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ORIGINAL RESEARCH PAPER

Classification of scale-sensitive telematic observables for riskindividual pricing

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Abstract Using modern telematics technologies for car insurance, it is no particular challenge to produce an intractably large amount of kinematic and contextual information about driving profiles of motor vehicles. In order to evaluate this data with respect to both efficient and effective use in scoring and subsequent actuarial pricing, we propose a scale-sensitive approach that treats observations on semantically different levels. Furthermore we discuss the application of methods necessary to assess the information of different scale levels including signal processing, pattern recognition and Fourier analysis. In this way we show how maneuvers, trips and trip sections as well as the total insurance period can be analyzed to individually or collectively gain significantly scoreable insights into individual driving behaviour.

Keywords Telematics technologies · Car insurance · Driving behaviour · Pattern recognition · Actuarial pricing

1 Introduction

With the recent increase of dynamics in the field of research regarding telematics technologies for car insurance, it has become ever clearer that sensible assessment of kinematic driving profiles is indeed possible. While tariffs based on the telematic

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measurement of mileage, vehicle speed, frequency of hard starts and stops and time of day are well-established in the U.S. and several European countries [1, 3, 12], the German car insurance market only sees first insurers introducing such offers. Since car insurance market is the largest line of business in the property and casualty insurance industry [10], innovations and market dynamics are expected to stay high and lead to further product differentiation [11]. To this end, insurers aim to analyze both the kinematic driving behaviour of the vehicle user as well as the context, i.e. the circumstances of the driving situation, including information about road type, lighting, weather conditions etc. [12].

The question, however, what kind of inferences can be drawn from the vast amount of data collected over real portfolios is met by the computational challenges of storing and processing these large amounts of information. Combining basic findings of the still-new science of data mining with the preliminary results of actuarial investigations of telematic test portfolios proved fruitful, however, leave open which variables of telematic assessment are precise and semantically correct [14, 16]. It is clear that univariate exposure-related driving measures can act as first estimators [12]. However, there are also studies suggesting that in order to fully represent the insured risk adequately, one has to work with multivariate models assessing driving styles [23].

A fundamental finding of this contribution is expanding the former approach by analyzing all substantial granularity levels of telematic measurements, both individually as well as collectively, in order to fully capture the complete driving situation. Therefore, we take into account a framework where different observations are treated on various levels of time resolution ranging from maneuvers taking place on a scale of seconds up to a complete insurance period.

The present article aims to treat the topic with regard to two elementary methods of data processing—pattern recognition and Fourier analysis. Up to now, a feature-based recognition of patterns in data is the goal of a rapidly growing number of fields, in particular in computer science, information theory and optical processing techniques [21], where a wide variety of techniques have been invented, ranging from text- and speech recognition and fingerprint analysis over artificial learning algorithms to computer-aided diagnosis systems [17, 26]. Besides, it can be assumed that those data science techniques will be also applied in the insurance industry in the foreseeable future [12, 20]. In this paper, we focus on distinguishing images that are transformed from original observations by discrete Fourier analysis (DFT). The DFT is known for both its computational efficiency [4] as well as its power to represent and parametrize a signal into its constituent frequency components [27], posing itself a very useful input to pattern recognition and multivariate cluster analyzes [18].

The main contribution of this paper is to show that the presented methods of pattern recognition and Fourier analysis are suitable to find new insights regarding differentiation potential associated with driving behaviour and vehicle use on various scale levels of telematic driving profiles. In particular, these results, given a priori risk experience, allow incorporating telematic risk factors into actuarial pricing decisions. Moreover, it can be concluded that overarching driving behaviour trends, observed over long periods in the whole telematic portfolio or homogenous segments thereof, can be used to benchmark or crosscheck classical tariff assumptions.

The remainder of this paper is divided into three sections. Following the definition and explanation of methods in Sect. 2, Sect. 3 deals with the application to exemplary, albeit real telematic data sets of all introduced scale levels. Section 4 concludes by pointing out the relevant usage scenarios with respect to implementing telematic scoring into car insurance tariffs.

2 Method

2.1 Scales of driving profiles

Collected telematic driving profiles supply comprehensive information on vehicle use and driving behaviour, provided that the measurand selection is adequate in terms of both quality and quantity. However, in order to extract the essential information for identification and evaluation of a risk profile, it is important to distinguish between different scales [23]. If, for example, an overtaking maneuver is performed, high acceleration values need to be handled in a different way from an entire sporty driving style. Therefore, the assessment of telematic values is to be made by taking into account the general situation characterized by the road condition, the traffic state and, finally, the specific driving maneuver, which are only recognized completely on the basis of various sets of segments at the same time.

Furthermore, we need different degrees of granularity of the measured data depending on the problem, as not every situation can be presented equally effectively at the same level. To answer certain questions e.g. concerning signs of fatigue, we have to examine a whole measurement series, whereas questions concerning specific maneuvers can not be satisfactorily answered on the basis of aggregates.

