Artificial Ground Freezing of Fully Saturated Soil: Viscoelastic Behavior

Roman Lackner¹; Christian Pichler²; and Andreas Kloiber³

Abstract: The transport and mechanical properties of saturated soil drastically change when temperatures drop below the freezing temperature of water. During artificial ground freezing, this change of properties is exploited in order to minimize deformations during construction work and for groundwater control. Whereas for the latter only the size of the frozen-soil body is relevant, which is obtained by solving the thermal problem, the design of the ground-freezing work for support purposes requires information about the mechanical behavior of frozen soil. In addition to the quantification of the improvement of mechanical properties during freezing, information about the dilation associated with the 9% volume increase of water during freezing is required in order to assess the risk of damage to surface infrastructure caused by frost heave. In this paper, a micromechanics-based model for the prediction of both the aforementioned phase-change dilation and the elastic and viscous properties of freezing saturated soil is presented. Hereby, the macroscopic material behavior is related to the behavior of the different constituents such as soil particles, water, and ice. Combined with the solution of the thermal problem, the proposed model provides the basis for predictions of the performance of support structures composed of frozen soil.

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Introduction

The characterization of mechanical properties of frozen ground provides important information for (1) the design specification for artificial ground freezing (AGF) [dating back to Poetsch (1883)] used for ground improvement and (2) the assessment of the long-term performance of permafrost supports. As the temperature in soils drops below 0°C the pore water starts to freeze. However, the interaction between the soil particles and the surrounding water leads to a continuous change of morphology as the temperature decreases: starting with the nucleation of ice crystals in the center of the saturated pore space, the ice phase grows toward the particle surface as the temperature further decreases. Even for temperatures significantly lower than 0°C, a thin layer of unfrozen water around the soil particles remains unfrozen, rendering frozen saturated soil a three-phase composite, consisting of soil particles, water, and ice. From a micromechanics point of view, the continuous change of morphology, characterized by the increase of the volume fraction of the ice phase while the amount of unfrozen water decreases, gives rise to two different soil morphologies: at temperatures slightly below the freezing temperature of water, the formed ice crystals in the pore system are embedded within the pore water. At this stage, the latter may be considered the matrix phase. When the different ice particles in the pore system continue to grow and finally interconnect (percolate), a load-carrying system made of the ice phase is established. In terms of micromechanics, the matrix is then given by the now interconnected ice phase.

In the past, research on determination of mechanical properties of frozen ground mainly focused on the improved strength properties. Results from triaxial and uniaxial tests are reported in Orth (1986), Da Re (2000), and Da Re et al. (2003). In these experiments, characterized by the increase of load until failure, the frozen-soil specimen experiences elastic, viscous, and plastic (fracture) deformations. The same authors conducted creep tests under constant loading conditions and encountered a direct transition from primary creep to tertiary creep (Fig. 1). The fracture processes occurring during the latter finally cause failure of the specimens.

In contrast to material models for the description of the mechanical behavior of frozen soil formulated exclusively at the macroscale, i.e., the scale of analysis with the materials assumed to be homogeneous, a micromechanics-based model for the prediction of mechanical properties is presented in this paper. So far, fracture is not taken into account, restricting the presented model to elastic and primary-creep response only. In order to cover the complex behavior of frozen soil in this loading regime, the dilation during ground freezing caused by the 9% volume increase of liquid water during freezing is included in the viscoelastic model. In this paper, the macroscopic strain associated with phase-change dilation will be denoted as “freezing” strain. For all mechanical properties and the phase-change dilation, the macroscale

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behavior of frozen soil is related to the properties of the material phases via upscaling. In the following section, dealing with elastic properties of frozen soil, the choice of an appropriate upscaling scheme is briefly discussed.

**Micromechanical Model for Determination of Elastic Properties**

In the past decades, micromechanical models were developed and continuously improved for the estimation of elastic properties of composites. The choice of the appropriate micromechanical approach depends on the characterization of the material under investigation where two classes of micromechanical models can be distinguished:

1. If the characterization reveals a spatial variation of physical quantities that can be represented by an elementary cell, characterized by different material phases, and a drift at the level of the macroscopic structure, the periodic media theory may be employed.

2. The effective media theory, on the other hand, is based on the introduction of a representative volume element (RVE), stipulating the separation of observation scales. The size of an RVE must be (1) considerably larger than the characteristic dimension of the material phases forming the material at the considered scale and (2) significantly smaller than the material or material phase built up by the RVE.

In contrast to the periodic media theory, recently employed for determination of elastic properties of frozen Manchester sand (Da Re et al. 1999; Da Re 2000), the use of continuum micromechanics within the effective media theory is proposed in this paper. Accounting for the distinct matrix-inclusion morphology of saturated granular soil, the Mori-Tanaka scheme (see Appendix) gives access to the effective shear modulus and bulk modulus, \( \mu_{\text{eff}} \) and \( k_{\text{eff}} \), reading for the case of spherical inclusions

\[
\frac{\mu_{\text{eff}}}{\mu_i} = 1 + f_p \frac{1 - \mu_i / \mu_p}{\mu_p / \mu_i + (1 - f_p) \beta (1 - \mu_i / \mu_p)} \tag{1}
\]

and

\[
k_{\text{eff}} = 1 + f_p \frac{1 - k_i / k_p}{k_p / k_i + (1 - f_p) \beta (1 - k_i / k_p)} \tag{2}
\]

where \( f_p \) represents the volume fraction of the soil particles, and subindices \( p \) and \( i \) refer to properties of the soil particles and the ice, respectively. In Eqs. (1) and (2), the constants \( \alpha \) and \( \beta \) are given by (Appendix)

Eqs. (1) and (2) were employed for determination of the Young’s modulus of frozen granular material, with \( E_{\text{eff}} = 9 k_{\text{eff}} \mu_{\text{eff}} / (3 k_{\text{eff}} + \mu_{\text{eff}}) \). Even though the amount of unfrozen water is not considered in Eqs. (1) and (2), the model prediction agrees very well with the experimental results, see Fig. 2 (the parameters of the constituents are given in Table 1).

