8 STIFFNESS FORMATION OF EARLY AGE CONCRETE

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Abstract
This paper presents a multiphase model for the determination of stiffness moduli (E,K,G,ν) in early age concrete. The model is based on the kinetics of cement hydration and on the knowledge of the elastic properties of the mix design materials. The most important factors are the internal volume changes in early age concrete, especially that of free water. These changes and the degree of hydration are measured with the hydrostatic weighing method. The stiffness parameters of the original mix materials are found by determining their elastic and bulk moduli. Comparative calculations demonstrate not only the influence of stiffness, but also the applicability of the model to concrete.
Keywords: Bulk Modulus, Elastic Modulus, Multiphase Model

1 Introduction

Stiffness formation of concrete is of interest in evaluating the risk of cracking in early age concrete as a result of internal stresses or building conditions. In standards the elastic modulus is usually defined as a variable of the compressive strength:

CEN EC2	$E_c = 9500 \cdot (f_c + 8)^{1/3}$	
France BAEL	$E_c = 11000 \cdot f_c^{1/3}$	
USA ACI 318	$E_c = 5000 \cdot f_c^{1/2}$	E_c, f_c in MPa

These formulas consider neither the different stiffnesses of cement paste and aggregates nor fractional mix volumes. They also result in very differing values for early age concrete lower then 20 MPa compressive strength. They are therefore only helpful as an assessment for commonly used aggregates and mix designs.

Attempts to describe effective elastic material constants are often based on a two phase model and isotropic homogeneous materials. Reuss´ model of equal stresses provides the lower bound for the elastic modulus. Voigt´s Model of equal deformations the upper bound. Because both bounds cover a broad range they are not very precise. This was improved by combining both models for fractional volumes and by introducing empirical factors to take the compound effect into account. Several

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authors applied these models to concrete rating material constants for the CSH-matrix and aggregates [1-4]. All these models consider only the elastic modulus and not the different poisson ratios. However, full identification of elastic materials is only possible with two of the four elastic constants E, K, G, ν.

The introduction of multiphase models with a second elastic constant further defined the upper and lower bounds. This development occurred in solid state mechanics and was first applied to metals and compound materials [5-7]. The results were obtained using the variational principle of linear elastic theory for composite sphere geometry [6]. The Hashin-Shtrikman limits (H-S) were also deduced for multi phase models with random geometry [7]. While the description of the effective bulk modulus is possible with simple energy methods, it is more difficult for the effective shear modulus. Christensen und Lo solved the problem for different geometries with results within the H-S-limits [8,9].

Using these improved models on concrete, measured results may also lie beyond the H-S-limits [10]. Though the more accurate models have been used for calculations they were applied only to the two phase model matrix-aggregate. Therefore, Monteiro recently proposed a three phase model for concrete, proposing a transition zone [10, 11]. However, its volume is difficult to determine. This paper takes this approach further on the basis of a composite model which is applied successively to all concrete mix materials. To that end, two elastic constants have to be determined for each mix material.

2 Composite Model of Christensen and Lo

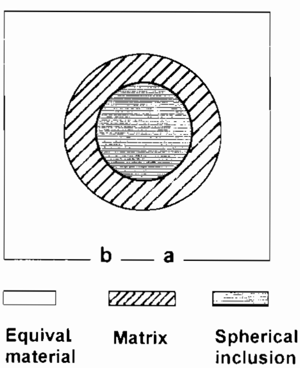

Fig. 1. Three phase model

The composite model is based on a spherical inclusion surrounded by a matrix. Both media are enclosed by an equivalent homogeneous material whose unknown effective properties have to be determined. Therefore it is required that the equivalent material of Fig.1 should store the same strain energy as the combined sphere model. For this model elastic stress-strain states and elastic strain energy have been determined [8].

