A new $K-\varepsilon$ turbulence parameterization for mesoscale meteorological models

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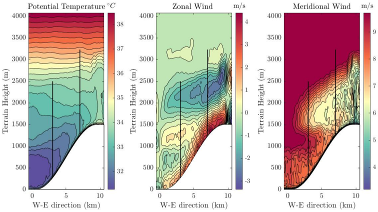

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Idea!

Employing a $\boldsymbol{K - \varepsilon}$ closure in order to avoid to define a mixing length scale

The $K - \varepsilon$ turbulence closure (1.5 order)

Mixing coefficient

$$\nu_\chi \sim \ell_\chi \sqrt{\mathbf{K}} \qquad \nu_\chi = \mathbf{c}_\mu \frac{\mathbf{K^2}}{\varepsilon}$$

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☺ PROGNOSTIC Turbulent Kinetic Energy (K) equation

$$\frac{\partial K}{\partial t} = \underbrace{-\frac{\partial \overline{wk}}{\partial z}}_{\text{v. diffusion}} \quad \underbrace{-\overline{uw}\frac{\partial U}{\partial z} - \overline{vw}\frac{\partial V}{\partial z}}_{\text{shear prod.}} + \underbrace{\frac{g}{\Theta_0}\overline{w\theta}}_{\text{buoy. prod./destr.}} - \underbrace{\varepsilon}_{\text{dissipation}}$$

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☺ PROGNOSTIC Dissipation Rate (ε) equation

$$\frac{\partial \varepsilon}{\partial t} = \underbrace{-\frac{1}{\sigma_\varepsilon}\frac{\partial \overline{\varepsilon w}}{\partial z}}_{\text{v. diffusion}} - \underbrace{\left[c_1\left(\overline{uw}\frac{\partial U}{\partial z} + \overline{vw}\frac{\partial V}{\partial z}\right) - c_3\frac{g}{\Theta_0}\overline{w\theta}\right]}_{\text{shear+buoy. prod./destr.}} \underbrace{\frac{\varepsilon}{K}}_{1/\tau} - \underbrace{c_2\frac{\varepsilon^2}{K}}_{\text{dissipation}}$$

Tuning the standard $K - \varepsilon$ closure

a) Prandtl number for the mixing coefficient (Hong et al., 2006)

$$\nu_H = \frac{\nu_M}{Pr} \qquad Pr = 1 + (Pr_0 - 1) \exp\left[\frac{-3\,(z - 0.1h)^2}{h^2}\right]$$

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c) Counter-gradient term for the heat flux

1) $\overline{w\theta} = -\nu_H\left(\frac{\partial\Theta}{\partial z} - \gamma\right) \qquad \gamma = C\frac{\overline{w\theta}_s}{w_\star h} \qquad$ (NL)

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2) $\overline{w\theta} = -\nu_H \frac{\partial \Theta}{\partial z} + \Phi_{cg}(K_\theta) \qquad \frac{\partial K_\theta}{\partial t} = -\frac{\partial \overline{wK_\theta}}{\partial z} - \overline{w\theta}\frac{\partial \Theta}{\partial z} - \varepsilon_\theta \qquad$ (L)

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Neglecting vertical diffusion + variable change

$$X = \frac{K}{\varepsilon} \qquad Y = \varepsilon^{\alpha} K^{\beta} \qquad A = c_\mu (S^2 - \frac{N^2}{Pr}) \qquad B = c_\mu (c_1 S^2 - c_3 \frac{N^2}{Pr})$$

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$$\frac{\partial \ln Y}{\partial t} = (\alpha A + \beta B)\, X - (\alpha + \beta c_2)\, \frac{1}{X} \tag{1b}$$

