Bachelor / Masters / PhD thesis

Numerical analysis of wave equations

This project considers accurate and efficient numerical methods for sound propagation, often steered by hard analysis. Some key problems of the numerical analysis: High frequencies, nonlinear wave propagation and boundary conditions, complex domains and sound emission.

Depending on the interests of the student, a project could consider adaptive finite element methods, time stepping methods or the pure analysis of hyperbolic equations. Also applications are of interest, e.g. to environmental noise and the design of silent roads and tires. A specific example is given by the work of Patrice Hauret on the modelling and adaptive computation of sound emission of tires and other complex sound sources [3], for which he was awarded the Felix Klein Prize of the European Mathematical Society. See also [1].

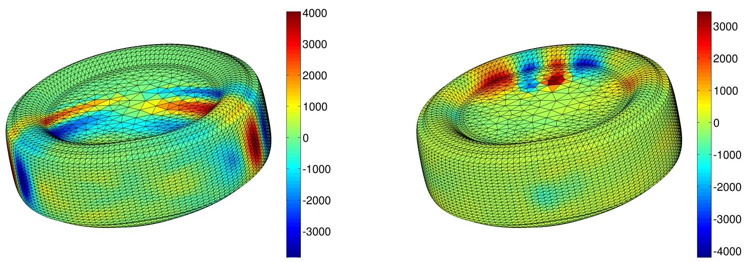
Background

Let Ω be a bounded polygonal or smooth domain in $\mathbb{R}^3$. The project considers the linear wave equation for the sound pressure

$$\partial_t^2 p(t,x) - \Delta_x p(t,x) = 0,\ \ p(0,x) = \partial_t p(0,x) = 0 \qquad (x \in \Omega)\ ,$$

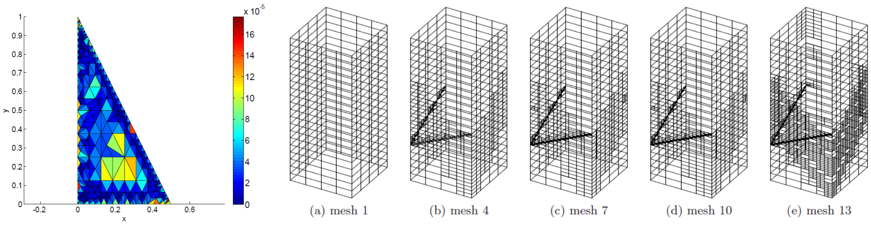
with challenges posed by the acoustics at the boundary $\partial\Omega$: Coupling to the dynamics of a nonlinear sound emitter (such as a tire), partially absorbing boundaries, complicated geometries.

The numerical analysis of such problems allows for independent subprojects in areas of current interest, numerical and analytical:



(a) (b)

Figure 1: Density of sound emitted by a vibrating tire (snapshots at two times) [1].



(a) (b)

Figure 2: Mesh refinement at a time step (a), space-time meshes (b) [1].

a) The analysis of partial differential equations gives rise to stable and efficient numerical methods. Computable a posteriori estimates for the numerical error then lead to efficient adaptive and high-order algorithms: The mesh or polynomial degree is precisely refined in regions of large numerical errors [1].
b) We would be particularly interested in interface problems with nonlinear wave equations, when different physics are coupled, such as in sound emission. There has been much recent work on time stepping methods for nonlinear hyperbolic model problems [5], but the coupling and a posteriori error estimation are widely open.
c) Stabilized finite element methods for time dependent problems give rise to simple, provably stable methods. We are at the moment particularly interested in variational inequalities [2], such as contact boundary conditions for non-penetrable obstacles or friction. They arise when a tire meets the road (or a car crashes into another).

References

[1] H. Gimperlein, M. Maischak, E. P. Stephan, *Adaptive time-domain boundary element methods and engineering applications*, Journal of Integral Equations and Applications 29 (2017), 75-105, survey article.

[2] H. Gimperlein, F. Meyer, C. Özdemir, E. P. Stephan, *Time domain boundary elements for dynamic contact problems*, Computer Methods in Applied Mechanics and Engineering 333 (2018), 147 - 175.

[3] F. Hauret, *Two-Scale Space-Time Methods for Computational Solid Mechanics*, Felix Klein Prize, European Congress of Mathematics, 2016.

[4] G. Lebeau, M. Schatzman, *A wave problem in a half-space with a unilateral constraint at the boundary*, Journal of Differential Equations 53 (1984), 309-361.

[5] A. Ostermann, K. Schratz, *Low regularity exponential-type integrators for semilinear Schrödinger equations*, Found. Comput. Math. 18 (2018), 731-755.

Prerequisites

Interest in numerical methods for partial differential equations.

Contact

Univ.-Prof. Dr. Heiko Gimperlein (heiko.gimperlein@uibk.ac.at)