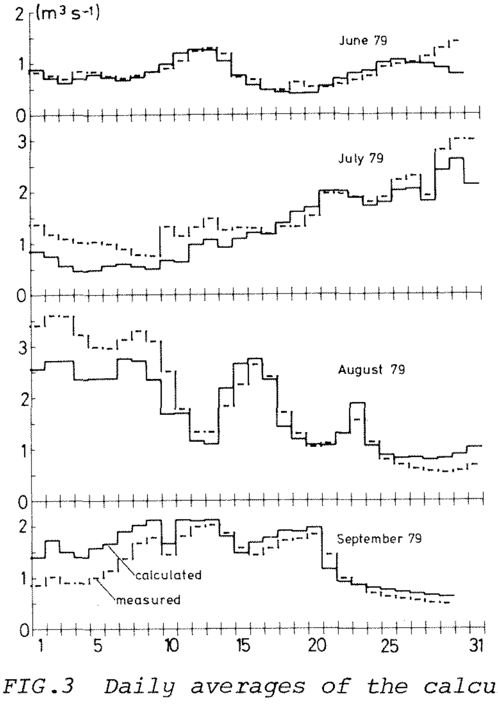

FIG.3 Daily averages of the calculated and measured runoff values for the whole modelling period in 1979.

measured runoff are shown only for the same 14 day period as for the data presented in Fig.1. One of the pleasing aspects of the model results was that they followed not just the daily averages (Fig.3) for measured runoff quite well, but also the variations that take place over periods of several hours. This is reflected by the fact that, for the entire model period from 1 June to 29 September 1979, the average error between the hourly values of model runoff and measured runoff is only 21.6%.

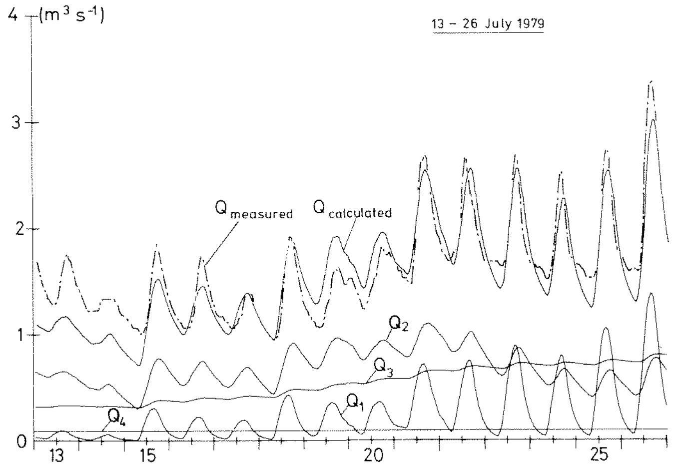

FIG.4 Hourly averages of the calculated and measured runoff values for the time period 13-26 July 1979. In addition to the total runoff, the discharge components Q_1, Q_2, Q_3 of the different reservoirs and the groundwater discharge Q_4 are plotted.

During this time there are two periods of twelve days each when the model results are worse than otherwise. Excluding these two periods, the average error becomes 15.1%. From 30 June to 11 July the model runoff is less than the measured runoff by a fairly constant amount of about 0.5 $m^3\ s^{-1}$ or about 40%. The error during this period is probably due to incorrect energy balance data, which determines the meltwater input functions $R_i(t)$ for the model. From the meteorological data, one sees that no melting took place from 2 July to 5 July and on 8 July. On the other days the meltwater production was small, as the weather was quite bad, with several periods of snowfall. It must be assumed that the energy balance data becomes worse during times of bad weather because the assumption of 0^oC surface temperature on the glacier is not fulfilled. This assumption affects all the terms of the energy balance except the shortwave radiation balance, and it was used throughout the whole calculation of meltwater production values. From 27 August to 7 September the model runoff is more than the measured runoff by a fairly constant amount of about 0.45 $m^3\ s^{-1}$ or about 50%. We do not know whether this error is due to incorrect energy balance data or to some other shortcoming of the model.