The marginal influence of the amount of unfrozen water is a consequence of the rapid decrease of the unfrozen water content in the case of granular materials as the temperature drops below 0°C. This is explained by the low specific surface of granular soil, with the specific surface mainly influencing the amount of unfrozen water.

**Micromechanical Model for Determination of Freezing Strains**

In contrast to the determination of elastic properties, determination of the freezing strains associated with the 9% volume increase of water during freezing requires coverage of the entire freezing process and, hence, consideration of the amount of unfrozen water. For this purpose, the degree of freezing is introduced in the form (Lackner et al. 2005)

\[
\xi = \frac{w_f}{w_{f,\infty}} \quad \text{with } 0 \leq \xi \leq 1 \tag{4}
\]

where \( w_f \) represents the actual amount of frozen water and \( w_{f,\infty} \) = amount of unfrozen water for \( T \approx 0°C \). For the development of a micromechanical model in the framework of continuum micromechanics, the theoretical case of the entire pore water

**Table 1. Properties of Constituents Used in Prediction of Elastic Material Properties of Frozen Granular Material**

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Young’s modulus ( E ) (GPa)</th>
<th>Poisson’s ratio ( \nu )</th>
<th>Bulk modulus ( k ) (GPa)</th>
<th>Shear modulus ( \mu ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice</td>
<td>9.0</td>
<td>0.325</td>
<td>8.6</td>
<td>3.4</td>
</tr>
<tr>
<td>Plastic</td>
<td>3.3</td>
<td>0.4</td>
<td>5.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Sand</td>
<td>90.0</td>
<td>0.2</td>
<td>50.0</td>
<td>37.5</td>
</tr>
</tbody>
</table>
being frozen at $T<0^\circ C$, i.e., when $w_{f,\infty}$ is equal to the water content $w$, is considered first. Thereafter, the unfrozen water content will be introduced in order to account for the three-phase composition of frozen soil.

**Freezing Strains of Ice/Soil-Particle Composite**

Starting with the theoretical case of an ice/soil-particle morphology (disregard of unfrozen water content), the macroscopic dilatation associated with the 9% volume increase of water in consequence of freezing is determined, by the repeated use of Hill’s lemma, in the framework of continuum micromechanics (Zaoui 2002). According to Hill’s lemma

$$\Sigma \cdot E = \langle \sigma(x) \rangle_y \cdot \langle \varepsilon(x) \rangle_y = \langle \sigma(x) \cdot \varepsilon(x) \rangle_y$$  \hspace{1cm} (5)

where $\sigma(x)$ is equilibrated and $\varepsilon(x)$ is compatible, i.e., derived from a displacement field, and either the former or the latter obeys homogeneous boundary condition (Zaoui 2002). In Eq. (5), $\langle \cdot \rangle_y = 1/V \int_V \cdot dV$. The total strain tensor at a point $x$ is divided into the elastic strain tensor $\varepsilon^e(x)$ and the eigenstrain tensor $\varepsilon(x)$

$$\varepsilon(x) = \varepsilon^e(x) + \varepsilon(x)$$  \hspace{1cm} (6)

The microscopic stress-strain law links $\varepsilon^e(x)$ to the stress tensor $\sigma(x)$

$$\sigma(x) = \varepsilon^e(x) : \varepsilon(x)$$  \hspace{1cm} (7)

with $\varepsilon(x)$ denoting the material tensor. The elastic strain tensor is linked to the macroscopic or homogenized strain $E$ via the strain localization tensor:

$$\varepsilon^e(x) = \Lambda(x) : E$$  \hspace{1cm} (8)

On the other hand, the macroscopic stress-strain law links $\Sigma$ and $E$ via the effective material tensor $C_{\text{eff}}$

$$\Sigma = C_{\text{eff}} E$$  \hspace{1cm} (9)

where $\Sigma=(\sigma(x))_y$ and $E=(\varepsilon(x))_y$. Inserting Eq. (8) into Eq. (7) and computing the volume average gives

$$\langle \sigma(x) \rangle_y = \Sigma = \langle \varepsilon(x) \rangle_y : \Lambda(x) : E$$  \hspace{1cm} (10)

Inserting Eq. (9) into Eq. (10) and applying Hill’s lemma [Eq. (5)] two times gives

$$C_{\text{eff}} : E = \langle \varepsilon(x) \rangle_y : \Lambda(x) : \varepsilon^e(x) : \Lambda(x) : \varepsilon(x)$$

$$C_{\text{eff}} : E = \langle \varepsilon(x) \rangle_y : \Lambda(x) : \varepsilon(x)$$  \hspace{1cm} (11)

Inserting Eq. (6) into Eq. (11) leads to

$$C_{\text{eff}} : E = \langle \varepsilon(x) \rangle_y : \Lambda(x) : \varepsilon^e(x) + \langle \varepsilon(x) \rangle_y : \Lambda(x) : \varepsilon(x)$$  \hspace{1cm} (12)

Accounting for the symmetry of $\varepsilon(x)$ and $\Lambda(x)$, with $c_{ijkl}=c_{klij}$ and $A_{ijkl}=A_{klij}$, the first term on the right-hand side of Eq. (12) becomes, after applying Hill’s lemma a third time

$$\langle \lambda(x) : \varepsilon^e(x) \rangle_y = \langle \lambda(x) : \sigma(x) \rangle_y = \langle \lambda(x) \rangle_y : \langle \sigma(x) \rangle_y = 1 : \Sigma$$  \hspace{1cm} (13)