We therefore propose scales ranging from styles per insurance period to individual driving styles per driving maneuver (see Fig. 1). At the highest level, the

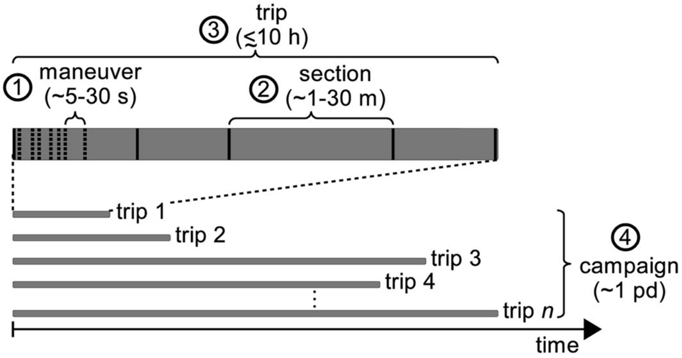

Fig. 1 Qualitative specification of the scale level: campaign > trip > section > maneuver

data is gathered over up to a whole policy period. This is the scale at the highest aggregation level—consisting of the least detail of all the scales. It offers information on general driving behaviour within defined time horizons. In particular, in this manner, a gradual change in driving behaviour during the policy period can be detected, e.g. due to changed driving ability of individual vehicle users. Another option is to investigate the driving one level below, for each individual vehicle trip. In particular, this will enable insurers to measure shares of different users [24][1]. For an even more detailed evaluation it is useful to divide driving trips into separate sections. The section level allows us to analyze individual driving habits under specific conditions such as several road types, i.e. national roads, urban roads and highways, or various road and viewing conditions due to weather influences, i.e. fog, rainfall, snowfall, ice, darkness glare from direct sunlight [23]. Moreover, it may sometimes be of interest to compare a series of sections among themselves, e.g., for identification of less reactivity on long trips or due to distraction. Lastly, at the most fundamental and granular level, the recognition of single maneuvers forms the basis for high-resolution scoring. Not only do the possible characteristics of the measured values depend strongly on the maneuver type (see Sect. 3), but also the type of accident [5, 8, 9]. If overtaking maneuvers, parking maneuvers or turning maneuvers (considering contextual information such as local weather conditions or current traffic flow) are not identified, there may not be enough precision to assess telematic data risk adequately.

It is important to bear in mind that—on every level above the most granular—we can perform statistical analysis of the findings of the (lower) level, i.e. examine the frequency of section or maneuver types and their respective classification. In this way a full score of full driving profiles merges information accessible from different time scales.

Note that there are not just specific driving situations, e.g., the already mentioned overtaking maneuvers, which are properly recognized if the finest granularity is ignored, but also situations which are not processed correctly if only the maneuver level is considered. Certain road conditions or traffic volume, for example, can force the driver to understeer or oversteer the vehicle in curves, so that general statements about individual driver characteristics can only be made until detailed analyzes of longer time periods have been carried out. This indicates that an adequate risk projection of risk does require a closer look at measured data on various scales discussed in more detail below.

2.2 Pattern recognition

A complete movement period process of a particular segment k—e.g. a single maneuver or a whole trip (see Sect. 2.1)—is given by a signal sequence $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}$ of a multi-dimensional vector of all physical measurable factors of driving behaviour as well as driving situation. Whereby the parameter j indicates a selected

[1] Not the insured risk, i.e. the vehicle, but the vehicle driver causes insurance losses. A driver recognition is therefore essential for an adequate assessment of the factual, underlying risk.

vehicle and the discrete time parameter t denotes the scanning signal of the vehicle movement. The task is to compare different sequences and to decide whether they are correlated or not in order to rate them with regard to its inherent risk. First of all, the prerequisite for that is pattern recognition.

Definition 1 (following [15]) A pattern is defined by the set of sequences $\Omega = \{(m_{k,t}^{(j)})_{t=1,\ldots,n_k}\}$, for all observed vehicles $j = 1, 2, \ldots, v$ and measured segments $k = 1, 2, \ldots, s_j$, that are characteristic of a driving style.

The dimension of the feature vector is the same for all sequences, which, however, is different for various purposes. It is important to bear in mind that driving maneuvers can only be fully recorded if the feature vector contains all multiple interdependent variables. Prematurely excluding dependent variables from the sequences, information is lost and false conclusions are drawn concerning the risk (see [23]).