The effective bulk modulus K_c of the three phase model is the same as in the composite sphere model (Eq.1)

$$K_c = K_m + \frac{c\,(K_i - K_m)}{1 + \dfrac{(1-c)\,(K_i - K_m)}{K_m + \frac{4}{3} G_m}} \qquad (1)$$

where the index m stands for matrix, i for inclusion and $c=(a/b)^3$ means the concentration of the included volume. The effective shear modulus G_c is found using Eshelby's formula

$$U_c = U_0 + \frac{1}{2}\int_S (\sigma_i u_i^0 - \sigma_i^0 u_i)\, ds \tag{2}$$

where U_c stands for strain energy of composite three-phase model. U_0 for strain energy outside the sphere b. $\sigma_i u_i$ for the stress-strain state inside the sphere b and $\sigma_i^0 u_i^0$ for the stress-strain state of the equivalent material inside the sphere b. The integration proceeds on the surface S of sphere b. The criterion for the effective shear modulus is the condition $U_c = U_0$ which leads to

$$\frac{1}{2}\int_S (\sigma_i u_i^0 - \sigma_i^0 u_i)\, ds = 0 \tag{3}$$

G_c follows from the solution of the quadratic equation (Eq.4)

$$A\left(\frac{G_c}{G_m}\right)^2 + 2B\left(\frac{G_c}{G_m}\right) + C = 0 \tag{4}$$

with the constants [9]

$$A = 8\left(\frac{G_i}{G_m}-1\right)(4-5\nu_m)e_1 c^{\frac{10}{3}} - 2\left[63\left(\frac{G_i}{G_m}-1\right)e_2 + 2e_1e_3\right]c^{\frac{7}{3}} + 252\left(\frac{G_i}{G_m}-1\right)e_2 c^{\frac{5}{3}} - 50\left(\frac{G_i}{G_m}-1\right)(7-12\nu_m+8\nu_m^2)e_2 c + 4(7-10\nu_m)e_2e_3 \tag{5}$$

$$B = -2\left(\frac{G_i}{G_m}-1\right)(1-5\nu_m)e_1 c^{\frac{10}{3}} + 2\left[63\left(\frac{G_i}{G_m}-1\right)e_2 + 2e_1e_3\right]c^{\frac{7}{3}} - 252\left(\frac{G_i}{G_m}-1\right)e_2 c^{\frac{5}{3}} + 75\left(\frac{G_i}{G_m}-1\right)(3-\nu_m)e_2\nu_m c + \frac{3}{2}(15\nu_m-7)e_2e_3 \tag{6}$$

$$C = 4\left(\frac{G_i}{G_m}-1\right)(5\nu_m-7)e_1 c^{\frac{10}{3}} - 2\left[63\left(\frac{G_i}{G_m}-1\right)e_2 + 2e_1e_3\right]c^{\frac{7}{3}} + 252\left(\frac{G_i}{G_m}-1\right)e_2 c^{\frac{5}{3}} + 25\left(\frac{G_i}{G_m}-1\right)(\nu_m^2-7)e_2 c - (7+5\nu_m)e_2e_3 \tag{7}$$

$$e_1 = (49-50\nu_i\nu_m)\,(\frac{G_i}{G_m}-1) + 35\frac{G_i}{G_m}(\nu_i-2\nu_m) + 35(2\nu_i-\nu_m) \quad (8)$$

$$e_2 = 5\nu_i\,(\frac{G_i}{G_m}-8) + 7\,(\frac{G_i}{G_m}+4) \quad (9)$$

$$e_3 = \frac{G_i}{G_m}\,(8-10\nu_m) + (7-5\nu_m) \quad (10)$$

The effective elastic modulus E_c and the poisson ratio ν_c can be calculated with solutions K_c and G_c using Eq. 11 and 12.

$$E_c = \frac{9\,K_c\,G_c}{3K_c + G_c} \quad (11) \qquad \nu_c = \frac{3K_c - 2G_c}{2\,(3K_c + G_c)} \quad (12)$$

With the elastic constants from this composite three phase model we can also describe a multiphase material such as concrete. Starting with two materials and successively adding the remaining mix materials according to their fractional volumes we receive a multiphase model.

