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AB Technische Mathematik

Bachelor / Masters / PhD thesis

Individual and Collective Behaviour in Biological or Robotic Systems

A fundamental problem across different disciplines is understanding how functional complexity at a macroscopic scale results from the actions and interactions among the individual components. Animal groups such as insect swarms, fish schools and bird flocks frequently display coordinated behaviours that result from individual interactions. The resulting collective movement is observed across different scales, from cells forming biological tissues to fish schools moving in synchrony. This coordinated movement has been adopted in robotics as a fundamental idea for the design of swarms of interacting robots. The control available for swarm robotic systems opens up new challenges about the optimal design and learning of self-organised systems, as well as their modelling and efficient simulation.

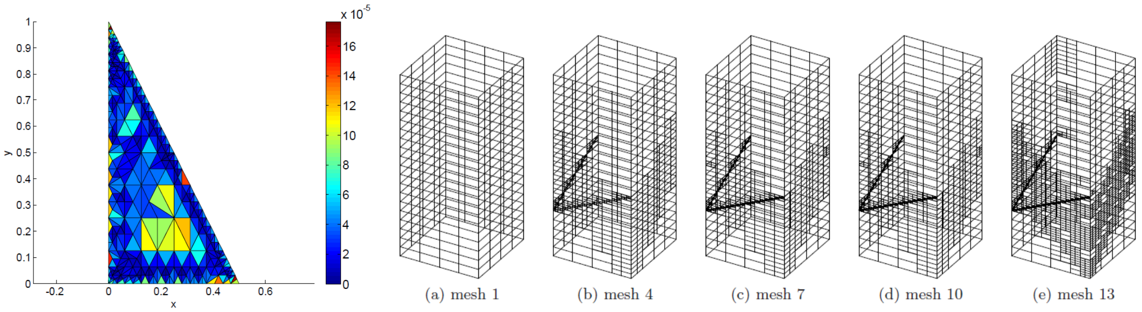
The proposed project considers the modelling and computational challenges of self-organization across scales, depending on the interests of the student:

- **Modelling:** macroscopic PDE descriptions of migration, swarming and flocking, based on microscopic movement laws [1, 9]. This may include simple cells [5], complex organisms [4] or learning robotic systems [2].
- **Numerics:** finite elements and their numerical analysis for the resulting, often nonlocal, macroscopic reaction-diffusion equations [5, 7].
- **Machine learning and inverse problems:** Ongoing work with computer scientists tries to design or learn microscopic interactions to achieve a prescribed macroscopic behaviour, combining machine learning with finite elements.

The slides [1] provide a good introduction to biological models for swarming.

(a) (b) (c)

Figure 1: (a) Swarming of birds, (b) Swarming of robots / drones, (c) Coverage of an area computed from macroscopic simulations vs. simulation of individual robots.



(a) mesh 1 (b) mesh 4 (c) mesh 7 (d) mesh 10 (e) mesh 13

(a) (b)

Figure 2: Finite elements with adaptive mesh refinements, space-time meshes.

References

[1] J. A. Carrillo, *Swarming models with local alignment effects*, slides available at https://mat1.uibk.ac.at/heiko/Edinburgh2018.pdf .

[2] S. Duncan, G. Estrada-Rodriguez, J. Stocek, M. Dragone, P. A. Vargas, H. Gimperlein, *Efficient quantitative assessment of robot swarms*, Bioinspiration & Biomimetics 17 (2022), 036006.

[3] G. Estrada-Rodriguez, H. Gimperlein and K. Painter, *Fractional Patlak-Keller-Segel equations for chemotactic superdiffusion*, SIAP 78 (2018), 1155 - 1173.

[4] E. Estrada, G. Estrada-Rodriguez, H. Gimperlein, *Metaplex networks: Influence of the exo-endo structure of complex systems on diffusion*, SIAM Review 62 (2020), 617-645.

[5] G. Estrada-Rodriguez, H. Gimperlein, K. Painter, J. Stocek, *Space-time fractional diffusion in immune cell models with delay*, M3AS 29 (2019), 65 - 88.

[6] H. Gimperlein, J. Stocek, *Space-time adaptive finite elements for parabolic fractional variational inequalities*, CMAME 352 (2019), 137 - 171.

[7] H. Gimperlein, J. Stocek, C. Urzua-Torres, *Optimal operator preconditioning of pseudodifferential boundary problems*, Numerische Mathematik 148 (2021), 1 - 41.

[8] S. Gomes, A. Stuart, M.T. Wolfram, *Parameter estimation for macroscopic pedestrian dynamics models from microscopic data*, SIAP 80 (2019), 1475 - 1500.

[9] T. Hillen, K. J. Painter, *A user's guide to PDE models for chemotaxis*, J. Math. Biol. 58 (2009), 183-217.

Prerequisites

Interest in mathematical modelling, numerical methods or machine learning.

Contact

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