Following up on the extraction of patterns $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}$, we need to classify them:

Definition 2 (following [15]) For pattern classification, every sequence $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}, j = 1, 2, \ldots, v$ and
$k = 1, 2, \ldots, s_j$, is assigned to exactly one class Ω_λ, $\lambda = 1, 2, \ldots, \tau$:

$$(m_{k,t}^{(j)})_{t=1,\ldots,n_k} \mapsto \Omega_\lambda, \tag{1}$$

whereby the following conditions are applied for the partition $\Omega = \{\Omega_1, \Omega_2, \ldots, \Omega_\tau\}$:

$$\Omega_\lambda \neq \emptyset, \quad \lambda = 1, 2, \ldots, \tau, \tag{2}$$

$$\Omega_\iota \cap \Omega_\kappa = \emptyset, \quad \forall \iota \neq \kappa, \tag{3}$$

$$\Omega = \bigcup_{\lambda=1}^{\tau} \Omega_\lambda. \tag{4}$$

There is a variety of ways to achieve a partition of patterns complying with these conditions, but only a few of them are of real interest for practical applications. An interesting partition is characterized by the fact that the classification criteria are likely to result in classes that meaningfully differentiate the risk. This will require an immediate correlation between various patterns of specific classes and historical claims experience. The precondition is that different accident causes and types can be linked with patterns of different classes. Consequently, patterns in the same class are as similar as possible and patterns in different classes are as different as possible. However, in order to satisfy the condition of disjointness, not only the classifier but already the segmentation is the decisive factor. In particular, it is possible that a maneuver shows different patterns, that is, different driving styles. If various patterns are part of a single maneuver, then we can take into account two aspects: a finer division of several segments on the one hand and the introduction of mixed

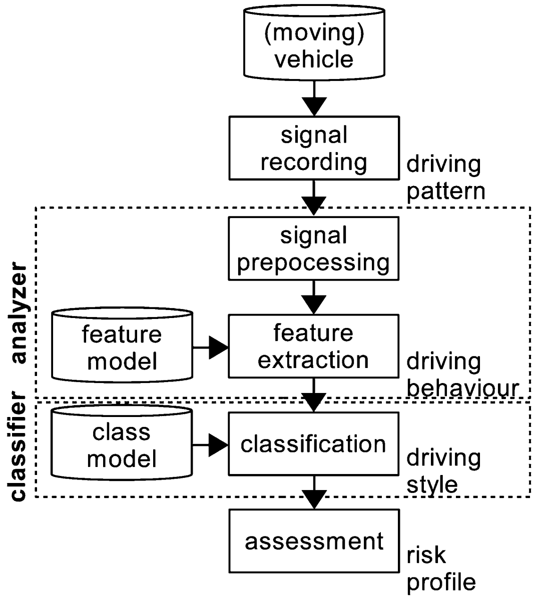

Fig. 2 A pattern recognition principle (following [6, 15])

classes on the other hand [15]. It is nevertheless considered appropriate to require disjoint classes per scale, as the driving behaviour cannot be simultaneously assigned to more than one driving style.

Finally, please note that classes of different scale levels are not comparable. On each scale, specific driving characteristics are to be classified. Hence, at each level, we characterize scale-sensitive patterns for information extraction, such as overtaking maneuvers, parking maneuvers or turning maneuvers at the maneuver level, series of maneuvers at the section level etc. The non-comparability of course is true only for pattern classification. Concerning risk assessment, the statistical dependence of classes of various scales cannot be ruled out.

Figure 2 shows the fundamental procedure in recognition of patterns, broadly splitted into three steps: preprocessing, feature extraction and classification [6, 15]. The key idea within telematic tariff calculation is the data collection and processing, in particular, for the building of driving profiles, which represent the patterns to analyze.[2] Thus, there will be large data sets which have to be preprocessed in order to reach both a qualitatively good database and a tractable data volume. Essential operations in preprocessing are verification of data plausibility[3], data cleansing and reduction without loosing relevant information as well as data integration and transformation in order to adapt the measured data to a target structure. This

[2] Therefore, a sophisticated logic for measuring and transmission is the decisive quality factor, clarified further in Weidner and Transchel [23].

[3] At this point fraud prevention, i.e. an extensive detection of data manipulation, should already be started. For example, a comparison of the measured data of a single vehicle and measurements of the whole ensemble is conceivable for this.

includes a segmentation of driving profiles in different scale levels in which driving situations are separated from each other (see Sect. 2.1).

The extraction of features from the conditioned data takes place in the next stage. Here, the aim is to uncover the essential structure of individual patterns or parts thereof and to make them understandable in order to achieve an assigning to special classes. What kind of features embodied in each pattern is relevant depends on the respective information and situation to be probed. Possible features are measures of central tendencies and dispersion, e.g., moments of the distributions such as the expected value or variance, the full ranges of the distribution, i.e. the minimum and maximum values as well as percentiles of the distributions such as the median or the interquartile range, which combine the characteristics of the measured patterns. In addition, information on the absolute frequencies such as the number of modes of the distributions, may also be of interest. All of these figures can be estimated using statistical methods in the feature extraction stage.