3 Multiphase Model for Concrete

Concrete is a multiphase material, as a minimum consisting of CSH-gel as matrix phase, and the inclusions: air pores, free capillar water, unreacted cement and aggregates. The volume fractions of these materials change on a linear basis during the reaction depending on the degree of hydration α (Fig.2). As a result of chemical bond forces the volume of CSH-gel is smaller than the starting volume. This chemical shrinkage volume V_{cs} is filled with gel pores and causes a suction effect with the same dimension on the concrete surface. Water or air is sucked in, according to the conditions of storage. This condition also determines the drying of free capillar water (drying curve). The water demand for full hydration and the chemical shrinkage volume follow the characteristic cement parameters specified in [12]. Proceeding from the original mixing values V_c, V_w and V_a the variable volumes of cement paste result in:

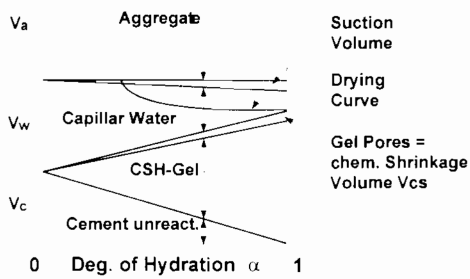

Fig. 2. Fractional concrete volume

unreacted cement: $V_{cu} = V_c\ (1-\alpha)$ (13)

CSH-gel: $V_{csh} = V_c\ (1 + F_{wv}\ (1-F_s))\ \alpha$ (14)

gel pore volume:
shrinkage volume:
suction volume: $V_{cs} = V_{gp} = V_c\ F_{wv}\ F_s\ \alpha$ (15)

capillar pore volume: $V_{cp} = V_w - V_c\ F_{wv}\ \alpha$ (16)

Where air is sucked in the capillar pore volume (Eq.16) is reduced by the sucked air volume (Eq.15). The volumetric water demand ratio F_{wv} amounts to 0.80-0.82, the shrinkage ratio F_s to 0.23-0.25, depending on the cement composition [12]. The variable CSH volume and therefore the material concentrations c_i follow from Eq. 13-16. The formulas for effective moduli can now successively be applied to the different mix volumes. In view's of the model assumption that an inclusion is being added to a connected matrix, the order of succession of the materials is of importance. The computing programme here followed the order specified above. The degree of hydration can be measured by one of the known methods. For measuring the shrinkage volume V_{cs} the hydrostatic weighing method described in [12,13] was used.

4 Elastic Moduli of Mix Materials

The water fraction and the moduli of the aggregates are of dominant influence on the concrete stiffness [1,3]. For model application, two elastic constants of each material have to be determined. For air and water these values can be taken from the literature. For cement, CSH-gel and aggregates, the material moduli have to be measured. Drilling cores can usually be used for aggregates to determine the elastic modulus and poisson ratio. Measurements of elastic moduli are particularly difficult for fine materials. In order to obtain a nondirectional value for sand and cement, a compression unit was developed which allows the determination of bulk modulus. Fig. 3 shows a diagram of this unit.

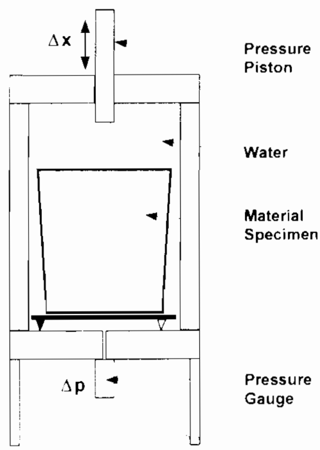

Fig. 3. Compression unit for bulk modulus

A material specimen of known volume is placed in the pressure vessel which is filled with water. Hydrostatic pressure is applied by a round smooth polished pressure piston. Its displacement Δx and the water pressure Δp are the measured values. The material compressibility is deduced from a pressure -volume change graph. The material bulk modulus can be calculated by comparing the compressibility of a control and a material measurement. First of all, the elastic compressibility of the pressure vessel itself -within the sample container- has to be measured and taken into account. For useful results a high material filling ratio is necessary. Materials with high bulk modulus give insufficiently accurate measurements. The unit is well suited for soft materials like early age concrete. The rise of bulk modulus can be measured at different times on the same material specimen. By measuring the displacement of the pressure piston without any load, the shrinkage volume as well as the degree of hydration can be determined.