Accounting for the stress-free boundary for determination of freezing strains, with $\Sigma = 0$, Eq. (12) reduces to

$$C_{\text{eff}} : E = \langle \lambda(x) \rangle_y : \langle \varepsilon(x) \rangle_y$$  \hspace{1cm} (14)

Finally, Eq. (14) is specialized for the case of freezing soil considering no eigenstrains in the particles ($\varepsilon_{p,\infty} = 0$) and eigenstrains $\varepsilon_{i} = 1/3 E_{i}^{\text{eff}}$ in the matrix material (ice), giving access to the homogenized freezing strains in the form

$$\bar{E}_i = f_i C_{\text{eff}} : \langle \lambda \rangle_y : \varepsilon_i$$  \hspace{1cm} (15)

where $f_i$ and $V_i$ denote the volume fraction and the volume of the ice phase. In Eq. (15), the freezing strains acting only in the ice phase (index $i$) were accounted for by $\langle \cdot \rangle_y = 1/V \int_{V_i} \cdot dV = (V_i/V) \int_{V_i} \cdot dV = f_i (V_i/V) \int_{V_i} \cdot dV = f_i \langle \cdot \rangle_{V_i}$: the case of spherical inclusions. In case of spherical inclusions, Eq. (15) may be written in volumetric form, with the volumetric freezing strain $\bar{e}_i^{\text{vol}}$ reading

$$\bar{e}_i^{\text{vol}} = \frac{E_i}{k_i} f_i C_{\text{eff}} : \langle \lambda \rangle_y : \bar{e}_i$$  \hspace{1cm} (16)

where $\langle \lambda \rangle_y$ is given for the Mori-Tanaka scheme as

$$\langle \lambda \rangle_y = \left( \sum_r \frac{f_r}{1 + \alpha(k_r/k_i - 1)} \right)^{-1}$$  \hspace{1cm} (17)

where $r \in \{i=\text{ice (matrix)}, p=\text{particles} \}$ and $\alpha = 3k_i/(3k_i + 4\mu_i)$ (Appendix). Eq. (17) is obtained by specializing Eq. (42) (Appendix) to the case of spherical inclusions.

Fig. 3 shows the (macroscopic) volumetric freezing strain $\bar{e}_f^{\text{vol}}$ computed from Eq. (16) as a function of the volume fraction of the soil particles $f_p$ for different bulk-moduli ratios $n$, with $n = k_i/k_p$. Based on the obtained relations, the effect of the stiffness properties of ice and the soil particles, represented by $n$, on the freezing strain $\bar{e}_f^{\text{vol}}$ may be assessed:

- For the theoretical case of $n=1$, i.e., for the case the ice and the soil particles had identical elastic properties, a linear decrease of $\bar{e}_f^{\text{vol}}$ is observed for increasing values of $f_p$, with $\bar{e}_f^{\text{vol}} = (1-f_p) \bar{e}_f^{\text{vol}}$.

For this case, $\bar{e}_f^{\text{vol}}$ is found to be equal to the *arithmetic* average of the freezing strains in the two material phases.

- For the theoretical case of $n>1$, the stiffness in the soil particles is lower than the stiffness of the matrix material, giving a freezing strain larger than the *arithmetic average* [Eq. (18)]. For the particle stiffness approaching zero (i.e., for the case of spherical voids, with $n = \infty$), the freezing strain $\bar{e}_f^{\text{vol}}$ becomes equal to the phase-change strain of the ice, $\bar{e}_i^{\text{vol}}$.

- For $n<1$, the stiffness of the soil particles exceeds the stiffness of the ice, representing the real situation; e.g., $n=0.17$ for bulk moduli of the ice and the soil particles of $k_{i,=8.6 \text{ GPa}}$ and $k_{p,=50 \text{ GPa}}$, respectively (Fig. 3). The presence of the rather
stiff inclusions results in a freezing strain lower than the arithmetic average obtained for $n=1$.

- For the two limiting cases, $f_p=0$ (ice only) and $f_p=1$ (theoretical case of soil particles only), the freezing strain becomes equal to the respective phase-change strain, i.e., $\bar{\varepsilon}_i=\varepsilon_i^{\text{vol}}$ for $f_p=0$ and $\bar{\varepsilon}_i^{\text{vol}}=0$ for $f_p=1$.

For $\bar{\varepsilon}_i^{\text{vol}}=\sqrt[1.09-1]{0.029}$ and considering sand with a particle content of $f_p=\pi/6$ (cubic-lattice packing), Eq. (16) predicts a freezing strain for $\xi=1$ and $w_f=\omega$ or $\bar{\varepsilon}_i^{\text{vol}}=1.15\%$. Thus, the presence of soil particles reduces the freezing strain from 2.9\% to 1.15\%.

**Extension to Consideration of Unfrozen Water Content**

In the previous subsection, an explicit expression for the determination of the macroscopic volumetric freezing strain was derived on the basis of an ice/soil-particle morphology. In order to cover the entire freezing process of saturated soil, the water phase needs to be taken into account. Hereby, a layer of water surrounding the soil particles, with its thickness decreasing as the temperature decreases, is introduced [Fig. 4(a)]. The additional water layer around the soil particle is considered by means of a two-step homogenization procedure:

1. Homogenization step I: First, the bulk modulus of the water/soil-particle composite [Fig. 4(b)] is computed according to the lower Voigt-Reuss bound [harmonic average, see, e.g., Torquato (2002)], is computed from

$$k_{pw} = \left[ \frac{1}{f_p + f_w} \left( \frac{1}{k_p} + \frac{1}{k_w} \right) \right]^{-1}$$

where $k_p$ and $k_w =$ bulk moduli of the soil particles and of water, respectively, and $f_w$ denotes the volume fraction of the unfrozen water.