Apart from that, driving sequences can be dismantled into components and their relationships, i.e. into the frequency spectrum, using a transformation such as the discrete Fourier transformation (DFT). In contrast to classification methods such as cluster analysis the Fourier transformation is linear in its input and maps the signal to a dual space, i.e. from spatial coordinates to phase space, or the frequency domain [2]. Due to its linearity, measurement operators like the derivative can also be applied on the level of phase space, such that intrinsically, no information is lost in the transformation itself. Therefore, we resolve to the study of frequency components with the intention to investigate whether, at certain time scales, there are statistically significant outliers in driving behaviour when comparing discrete Fourier spectrums of comparable processes. For example, we could ask the question whether we can deduce driver reaction times from pronounced Fourier coefficients (see Sect. 3).

Definition 3 (following [27]) Let $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}$, with $j \in \{1, 2, \ldots, v\}$ and $k \in \{1, \ldots, s_j\}$, be a measured pattern. The discrete Fourier transformation (DFT) of $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}$ is defined by

$$\begin{pmatrix} \mu_{k,1}^{(j)} \\ \mu_{k,2}^{(j)} \\ \vdots \\ \mu_{k,n_k}^{(j)} \end{pmatrix} = DFT \begin{pmatrix} m_{k,1}^{(j)} \\ m_{k,2}^{(j)} \\ \vdots \\ m_{k,n_k}^{(j)} \end{pmatrix}, \tag{5}$$

where

$$(\mu_{k,t}^{(j)}) = \frac{1}{n_k} \sum_{l=0}^{n_k-1} m_{k,l+1}^{(j)} \cdot \left[\cos\left(\frac{2\pi \times t \times l}{n_k}\right) - i \sin\left(\frac{2\pi \times t \times l}{n_k}\right) \right], \quad t = 1, \ldots, n_k. \tag{6}$$

The frequency spectrum consists of the DFT-coefficients $(\mu_{k,t}^{(j)})_{t=1,\ldots,n_k}$ with period n_k of standardized frequencies $\frac{l}{n_k}, l = 0, 1, \ldots, n_k - 1$.[4] Basic properties of the DFT can be found in [7, 22, 27]. However, we would like to make the observation that, with respect to the Hermitian symmetry [2] of driving profiles, consisting of real numbers only, the DFT simplifies to

$$\mu_{k,n_k-t}^{(j)} \equiv \mu_{k,-t}^{(j)} = \overline{\mu_{k,t}^{(j)}} \; \forall \; m_{k,l+1}^{(j)} \in \mathbb{R}, \quad t = 1, \ldots, n_k, \; l = 0, \ldots, n_{k-1}. \tag{7}$$

It follows that in the frequency domain, there exist independent complex coefficients for only $\frac{n_k}{2} + 1$ frequencies, whereby the zero and folding frequency always are real numbers.

The last step for pattern recognition is to either classify the features in classes or to assign the features to known patterns. The classes can be determined differently [6, 15]: On the one hand the generation of classifiers may be possible in advance for some cases; this can be done based on either a representative sample or wide-ranging expertise. These classifiers subsequently can be applied to assign the patterns. On the other hand classes can be formed directly from measured, classifiable data. Classes for driving behaviour can in each case be build automatically using clustering algorithms regardless of prior knowledge [24].

Following to the classification, the risk of individual patterns can be assessed including supplementary information to get risk profiles (see Sect. 2.3 for further details). Is there no other alternative as to process risk information heuristically, i.e. without any statistical confirmed claims experience, in addition, a pattern analysis should be carried out in order to facilitate a more illustrative understanding of the various class properties.

2.3 Scoring of driving behaviour

Currently, there is neither in science, nor in practice a standardized method to achieve a clear scoring of driving behaviour. Thus, for risk assessment, we define a scoring function which maps a set of random variables (hereafter referred to as "driving style"), representing an individual pattern $(m_{k,t}^{(j)})_{t=1,\ldots,n_k}, j \in \{1, \ldots, v\}$ and $k \in \{1, \ldots, s_j\}$ derived from measured data recorded by telematic systems, to the real numbers.

Definition 4 Let $X = (X_1, \ldots X_n)^T$ be a vector of random variables. The scoring function[5] is a differentiable function $s(X) : X \rightarrow [0, 1]$.

[4] Since we are not interested in back-transforming examined frequency data, the choice of normalization is somewhat arbitrary. When looking at the self-duality of $F(F(f(x))) = f(x)$, then a choice of $1/\sqrt{(}n_k)$ for both the transform itself and the conjugate transform respectively is more reasonable from a mathematical point of view.

[5] Although the random variables generally represent discrete signals whose information parameters take only a finite number of values, the differentiability of the scoring function $s(X)$ can be guaranteed by means of interpolation, provided that sufficient values within certain limits are available.

W.l.o.g. we consider the case in which the maximum desirable behaviour, the driving style with the lowest risk, is assigned the value 1 and the maximum adverse behaviour, the driving style with the highest risk, the value 0 [23]. Furthermore note, that the differentiability (and thereby implicitly the continuity) in the domain of the function s(X) is required for applicability of functional analytic methods without restrictions. In particular, such a scoring function s(X) is designed to stretch locally convex vector spaces, as used in the following.