5 Model Results

The multiphase model was programmed and tested in BASIC and EXCEL. Some results of these calculations are shown, making clear the influence of mixing parameters and material constants. The calculations are based on the mix design and material properties shown in table 1.

Tab. 1. Standard mix design and material constants

Material	Mass kg	Density kg/m3	Volume m3	E-Mod. GPa	K-Mod. GPa	Poisson Ratio
CSH-Gel	--	2340	--	40	23.0	0.21
Air	--	--	0.020	--	1. 10-4	0.50
Water	150	1000	0.150	--	2.04	0.50
Cement	300	3000	0.100	70	50.7	0.27
Aggreg.	1971	2700	0.730	50	33.3	0.25
Total	2421		1.000			

Changes in material fractions resulting from parameter variation are counted to the debit of the aggregates. Where the w/c ratio changed, the cement content was kept constant; similarly, where the cement content changed, the w/c ratio was kept constant. Unless it was varied as a parameter, the degree of hydration was set at 1.0.

As shown in Fig. 4, the degree of hydration is a dominant factor in the stiffness formation of early age concrete. Within 24 hours, cement can reach a degree of

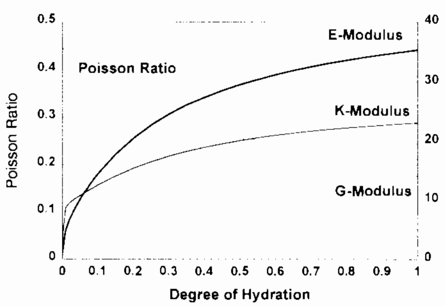

Fig. 4. Concrete moduli vs. hydration

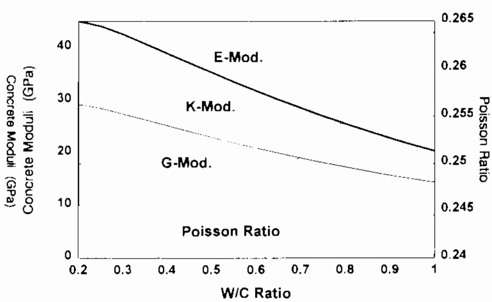

Fig. 5. Concrete moduli vs. w/c-ratio

hydration between 10 and 50% . Hydration kinetics thus constitute the controlling parameter for the stiffness formation of early age concrete. Contrary to what we find in the literature, the Poisson ratio of the model becomes monotonically decreasing. Fig. 5 shows the relationship between the w/c-ratio and the final stiffness of the concrete. The influence of the aggregate stiffness and fractional aggregate volume against the degree of hydration on elastic modulus development in concrete is shown in Fig. 6 and 7. As expected, increasing aggregate stiffness and fractional aggregate volume raise the stiffness of concrete.