2. Homogenization step II: In the second homogenization step, the explicit equation derived in the previous subsection [Eq. (16)] is applied to the ice/(water–soil-particle) composite, employing the results of homogenization step I. Hereby, the volume fraction and the bulk modulus of the composite; and

**Percolation of Ice Phase—Shift in Frozen-Soil Morphology**

So far, the ice phase was considered as matrix material, which, however, for small values of $\xi$, is certainly not the case. As regards the formation of the ice matrix, the appearance of a long-range connectivity (percolation) between the single ice formations within the pore structure is required. In the following, the value of the degree of freezing associated with the percolation of the ice phase is denoted as $\xi_0$, referred to as percolation threshold (Fig. 6).

In order to identify the value of the percolation threshold, separating free phase-change expansion, on the other hand, from restrained ice formation within the established ice matrix, on the other hand, different packing models are employed. Assuming the soil particles to be spherical, the packing situations shown in Fig. 7 correspond to granular materials with varying void ratio. Accounting for the nucleation of ice formation in the bulk of the pore space, propagating toward the soil-particle boundaries, the percolation threshold $\xi_0$ is defined as the minimum value for the degree of freezing showing an interconnected ice phase by assuming the thickness of the water layer surrounding the soil particles to be constant. For determination of $\xi_0$, the thickness of the water layer $d$ is continuously decreased until the first interconnected ice phase appears. The so-obtained values for the percolation threshold $\xi_0$ are listed in Table 2 for the different packing situations illustrated in Fig. 7.

From the percolation threshold $\xi_0$ and the volume fraction of the soil particles $f_p$, a dimensionless thickness of the water layer is computed as
where $r_p$ represents the radius of the soil particles and $r=r_p+d$.

The value of the dimensionless thickness of the water layer, $\bar{d}$, is shown in Fig. 8 as a function of $f_p$. From Fig. 8, a linear relationship between the dimensionless thickness $\bar{d}$ and $f_p$ is encountered. Except from the packing situation corresponding to $f_p=60\%$ (hexagonal lattice), which does not represent a natural packing situation (Fig. 7), the approximation given in Fig. 8 covers very well a wide range of packing situations, exhibiting quite different interconnected ice structures at percolation (Fig. 9).

Thus, for a certain type of granular soil, given by its volume fraction of soil particles $f_p$, the linear relation given in Fig. 8 gives access to the dimensionless thickness $\bar{d}$ and, hence, via Eq. (22) to the percolation threshold $\xi_0$.

**Table 2. Percolation Threshold $\xi_0$ for Different Packing Situations**

<table>
<thead>
<tr>
<th>Packing situation</th>
<th>$n$</th>
<th>$f_p$ (vol-%)</th>
<th>$\xi_0$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedral lattice</td>
<td>4</td>
<td>34</td>
<td>0.005</td>
</tr>
<tr>
<td>Cubic lattice</td>
<td>6</td>
<td>52</td>
<td>0.073</td>
</tr>
<tr>
<td>Hexagonal lattice</td>
<td>8</td>
<td>60</td>
<td>0.460</td>
</tr>
<tr>
<td>Centered sphere</td>
<td>8</td>
<td>68</td>
<td>0.022</td>
</tr>
<tr>
<td>Face-centered cubic</td>
<td>12</td>
<td>74</td>
<td>0.177</td>
</tr>
<tr>
<td>Different spheres</td>
<td>18</td>
<td>79</td>
<td>0.191</td>
</tr>
</tbody>
</table>

Note: $n=$Number of contact points.
Material Function for Freezing Strain

With the percolation threshold at hand, the material function for the freezing strain is obtained for 
\[ \bar{\varepsilon}_{f}^{\text{vol}} = (\gamma_{1.09} - 1)(1 - f_p - f_{w,\infty}) \times \xi \]  
where \( f_{w,\infty} \) represents the volume fraction of unfrozen water at \( T \leq 0 \)°C.

2. \( \xi > \xi_0; \) After percolation, the interconnected ice phase constrains the free expansion and the solution derived using continuum micromechanics is employed, see Eq. (20).

Finally, the material function for the freezing strain is obtained by combining both solutions, enforcing \( \bar{\varepsilon}_{f}^{\text{vol}} \) given in Eqs. (23) and (20) to match at \( \xi = \xi_0 \) (Fig. 10).

![Material function for freezing strain](image)

Fig. 10. Material function for freezing strain

Primary Creep of Frozen Sand—Experiments and Micromechanical Model

The viscoelastic behavior of freezing granular soil is dealt with in this section, restricting ourselves to primary creep in the low- and medium-loading regime. For identification of the viscous behavior of frozen soil, triaxial creep experiments were conducted at different subzero temperatures. The grading curve of the employed sand is depicted in Fig. 11. Prior to the creep tests, uniaxial compression tests were performed in order to specify the loading for the creep tests. Uniaxial compression tests, performed on cylindrical specimens with a diameter/height of 100/200 mm, revealed a failure load of about 110 kN. Based on this result, the maximum axial load considered for the creep experiments was set to 10 kN.

Creep Experiments—Presentation of Results

During the creep experiments, the frozen-sand samples were subjected to triaxial stress states, defined by the force \( F \) in the loading stud and the cell pressure \( p \) (Table 3). Based on \( F \) and \( p \), the axial and radial stress component in the soil sample are determined as

\[ \sigma_a = -\frac{F + p(A - A_0)}{A} \quad \text{and} \quad \sigma_r = -p \]

where \( A = 100^2 \pi/4 = 7,854 \text{ mm}^2 \) represents the cross-section area of the specimen and \( A_0 = 25^2 \pi/4 = 491 \text{ mm}^2 = \text{cross-section area of the loading stud.} \)

The creep-compliance function of frozen sand from the given displacement history, the viscoelastic strains

![Grading curve of sand considered for triaxial creep experiments](image)