In addition to the assessment of statistically distinguishable driving styles, also an assessment of different driving style combinations is of particular importance on each scale level for the practical application. The driving behaviour belonging to an insured vehicle and, therefore, the driving styles at any scale level differ not only by several users, but also by used road types, weather conditions or the traffic volume. This means that an insured vehicle can be assigned to different driving styles both during a trip and within a given period [23, 24]. The logical next step is to develop an overall scoring function for driving style combinations[6]

$$\phi_{total}^{sc} = \alpha_{sc,1}\phi_{sc,1} + \alpha_{sc,2}\phi_{sc,2} + \cdots + \alpha_{sc,n_{sc}}\phi_{sc,n_{sc}} \tag{8}$$

on each scale level $sc = 1, \ldots, m$ with shares $\alpha_{sc,l} \geq 0,\ \sum_{l=1}^{n_{sc}} \alpha_{sc,l} = 1$ and specified driving styles $\phi_{sc,l} \equiv \mathbf{e}_l \in \mathbb{R}^{n_{sc}},\ l = 1, \ldots, n_{sc}$.

For this, we can use the $(n_{sc} - 1)$-simplex determined by n_{sc} different driving styles. Specially, if we assume w.l.o.g. that $\phi_{sc,l} \equiv \mathbf{e}_l \in \mathbb{R}^{n_{sc}-1},\ l = 1, \ldots, n_{sc} - 1$, and $\phi_{sc,n_{sc}}$ is linearly dependent of the subset $\{\phi_{sc,1}, \phi_{sc,2}, \ldots, \phi_{sc,n_{sc}-1}\}$, the $(n_{sc} - 1)$-simplex is the simplest $(n_{sc} - 1)$-dimensional polytope which stretches the convex vector space of all scoring functions. With this, we make use of the fact that a bi-comparative score can be performed on pairs of driving styles instead of the whole ensemble [23].

Definition 5 Let V be a vector space given by

$$V = \bigoplus_{sc=1}^{m} \mathbb{R}^{n_{sc}}. \tag{9}$$

An overall scoring function across all scale levels is given by $S : V \to [0, 1]$, mapping convex linear combinations on a simplex to the unit interval:

$$S(\phi_{total}) = \frac{1}{\sum_{sc=1}^{m} d_{sc}} \sum_{sc=1}^{m} d_{sc} \binom{n_{sc}}{2} \sum_{<i,j>}^{n_{sc}} s\left(\frac{\alpha_{sc,i}}{\alpha_{sc,i} + \alpha_{sc,j}} \cdot \phi_{sc,i}, \frac{\alpha_{sc,i}}{\alpha_{sc,i} + \alpha_{sc,j}} \cdot \phi_{sc,j}\right) \tag{10}$$

for an entire driving style combination $\phi_{total} = d_1\phi_{total}^1 \oplus d_2\phi_{total}^2 \oplus \cdots \oplus d_m\phi_{total}^m$ across all scales, where

$$\phi_{total}^{sq} = \alpha_{sc,1}\phi_{sc}^1 + \alpha_{sc,2}\phi_{sc}^2 + \cdots + \alpha_{sc,n_{sc}}\phi_{sc}^{n_sc},\ sc = 1, \ldots, m \tag{11}$$

[6] In particular, this means that the classification is executed separately for each scale. Classes from different scales can resemble each other (despite the lack of comparability) by reason of similar contents, but the risk assessment of those classes can vary according to the context.

constitutes the driving style combination on each scale level with shares $\alpha_{sc,l} \geq 0, \sum_{l=1}^{n_{sc}} \alpha_{sc,l} = 1$ and specific driving styles $\phi_{sc,l} \equiv \mathbf{e}_l \in \mathbb{R}^{n_{sc}},\ l = 1, \ldots, n_{sc}$. In addition, the combination of different scales requires scale-specific weighting factors $d_{sc} \geq 0, \sum_{sc}^{m} d_{sc} = 1$.

3 Results

3.1 Maneuver scale

The research issue is to investigate the driving style at the most granular level. It is essential to this objective that specific driving maneuvers are extracted from driving profiles to analyze them. Fig. 3 shows the velocity processes together with the acceleration behaviour in the longitudinal respectively lateral direction deduced from single, albeit representative sample trips. A starting after a standstill is characterized by an increase in velocity as result of high longitudinal acceleration values (see Fig. 3a), whereas the velocity decreases in conjunction with high deceleration values during a braking process due to an obstacle (see Fig. 3b). The lateral acceleration, i.e. acceleration values to left or right when driving a turning maneuver, is of little interest in this context; for the sake of the clarity only the longitudinal acceleration is plotted in these both maneuvers. Note that the descending absolute values in the acceleration (deceleration) process at velocities of around 15, 25 and 35 km/h result from the change into a higher (lower) gear.