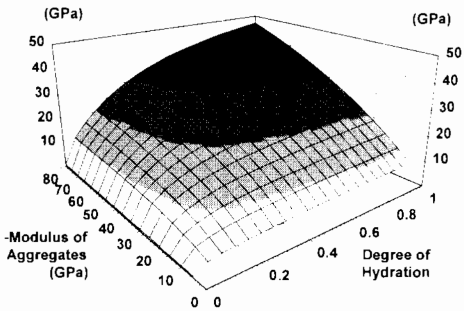

Fig. 6, Concrete E-modulus vs. hydration and E-modulus of aggregates

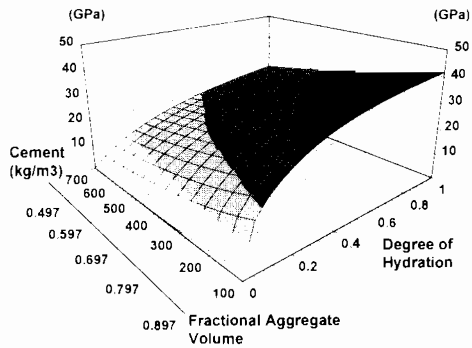

Fig. 7, Concrete E-modulus vs. hydration and cement content, Eagg=50 GPa

6 Comparison with measurements

Comparative measurements were made on 40/40/160 mm specimen with cement paste and concrete. The bulk modulus was determined as described earlier. The Poisson ratio was calculated from measured values of elastic and bulk modulus. Hydration of cement was recorded with the hydrostatic weighing method and the degree of hydration was calculated from chemical shrinkage volumes [12,13]. Fig. 8 shows the reaction process of a PZ 35F cement (w/c=0.35) and the recorded stiffness values. The starting reaction is characterized by a strong swelling period during the first day. The bulk modulus after one day is therefore lower than at the beginning. Fig. 9 shows the reaction behaviour of the same cement type, but with a swelling phase of

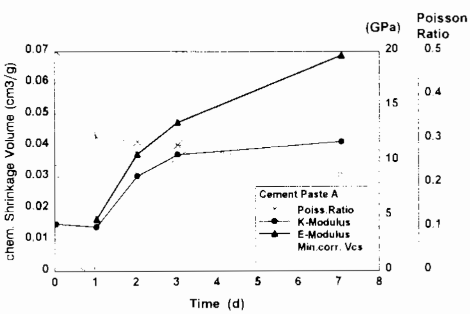

Fig. 8, Measured cement paste A reaction and moduli (w/c = 0.35)

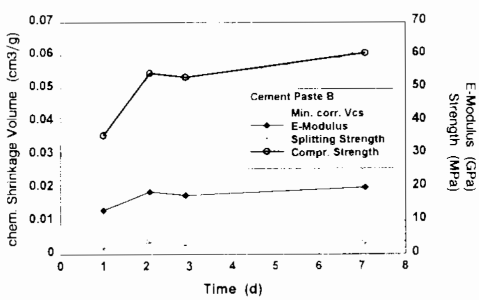

Fig. 9, Measured cement paste B reaction, E-modulus and strength, (w/c = 0.35)

10 hours. It is striking that, after three days, the measured strength and also the elastic modulus are lower than after two days. This effect can be explained by the unusual reaction process, which does not increase monotonically but shows counter reactions after 2, 4 and 5 days. In Fig. 8 and 9, the chemical shrinkage volume is based on the maximum swelling volume (initial period). Fig. 10 shows the calculated and measured stiffnesses of cement paste A and B against the degree of hydration.

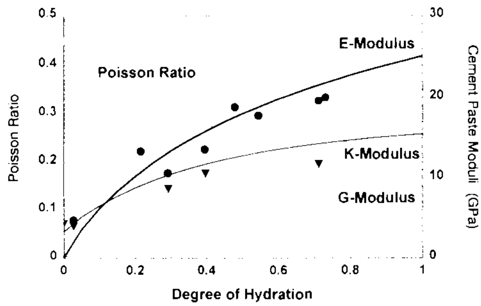

Fig. 10, Calculated and measured cement paste moduli vs. hydration

7 Summary

Based on the composite model of Christensen and Lo, a multiphase model for concrete was developed which allows the calculation of elastic moduli and Poisson ratio. Essential model parameters are the elastic constants of mix materials. For each of these materials, two of those constants have to be determined. The bulk modulus was measured with a compression unit. In early age concrete the changing free water content is decisive. The degree of hydration, calculated from hydrostatic weighing recordings, is used to take account of the variable volume fractions. Reaction kinetics of cement, mix design and material constants are essential for the stiffness formation of early age concrete. These influences are demonstrated by model calculations . The applicability of the multiphase model was demonstrated by comparing stiffness measurements of cement pastes with model results.

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