Fig. 11. Grading curve of sand considered for triaxial creep experiments

Table 3. Creep Experiments: Temperature, Loading Conditions\(^a\), and Volume Fractions\(^b\)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Temperature ( T (°C) )</th>
<th>Axial load ( (\text{kN}) )</th>
<th>Cell pressure ( (\text{bar}) )</th>
<th>( m ) (g)</th>
<th>( m_d ) (g)</th>
<th>( m_w ) (g)</th>
<th>( V_w ) (cm(^3))</th>
<th>( f_w ) (vol-%)</th>
<th>( f_p ) (vol-%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>06</td>
<td>-10</td>
<td>5</td>
<td>3</td>
<td>3,180</td>
<td>2,623</td>
<td>557</td>
<td>557</td>
<td>35.4</td>
<td>64.6</td>
</tr>
<tr>
<td>07</td>
<td>-10</td>
<td>10</td>
<td>3</td>
<td>3,110</td>
<td>2,577</td>
<td>533</td>
<td>533</td>
<td>33.9</td>
<td>66.1</td>
</tr>
<tr>
<td>08</td>
<td>-5</td>
<td>10</td>
<td>3</td>
<td>3,180</td>
<td>2,667</td>
<td>513</td>
<td>513</td>
<td>32.6</td>
<td>67.4</td>
</tr>
<tr>
<td>09</td>
<td>-5</td>
<td>10</td>
<td>3</td>
<td>3,171</td>
<td>2,654</td>
<td>517</td>
<td>517</td>
<td>32.9</td>
<td>67.1</td>
</tr>
<tr>
<td>10</td>
<td>-5</td>
<td>10</td>
<td>3</td>
<td>3,170</td>
<td>2,657</td>
<td>513</td>
<td>513</td>
<td>32.7</td>
<td>67.3</td>
</tr>
<tr>
<td>11</td>
<td>-15</td>
<td>10</td>
<td>3</td>
<td>3,165</td>
<td>2,660</td>
<td>505</td>
<td>505</td>
<td>32.1</td>
<td>67.9</td>
</tr>
<tr>
<td>12</td>
<td>-15</td>
<td>10</td>
<td>3</td>
<td>3,166</td>
<td>2,647</td>
<td>519</td>
<td>519</td>
<td>33.0</td>
<td>67.0</td>
</tr>
<tr>
<td>13</td>
<td>-15</td>
<td>10</td>
<td>3</td>
<td>3,144</td>
<td>2,637</td>
<td>507</td>
<td>507</td>
<td>32.2</td>
<td>67.8</td>
</tr>
<tr>
<td>14</td>
<td>-10</td>
<td>10</td>
<td>3</td>
<td>3,174</td>
<td>2,670</td>
<td>504</td>
<td>504</td>
<td>32.1</td>
<td>67.9</td>
</tr>
</tbody>
</table>

\(^a\)Unloading After 72 h.
\(^b\)\( m_w = m - m_d \), \( V_w = p_wm_w \) with \( p_w = 1.0 \text{ g/cm}^3 \), \( f_w = V_w/(\text{sample volume}) \) with sample volume = \( 100^2 \pi/4 \times 200 = 1,570 \text{ cm}^3 \), and \( f_p = 1 - f_w \).
are linked to the stress tensor by replacing the elastic material properties in the linear-elastic material law by the respective creep-compliance functions, reading

$$
e_{ij}^c = \frac{1}{3k} \sigma_{ii} \delta_{ij} + \frac{1}{2\mu} s_{ij} - e_{ij}^0 = \frac{1}{3} J^{\text{vol}} \sigma_{ii} \delta_{ij} + \frac{1}{2} J^{\text{dev}} s_{ij}$$  \hspace{1cm} (25)$$

where $\sigma_{ii}$ denotes the average stress and $s_{ij}$=stress deviator. In Eq. (25), $J^{\text{vol}}$ and $J^{\text{dev}}$ describe the evolution of viscoelastic deformations in consequence of hydrostatic and deviatoric loading, respectively. For the considered triaxial test, with $\sigma''=(\sigma_a+2\sigma_r)/3$ and $s_{ij} = \sigma_a - \sigma''$, Eq. (25) gives access to the axial creep strain, with

$$e_a^c = \frac{1}{9} J^{\text{vol}} (\sigma_a + 2\sigma_r) + \frac{1}{3} J^{\text{dev}} (\sigma_a - \sigma_r)$$  \hspace{1cm} (26)$$

The development of volumetric creep strains under hydrostatic pressure is associated with the closure of air-filled voids during the creep process. In fully saturated soil, almost no air voids are present in the material, and thus deformations associated with volumetric creep are significantly smaller than deformations related to deviatoric creep. Taking into account that the main part of creep deformations is associated with deviatoric loading, $J^{\text{vol}}$ is set equal to the elastic material response, with $J^{\text{vol}}=1/k$. Thus, the deviatoric creep-compliance function can be computed from Eq. (26) considering $e_a^c = u(t)/H$, where $H$=height of the cylindrical specimen.

$$e_a^c = \frac{u(t)}{H} = \frac{1}{9k} (\sigma_a + 2\sigma_r) + \frac{1}{3} J^{\text{dev}} (\sigma_a - \sigma_r)$$

$$\rightarrow J^{\text{dev}} = 3 \frac{u(t) - (\sigma_a + 2\sigma_r)/(9k)}{H(\sigma_a - \sigma_r)}$$  \hspace{1cm} (27)$$

With the monitored displacement history at hand, the creep-compliance rate $J^{\text{dev}}$ is obtained from Eq. (27) as

$$J^{\text{dev}} = 3 \frac{\dot{u}(t)}{H(\sigma_a - \sigma_r)}$$  \hspace{1cm} (28)$$

See $\log_{10} J^{\text{dev}} - \log_{10} t$ diagram in Fig. 13.