However, the lateral acceleration is the important parameter to determine steering or even a tack and turning maneuver. In the example (see Fig. 3c), positive acceleration values represent a right turn and negative acceleration values left

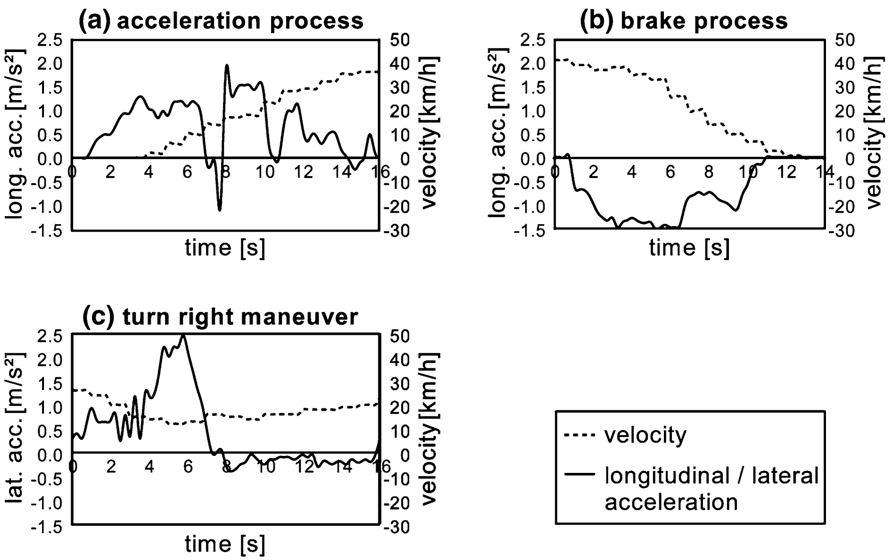

Fig. 3 Sample illustration of three specific driving maneuvers

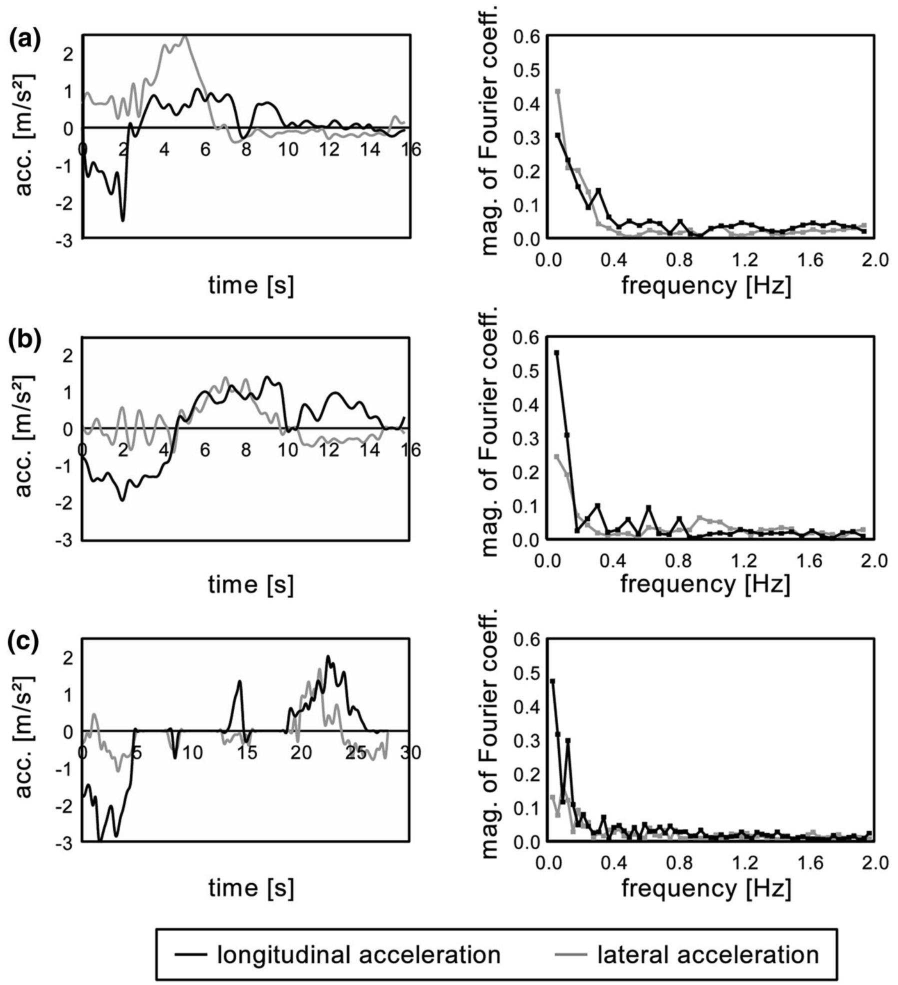

Fig. 4 Turn right maneuvers at the same crossroad (left) and their respective DFT spectrum (right)

steering respectively. The velocity (and accordingly, the longitudinal acceleration), which is reduced here before the steering force increase significantly, primarily provide additional information concerning curves in order to grade the level of risk.