...**analytical solution!!**

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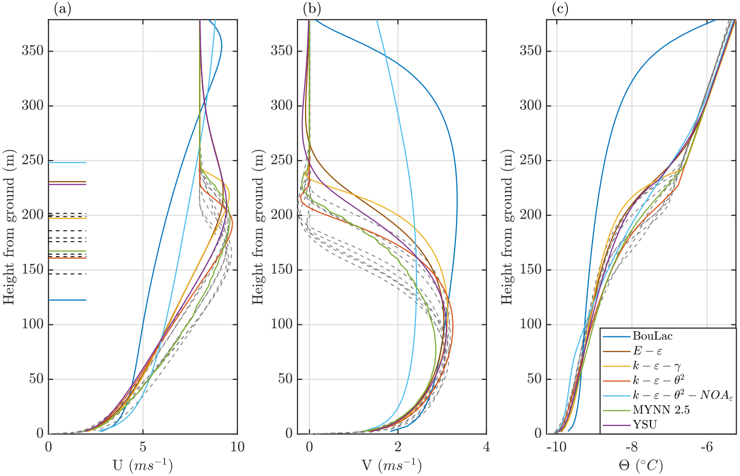
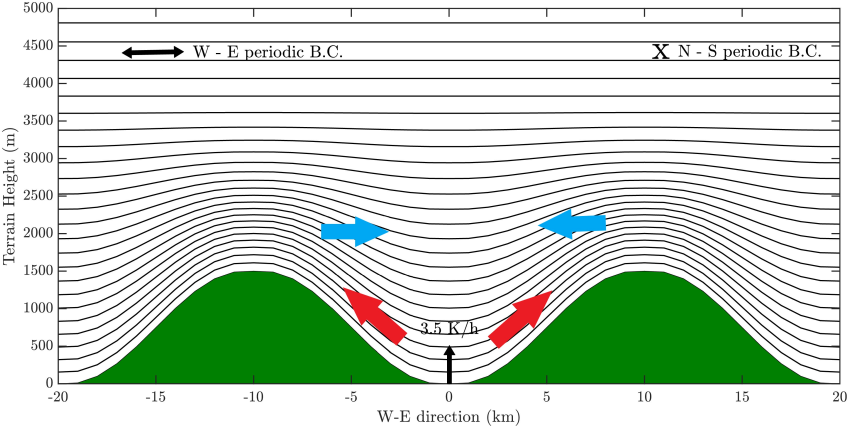
After solving the system, **diffusion** is calculated, and **advection** is applied to $\mathbf{K}$, ε and $\theta^{\mathbf{2}}$

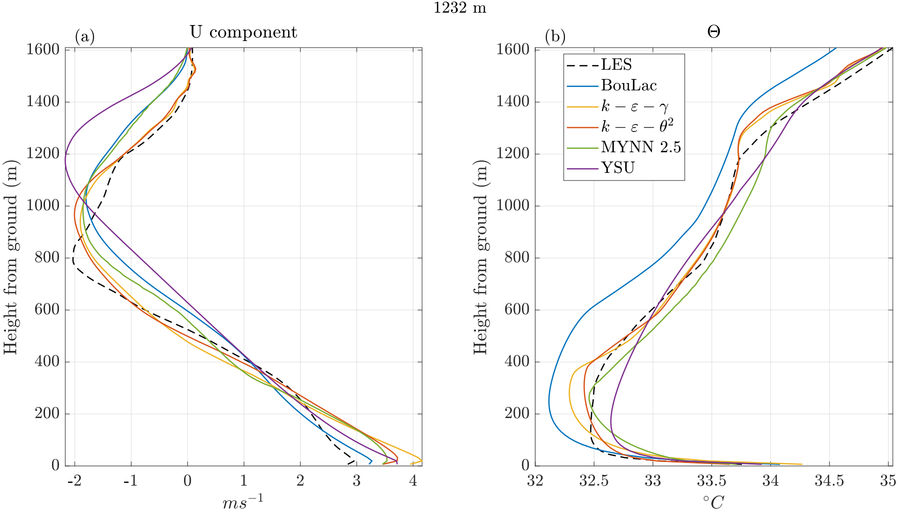
1D - Stable - GABLS2 - LES (2 m) vs RANS (1 km)



2D - LES and RANS simulation Set-Up



2D - Complex terrain - LES (50 m) vs RANS (1 km)