Fig. 12. History of axial deformation monitored during creep tests (unloading at $t=72$ h)

Fig. 13. Creep compliance rate $J^{\text{dev}}$ ($\mu$m/m/MPa/s) for $0 \leq t \leq 72$ h
Micromechanical Modeling of Creep

For determination of the viscoelastic behavior of the ice-water-particle composite, creep deformations are assigned to the ice phase only. The inclusions (particles surrounded by unfrozen water) are assumed to exhibit elastic deformations. Motivated by the experimentally-observed creep-compliance rate that is proportional to $r^{-1}$ (Fig. 13), a (deviatoric) creep-compliance rate for ice following logarithmic creep is proposed

$$J_i^{\text{dev}}(t) = \frac{1}{\mu_i} + J_i^0 \ln \left(1 + \frac{t}{\tau_i^v}\right), \text{ giving } J_i^{\text{dev}}(t) = J_i^0 + \frac{1}{\tau_i^v} + t$$

(29)

In Eq. (29), $J_i^0$ and $\tau_i^v$ are model parameters, with their effect of the creep-compliance rate illustrated in Fig. 14. Hereby, $J_i^0$ (MPa$^{-1}$) defines the value of the long-time asymptote ($J_i^0/r$) of the creep-compliance rate at $r=1$ s and $\tau_i^v(s)$ is referred to as the settling-down time [exhaustion mechanism of logarithmic creep, see Nabarro (2001)]. Similar to the interpretation of the triaxial test data, creep of ice in consequence of volumetric loading is omitted, setting $J_i^\text{vol}=1/k_i$.

For determination of the viscoelastic response of frozen soil from the soil composition and the creep properties of ice, continuum micromechanics is employed. Based on the so-called correspondence principle (Lee 1955; Mandel 1966), the solutions provided for the effective elastic properties of composite materials are used for determination of the respective effective viscoelastic properties by replacing the elastic parameters of the material phases by the Laplace-Carson transformed viscoelastic parameters; e.g., the shear compliance of ice $1/\mu_i$ appear in the elastic solution for the effective shear modulus obtained from continuum micromechanics is replaced by the Laplace-Carson transform of the deviatoric creep compliance $J_i^{\text{dev}} = \mathcal{L}[J_i^{\text{dev}}]$.

The Laplace-Carson transform of the deviatoric creep compliance of ice (matrix material), given in Eq. (29), reads

$$J_i^{\text{dev}} = \mathcal{L}[J_i^{\text{dev}}(t)] = \frac{1}{\mu_i} + J_i^0 \exp[\rho \tau_i^v] \Gamma[0,\rho \tau_i^v]$$

(30)

with $\Gamma$ denoting the incomplete gamma function. The incomplete gamma function $\Gamma[a,z]$ satisfies

$$\Gamma[a,z] = \int_z^\infty t^{a-1}e^{-t}dt$$

(31)

Considering the creep compliance of ice in the $\mathcal{L}[\cdots]$-transformed expression of the MT scheme, used earlier for determination of elastic properties, one gets

$$\frac{1}{J_i^{\text{eff}} + \rho / \tau_i^r} = \sum_r \frac{1}{J_i^{\text{dev}} + \rho / \tau_i^r} \left[1 + \beta^r \left(\frac{J_i^{\text{dev}}}{J_i^{\text{dev}} + \rho / \tau_i^r} - 1\right)\right]^{-1}$$

(32)

with $r \in \{i=\text{ice(matrix)}, p=\text{particle-water composite}\}$ and

$$\beta^r = \frac{6(1/J_i^{\text{vol}} + 2/J_i^{\text{dev}})}{5(3/J_i^{\text{vol}} + 4/J_i^{\text{dev}})}$$

(33)

where $J_i^{\text{vol}}=1/k_i$. In Eq. (32), elastic material response for the particle-water composite is considered by $J_i^{\text{vol}}=1/k_{pw}$ and $J_i^{\text{dev}}=1/k_{pw}$. Inserting Eq. (30) into Eq. (32) and performing the inverse Laplace-Carson transformation gives access to the effective deviatoric creep compliance of frozen soil in the time domain, $J_i^{\text{eff}}(t)=\mathcal{L}^{-1}[J_i^{\text{eff}}]$. Hereby, the inverse transformation is performed in a pointwise manner (for discrete values of $t>0$) by applying the Gaver-Stehfest algorithm (Stehfest 1970). Imposing an affine form of the creep compliance of ice, $J_i^{\text{dev}}$, [Eq. (29)], and the effective creep compliance, $J_i^{\text{eff}}(t)$, respectively, the discrete points from inverse transformation were approximated by

$$J_i^{\text{eff}}(t) = \frac{1}{\rho^{\text{eff}}} + J_i^{\text{eff}} \ln \left(1 + \frac{t}{\tau_i^{\text{eff}}}\right)$$

(34)

giving access to the effective creep-compliance function and thus to the parameters $J_i^{\text{eff}}$ and $\tau_i^{\text{eff}}$. In Eq. (34), $\rho^{\text{eff}}$=effective shear modulus of frozen soil [Eq. (1)].

While the presented continuum-micromechanics approach for determination of the viscoelastic properties of frozen soil accounts for the material microstructure, the temperature dependence of the creep process of frozen soil is considered by the creep behavior of ice. For this purpose, the dependence of $J_i(T)$ on the temperature is described by an Arrhenius law, reading

$$J_i(T) = \tilde{J}_i \exp\left[-\frac{Q}{R\left(\frac{1}{T} - \frac{1}{T_0}\right)}\right]$$

(35)

with $Q=67,000$ J/MOL as the creep activation energy [see, e.g., Sinha (1978); Abdel-Tawab and Rodin (1997)], $R=8.314$ J/(MOL K) as the universal gas constant, and $T=263$ K as the reference temperature. The parameter $\tilde{J}_i$ appearing in Eq. (35), representing $\tilde{J}_i$ at $T$, is determined by means of back analysis from the experimental results, giving $\tilde{J}_i=13,000$ $\mu$m/m/MPa. Finally, the performance of the micromechanical model is shown in Fig. 15. Taking into account the scatter in the experimental results and that all input parameters are measurable quantities, such as, e.g., the volume fraction of soil particles $f_i$, the proposed micromechanical model is well suited for the prediction of the viscoelastic behavior of frozen granular soil.