Not only do the maneuvers of various maneuver types differ significantly, but also samples of the same type, to which Fig. 4 gives an exemplary overview for the acceleration behaviour including the associated Fourier coefficients.[7] The processes (see Fig. 4, left) obviously show various measures of central tendency and

[7] Only half of the evolution coefficients are necessary for representing of DFT-results due to Hermitian symmetry (see Sect. 2.2). Therefore, since the driving maneuver is recorded with 4 Hz, Fig. 4 (right) represents the magnitude of Fourier coefficients for frequencies from 0 to 2 Hz. Both, zero and folding frequency are neglected here because they do not provide any information about the maneuver structure.

dispersion, which are summarized in the "Appendix". Furthermore, the Fourier analysis (see Fig. 4, right) provides direct information on the maneuver structure. The type and shape of oscillations with varying frequency and amplitude occurring in a concrete maneuver are specific: Basically, a balanced driving style with low short-periodic amplitudes is characterized by low Fourier coefficients at higher frequencies, whereas high Fourier coefficients at low frequencies reflect a generally increased level for acceleration values. In the example, the coefficients vary greatly with regard to the small frequency range. For instance, the weight of the coefficients of the five lowest frequencies concerning lateral acceleration of Fig. 4 (a) is twice as high as the weight in (b) and (c). Apart from the coefficient at the lowest frequency (0.0625 Hz, i.e. a period of 16.00 s)[8] this is due to the high coefficient at the third lowest frequency (0.1875 Hz, i.e. a period of 5.33 s), which reflects the period of the effective turn over the entire lateral acceleration process. With respect to the longitudinal acceleration, in particular, note that the relatively low coefficient for the frequency of 0.1875 Hz in Fig. 4b is compensated in the frequency range up to 0.8 Hz.

In the high frequency range, it must be kept in mind that the turn right maneuver illustrated in Fig. 4c takes up to 30 seconds because of a standstill of 10 seconds at a crossroad. From a purely technical viewpoint, this initially leads to frequency increments half the size of those in Fig. 4a, b. Moreover, the acceleration behaviour automatically seems to be more balanced due to the long standstill. However, for all three examples, it is clearly evident that the noise in small period is reflected in Fourier coefficients with higher frequencies. For example, the coefficients concerning the lateral acceleration around the 1 Hz frequency in Fig. 4b illustrate the oscillation of the lateral acceleration up to 4 seconds.

We can conclude that magnitudes of Fourier coefficients can be considered as relevant characteristics for differentiation both between and within specific maneuver types and thus allow a classification of maneuver structures, when combined with measures of central tendencies and dispersion. The final evaluation of specific maneuver structures concerning to the risk is to be made empirical on the basis of claims expectations obtained from intuition or real experience. It seems therefore useful to build up an extensive claims history in the long term, by analogy with meaningful risk statistics of the German Insurance Association across the market legalized by the block exemption regulation 2010 (insurance sector).[9] However, the analysis of general accident statistics [13, 19] is initially conceivable for a transitional period.

3.2 Section and trip scale

Driving behaviour analysis over longer time is the basis for determining a comprehensive driving style under specific environmental conditions. Therefore, driving profile structures divided into sections or even trips only provide the foundation. This chapter deals with the exemplary classification of sections by road

[8] The following relationship exists for frequency f [Hz] and period duration T [s]: $T = \frac{1}{f}$.

[9] VO (EU) Nr. 267/2010.

type.[10] To this end we observe the kinematic telematic data of a single, albeit representative trip that can be decomposed into three separate sections by road type discrimination; in Fig. 5 (above) there are two vertical lines indicating the section limits as well as labels for the respective road type. From the position space data we clearly observe that the three sections differ in each observed parameter, the velocity distribution as well as longitudinal and lateral acceleration behaviour.

Moreover, Fourier coefficients (see Fig. 5, below) provide significant information about the profile structure, which is not reflected in statistical measures describing the distribution in highly aggregated form only (see "Appendix").[11,12] However, it should be noted that we do not analyze intrinsically periodic signals (as in the case of maneuvers recurring periodically), but try to identify hidden periodicity. For that, it is useful to fit the measured data in the DFT plots as the deviations of the coefficients from the fits indicate these characteristics.[13]

As expected, driving in an urban environment again shows unsteady levels of traffic flow, indicated by high fluctuations in the acceleration behaviour, whereas the maximum speed is limited to roughly 50 km/h. The steering behaviour is fluctuating as well. With this in mind, it is not surprising that the DFT spectrum of the city sections is qualitatively bumpy, indicating unsteady driving according to the traffic environment of cities. By looking at this single instance of a section and its Fourier transform, we show that it deviates heavily from the expected average value (see the fitted lines). With a portfolio large enough it is therefore possible to obtain

[10] Since the driving behaviour naturally differs between different road types, we chose this particular selector to highlight the significance of the Fourier analysis of driving profiles. Other conditions and contexts are also possible, for example differentiation with respect to weather/light conditions or time of day.