For the three areas under consideration, ice, firn, and snow, the energy balance data for the ice seem to be the most reliable. There are several reasons for this. First of all, during bad weather with snowfall, there is no ice area. On the other hand, it has been observed that, if an ice area exists, melting will occur there during some part of the day. Part of this meltwater will remain on the glacier surface during the night as well, thus hindering any cooling of the surface substantially below 0°C. Other reasons for good ice data are that the albedo estimates for ice are more accurate than for firn and snow, and that the ice region, which is positioned in the lower parts of the glacier, shows higher air temperatures than the firn and snow areas. This leads to higher fluxes of sensible heat, making the energy balance more positive.

The good reliability of the ice data is also indicated by the following consideration: because the ice runoff from the Vernagtferner reaches the gauging station quicker than the runoff from the snow and firn areas, it accounts for the large daily variations in total runoff (Fig.4) (Behrens *et al.*, 1979; Oerter *et al.*, 1980). The ice reservoir output, $Q_1(t)$, follows these variations fairly well, even during the two above-mentioned periods with large differences between model and measured runoff. Runoff contributions from firn and snow, Q_2 and Q_3, tend to be more constant than Q_1 (Fig.4). The fact that the errors during these two periods also remain fairly constant seems to suggest that they originate mainly in the firn and snow data.

In the firn and snow areas one sometimes has to deal with negative energy values which are extremely high. They are too large to have any realistic physical significance, and are probably due to the fact that the glacier surface temperature at that time dropped below 0°C. As already mentioned, this introduces inaccuracies into the energy balance data. Modifications of the model which can take account of sub-zero temperatures are planned for the future. For now we have simply set reasonable lower limits for the energy balance and reset the data to these limits whenever it falls below them.

The storage constants for the linear reservoirs were held constant for the entire model period (k_1 = 4 h, k_2 = 30 h, k_3 = 430 h), even though it is reasonable to expect a certain amount of variation through the summer. In our first modelling attempts we used an iterative numerical method to determine these constants so as to minimize the standard deviation between model and measured runoff. It was found that large variations in the storage constants are often necessary to obtain trivial improvements in the standard deviation, and that errors in meltwater input data can force these constants to take on unrealistic values. A more suitable method for varying the storage constants as the summer progresses remains a goal for future work.

The remaining modelling parameters which have not yet been discussed are the initial values for the reservoirs, $Q_i(0)$. Equation (4) shows that these values affect $Q_i(t)$ only as coefficients in front of an exponential $\exp(-t/k_i)$. For Q_1 and Q_2 the storage constants k_1 and k_2 are small enough so that this term is practically zero within one or two days. Thus the choice of $Q_1(0)$ and $Q_2(0)$ exerts very little influence on the model values $Q_m(t)$, except during the first two days. The storage constant

k_3 = 430 h is large enough so that the effect of $Q_3(0)$ is noticeable for a longer time, perhaps the first two weeks. Raising or lowering the value of $Q_3(0)$ will appreciably raise or lower the values of $Q_m(t)$ during this time, so if $Q_3(0)$ is chosen badly, a comparison of the model and measured runoff for the first two weeks should make this apparent and indicate an appropriate correction.

Finally we remark that the model does not take into account rainfall during the model period. Total rainfall for this period was 60 mm, which correspond to about 4% of the total measured runoff. Such rainfall contributes to the runoff at the gauging station, although it does not contribute in any substantial way to the energy balance and thus cannot appear in the model runoff.

In conclusion, we believe that the discharge model described in this work agrees quite well with the measured runoff data. Possible model improvements already mentioned in the above discussion will be implemented and applied to subsequent data from Vernagtferner.