Conclusions and Outlook

In this paper, the mechanical behavior of frozen granular soil was investigated, focusing on the elastic and viscous material response as well as the volume increase associated with freezing of soil water. For this purpose, a micromechanics-based material model was proposed, employing the Mori-Tanaka scheme for determi-
nation of material properties of the three-phase (soil-particle/ice/water) composite. From the findings presented in this paper, the following conclusions can be drawn:

- Application of the Mori-Tanaka scheme for determination of the elastic properties of frozen granular material showed excellent agreement with experimental results taken from Da Re et al. (1999). Since the unfrozen water phase was disregarded during determination of the elastic properties, the good agreement suggests that the unfrozen water does not affect the elastic material response of frozen granular material (this argument certainly does not apply to frozen cohesive soils exhibiting a pronounced amount of unfrozen water at subzero temperatures).

- The investigation of the effect of the 9% volume increase of freezing water on macroscopic deformations required the consideration of the freezing process, using the degree of freezing introduced in Lackner et al. (2005). Hereby, the percolation of the ice phase represents the transition from unrestrained to restrained volume expansion. Finally, a micromechanics-based material function was derived that, on the one hand, defined the percolation threshold, and on the other hand, served as input for the micromechanical model describing constrained expansion after percolation.

- The lack of experimental data on primary creep of frozen soil was the motivation for the initiation of the experimental program presented in this paper. Based on the obtained experimental results, a logarithmic creep law was identified. The assumption of the temperature influencing creep of frozen soil exclusively via the creep properties of ice was proven by the good agreement between model results and experimental data for different subzero temperatures. For determination of the creep-compliance function of frozen soil, continuum micromechanics was combined with the elastic-viscoelastic correspondence principle, yielding good agreement between creep parameters predicted by upscaling and obtained from the creep experiments.

Future work will be devoted to the experimental investigation of creep of pure ice. Whereas the elastic properties of pure ice are well known, the creep behavior observed in experiments strongly depends on existing slip planes and heterogeneities in the ice sample, requiring intense work on sample preparation. This investigation should provide insight into both primary and tertiary creep of pure ice. The latter, which is associated with fracture processes, will serve as input for the extension of the material model proposed in this paper towards tertiary creep and failure of frozen granular material. In addition to test results reported in the open literature on this topic (Orth 1986; Da Re et al. 1999; Da Re 2000), uniaxial and triaxial compression tests of frozen sand will be performed in order to validate the effective failure criterion.

Having provided material parameters for the elastic material response, dilation in the course of freezing, and for primary creep in this paper, the material parameters describing failure of frozen granular material will provide the complete set of material functions required for macroscale numerical simulation of frozen-soil bodies.

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Appendix: Continuum Micromechanics for Ice-Particle Composite

Within continuum micromechanics, the local strain tensor $\varepsilon$ at the position $x$ is related to the homogenized strain tensor $E$ by the so-called localization tensor $\Lambda$, reading

$$\varepsilon(x) = \Lambda(x) : E$$

(36)

with the homogenized strain tensor $E$ being obtained from volume averaging of $\varepsilon$

$$E = (\varepsilon(x))_V = \frac{1}{V} \int_V \varepsilon(x) dV$$

(37)

From Eqs. (36) and (37) follows $(\Lambda(x))_V = 1$. For the case of a soil particle (index $p$), represented by an ellipsoidal inclusion and embedded in a matrix material made of ice (index $i$), the so-called Mori-Tanaka scheme (Mori and Tanaka 1973), accounting for this matrix-inclusion morphology yields the respective localization tensor $A_p$ for the soil particle as (Eshelby 1957)

$$A_p = \left[1 + S_p(c_i^p; c_p - 1)\right]^{-1} \sum_{r = 1}^{9} f_r [1 + S_r(c_r^i; c_r - 1)]^{-1}$$

(38)

In Eq. (38), $c_i$ and $c_p$ = material tensor of the ice and the soil particle, respectively, and $f_r$ denotes the volume fraction of the $r$th material phase. The so-called Eshelby tensor $S_p$ accounts for the geometrical properties of the soil particle and depends on the elastic properties (Poisson’s ratio) of ice (matrix material).
The homogenized stress tensor $\Sigma$ is obtained from volume averaging of the local stress tensor $\sigma(x)$, reading

$$\Sigma = \langle \sigma(x) \rangle_\nu = \frac{1}{V} \int_V \sigma(x) dV \quad \text{(39)}$$

Assuming linear-elastic material response in the $r$th material phase, $\sigma_r(x) = c_r \varepsilon_r(x)$ gives access to the effective material tensor

$$\Sigma = (\sigma(x))_\nu = (c(x) : e(x))_\nu = (c(x) : A(x))_\nu E \rightarrow C_{\text{eff}} = (c(x) : A(x))_\nu$$

Using

$$\langle A(x) \rangle_\nu = \frac{V_1}{V} \int_V A(x) dV + \frac{V_p}{V} \Lambda_p = 1 \rightarrow f_A(\langle A(x) \rangle_\nu) = 1 - f_p \Lambda_p$$

$$\text{(40)}$$

where $f_A$ (-) and $f_p$ (-) denote the volume fractions of the ice and soil particle, respectively, and Eq. (38), one gets the volume average of the localization tensor over the ice matrix as

$$\langle \Lambda(x) \rangle_\nu = \left\{ \sum_{i \neq p} f_i [1 + S_i (c_i / c_i - 1)] \right\}^{-1} \quad \text{(41)}$$