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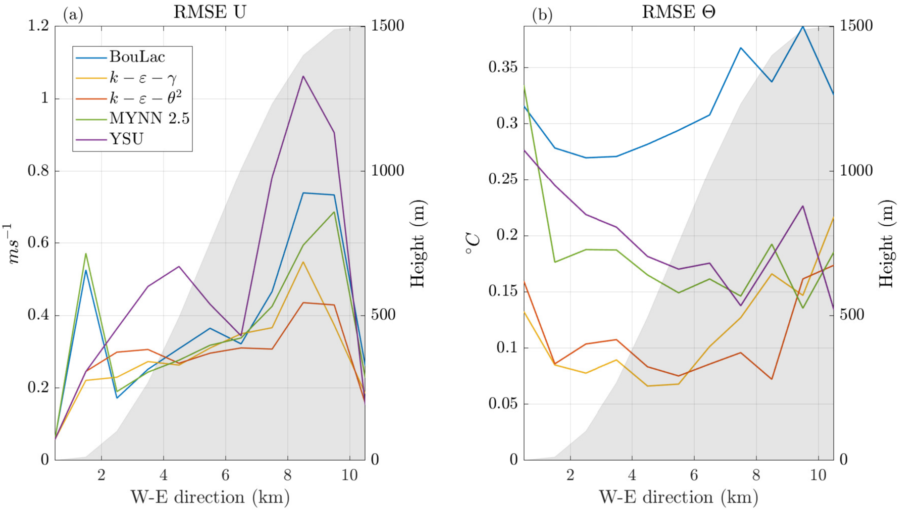
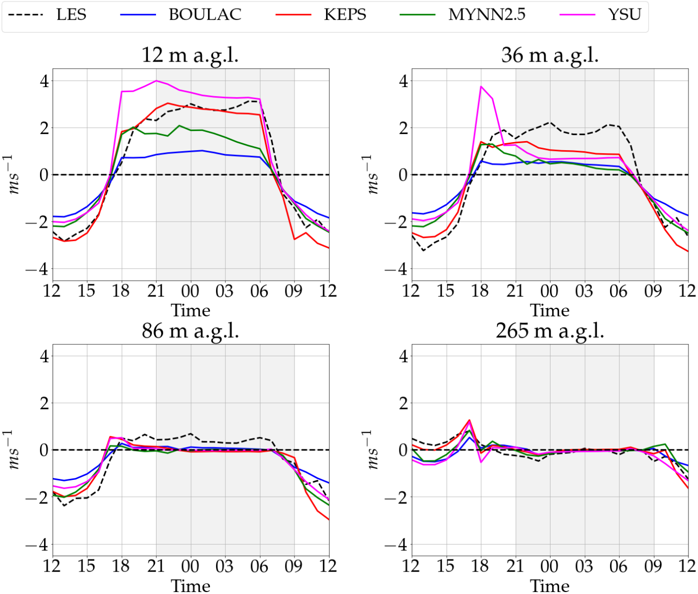
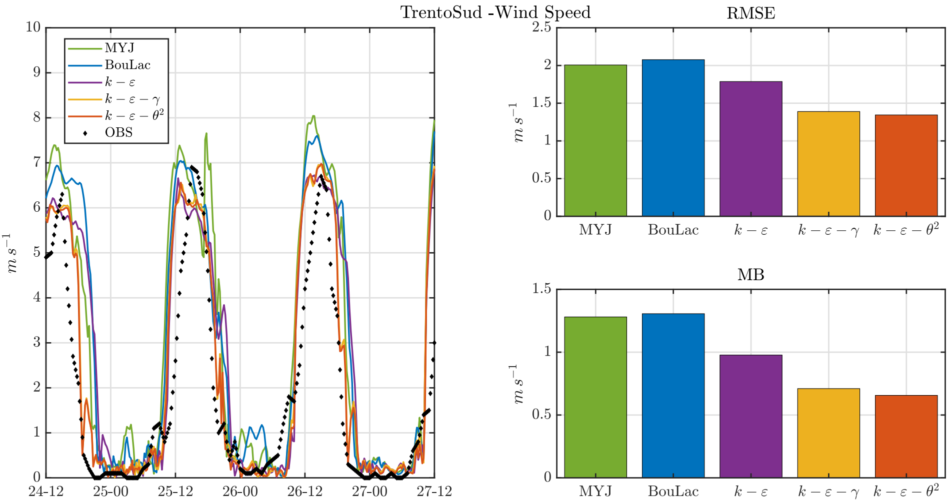


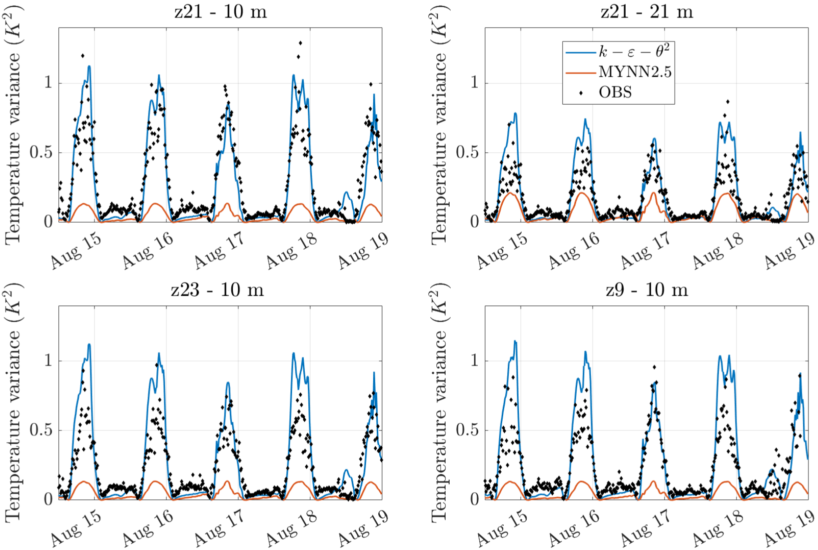
3D - Complex terrain - LES (100 m) vs RANS (1 km)



Real 3D - SIM vs OBS - 10-m wind speed



Real 3D - WFIP2 - Temperature Variance



3) $K-\varepsilon$ turbulence closure - Conclusions

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Reference: A. Zonato, A. Martilli, P. A. Jimenez, J. Dudhia, D. Zardi & L. Giovannini, *A new $K-\varepsilon$ turbulence parameterization for mesoscale meteorological models*, Monthly Weather Review, 2022.

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