ACKNOWLEDGEMENTS This work was supported by the Deutsche Forschungsgemeinschaft within the framework of the Sonderforschungsbereich 81 of the Technische Universität München. The members of the Meteorologisches Institut of the Ludwigs-Maximilians-Universität München contributed greatly to this project with their continuous help in calibrating the meteorological equipment. We are particularly indebted to Erich Heucke and Wilhelm Stadler of the GSF-Institut für Radiohydrometrie and to numerous other people who helped with the field-work.

REFERENCES

Behrens, H., Moser, H., Oerter, H., Rauert, W., Stichler, W., Ambach, W. & Kirchlechner, P. (1979) Models for the runoff from a glaciated catchment area using measurements of environmental isotope contents. In: *Isotope Hydrology 1978*, vol.2 (Proc. Neuherberg Symp. June 1978), 829-846. IAEA-SM-228/41, Wien, Austria.

Bergmann, H. & Reinwarth, O. (1976) Die Pegelstation Vernagtbach (Oetztaler Alpen). Planung, Bau und Messergebnisse (The gauge station "Pegelstation Vernagtbach" (Oetztal Alps). Planning, construction and results). *Z. Gletscherk. Glazialgeol.* 12(2), 157-180.

Campbell, W.I. & Rasmussen, L.A. (1973) The production, flow and distribution of meltwater in a glacier treated as a porous medium. In: *Hydrology of Glaciers*, (Proc. Cambridge Symp. Sept. 1969), 11-27, IAHS Publ. no. 95.

Derikx, L. (1973) Glacier discharge simulated by groundwater analogue. In: *Hydrology of Glaciers*, (Proc. Cambridge Symp. Sept. 1969), 29-40, IAHS Publ. no. 95.

Escher-Vetter, H. (1980) Der Strahlungshaushalt des Vernagtferners als Basis der Energiehaushaltsberechnung zur Bestimmung der Schmelzwasserproduktion eines Alpengletschers (The radiation balance of Vernagtferner as a basis of the energy budget computations for the determination of the meltwater production of

an Alpine glacier). *Wiss. Mitt. Met. Inst. München* no. 39.
Lang, H. (1973) Variations in the relation between glacier discharge and meteorological elements. In: *Hydrology of Glaciers* (Proc. Cambridge Symp. Sept. 1969), 85-94, IAHS Publ. no. 95.
Nilsson, J. & Sundblad, B. (1975) The internal drainage of Storglaciären and Isfallsglaciären described by an autoregressive model. *Geogr. Ann.* Ser. A, 57 (1/2), 73-98.
Oerter, H. (1981a) Der Abfluss an der Pegelstation Vernagtbach (Oetztaler Alpen) in den Jahren 1974-1980 (The discharge at the gauge station "Pegelstation Vernagtbach" (Oetztal Alps) in the years 1974-1980). *GSF-Bericht* R 260, Gesellsch. für Strahlen- und Umweltforschung München, Germany.
Oerter, H. (1981b) Untersuchungen über den Abfluss aus dem Vernagtferner unter besonderer Berücksichtigung des Schmelzwasserabflusses im Firnkörper (Investigations on the runoff from Vernagtferner with special respect to meltwater flow in firn). Dissertation, Techn. Univ. München, Germany.
Oerter, H., Rauert, W. & Stichler, W. (1980) Untersuchungen zum Abfluss eines Alpengletschers (Vernagtferner/Oetztaler Alpen) bei unterschiedlichen Ablationsbedingungen mittels Umweltisotopen (Investigations on the runoff from an Alpine glacier (Vernagtferner/Oetztal Alps) for various ablation conditions by means of environmental isotopes). In: Proc. XVIeme Congres Int. de Met. Alpine, Aix-les-Bains, Sept. 1980 (ed. by Soc. Met. de France), 285-290, Boulogne Billancourt Cedex, France.
Oerter, H., Baker, D., Moser, H. & Reinwarth, O. (1981) Glacial-hydrological investigations at the Vernagtferner glacier as a basis for a discharge model. *Nordic Hydrology*. 12(4/5), 335-348.