Considering Eqs. (42) and (38) in Eq. (40) yields the effective material tensor

$$C_{\text{eff}} = f_p c_p \langle A(x) \rangle_\nu + f_p c_p \Lambda_p = \left\{ \sum_{i \neq p} f_i [1 - S_i (c_i / c_i - 1)] \right\}^{-1}$$

$$\sum_{i \neq p} f_i [1 + S_i (c_i / c_i - 1)]^{-1} \quad \text{(43)}$$

For the case of spherical inclusions, Eq. (43) is rewritten in terms of the effective shear and bulk modulus, $\mu_{\text{eff}}$ and $k_{\text{eff}}$, reading

$$\mu_{\text{eff}} = \frac{\sum_{i \neq p} f_i \mu_i [1 + \beta (\mu_i / \mu_i - 1)]^{-1}}{\sum_{i \neq p} f_i [1 + \beta (\mu_i / \mu_i - 1)]^{-1}} \quad \text{(44)}$$

and

$$k_{\text{eff}} = \frac{\sum_{i \neq p} f_i k_i [1 + \alpha (k_i / k_i - 1)]^{-1}}{\sum_{i \neq p} f_i [1 + \alpha (k_i / k_i - 1)]^{-1}} \quad \text{(45)}$$

where the Eshelby tensor $S_i$ reduces to

$$\beta = \frac{6(k_i + 2\mu_i)}{5(3k_i + 4\mu_i)} \quad \text{and} \quad \alpha = \frac{3k_i}{3k_i + 4\mu_i} \quad \text{(46)}$$

Considering $f_i = 1 - f_p$ in Eqs. (44) and (45), one gets

$$\frac{\mu_{\text{eff}}}{\mu_i} = 1 + f_p \frac{1 - \mu_p / \mu_i}{\mu_p / \mu_i + (1 - f_p) \beta (1 - \mu_p / \mu_p)} \quad \text{(47)}$$

and

$$\frac{k_{\text{eff}}}{k_i} = 1 + f_p \frac{1 - k_p / k_i}{k_p / k_i + (1 - f_p) \alpha (1 - k_p / k_p)} \quad \text{(48)}$$

Notation

The following symbols are used in the paper:

- $A$ = fourth-order localization tensor;
- $A_0$ = cross-sectional area of sample in triaxial test;
- $A_{\text{eff}}$ = effective material tensor;
- $c_r$ = material tensor of material phase $r$;
- $d$ = thickness of layer of unfrozen water surrounding soil particles;
- $\bar{d}$ = dimensionless thickness of layer of unfrozen water;
- $E$ = homogenized strain tensor;
- $E_{\text{eff}}$ = effective Young’s modulus;
- $E_{\text{r}}$ = Young’s modulus of material phase $r$;
- $f$ = force in loading stud in triaxial test;
- $f_p$ = volume fraction;
- $H$ = height of cylindrical soil sample in triaxial test;
- $I$ = fourth-order unity tensor;
- $j_{\text{eff}}$ = creep compliance rate;
- $J_{\text{dev}}$ = creep compliance associated with deviatoric loading;
- $J_{\text{eff}}$ = effective creep compliance;
- $J_p$ = creep compliance of material phase $r$;
- $f_p$ = creep parameter defining long-term asymptote;
- $r_{\text{vol}}$ = creep compliance associated with hydrostatic loading;
- $k_{\text{eff}}$ = effective bulk modulus;
- $k_r$ = bulk modulus of material phase $r$;
- $m$ = mass of soil sample in triaxial test;
- $m_d$ = mass of dried soil sample in triaxial test;
- $m_w$ = amount of water in soil sample of triaxial test;
- $n$ = ratio between bulk modulus of ice and soil particles;
- $n_{\text{ct}}$ = number of contact points in packing models;
- $p$ = cell pressure in triaxial test;
- $Q_i$ = activation energy for creep of ice;
- $Q_i$ = gas constant;
- $q$ = radial coordinate;
- $r$ = radius of soil particles;
- $S$ = fourth-order Eshelby tensor;
- $T$ = temperature;
- $T_0$ = reference temperature;
- $t$ = time;
- $\bar{u}$ = axial deformation in triaxial test;
- $V$ = volume;
- $w$ = water content of unfrozen soil;
- $w_f$ = amount of frozen water;
- $w_{f,s}$ = amount of frozen water for $T < 0 \degree C$;
- $x$ = position vector;
- $\alpha, \beta$ = parameters representing Eshelby tensor in case of spherical inclusions;
- $\delta$ = Kronecker delta;
- $e$ = strain tensor;
- $e^c$ = elastic strain tensor;
- $e^v$ = viscoelastic strain tensor;
- $e_u$ = axial component of viscoelastic strain tensor in triaxial test;
- $\bar{e}_f$ = freezing strain tensor;
- $\bar{e}_s$ = phase-change strain tensor in ice;
- $\bar{e}_p$ = phase-change strain tensor in soil particles;
- $\bar{e}_{\text{vol}}$ = volumetric freezing strain;
- $\bar{e}_{\text{p}}$ = volumetric phase-change strain in ice;
- $\mu_{\text{eff}}$ = effective shear modulus;
- $\mu_r$ = shear modulus of material phase $r$;
- $\nu_r$ = Poisson’s ratio of material phase $r$;
\( \xi \) = degree of freezing;
\( \xi_0 \) = percolation threshold;
\( \pi \) = 3.14;
\( \rho_w \) = density of water;
\( \sigma \) = stress tensor;
\( \sigma_a \) = axial stress component in triaxial test;
\( \sigma_r \) = radial stress component in triaxial test;
\( \Sigma \) = homogenized stress tensor; and
\( \tau^c \) = parameter of creep-compliance function.

References


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