[11] For the DFT, we reduced the degree of detail to the mean value of 4 seconds, i.e. to a frequency of 0.25 Hz; we thus abstract the information to the required precision at the section level. Therefore, by reason of Hermitian symmetry (see Sect. 2.2), Fig. 5 (below) represents the magnitude of Fourier coefficients for frequencies from 0 to 0.125 Hz. The zero frequency, which generally reflects the homogeneity of the telematic measurements, is also shown.

[12] Instead of processing the signal data directly, one can resort to model the auto-correlation function instead, given by $R_{m_k^{(j)}}(\tau) = \frac{1}{n_k}\sum_{t=1}^{n_k} m_k^{(j)}(t) m_k^{(j)}(t+\tau)$, where $\tau \in \mathbb{N}$ is the probed period length. However, to address the question when to use which quantity, there is no sufficient portfolio data available to define figures of merit for this particular case and this comment should be considered for brevity.

[13] We fitted the DFT spectrum after the following considerations: Due to Parseval's theorem, the total energy of the process (and thus, in the discrete case, the sum of all Fourier coefficients) in the frequency domain matches the integral over the velocity data in positional space. As a consequence from the fact that frequency is proportional to the inverse period, we fit the velocity transform by $F(x) \sim \mu_{k,2}^{(j)} \cdot e^{-0.57 \cdot x^{0.5}}$, where the scaling factor $\mu_{k,2}^{(j)}$ is the coefficient of the greatest period length, due to the expectation that higher frequencies must contain exponentially less energy in order to get a bounded expression. It appears as an exponentially less steeper line in the log plots. Although the longitudinal acceleration transform depends linearly on the velocity transform, its fit is almost linear due to the fact that the range of the coefficients is reduced compared to the velocity transform. It is thus sufficient to fit it by a linear expression as mean value for each Fourier coefficient per a portfolio average to indicate the expected distribution according to the central limit theorem that appears as a slightly exponentially decreasing function in the log plots. The dependent part of the lateral acceleration transform shows, like the longitudinal dimension, a limited range. Therefore, even without direct connection with the energy as in the case of the longitudinal acceleration, we fit it by a linear expression too.

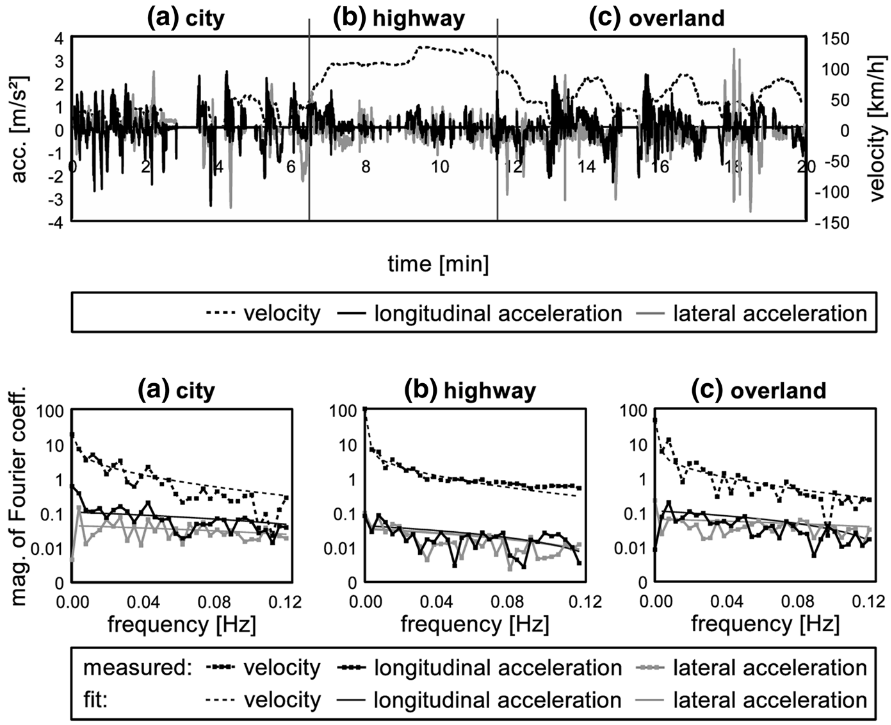

Fig. 5 Driving behaviour during a single trip separated according to road types (above) and their respective DFT spectrum (below)

significant differentiation of both measures of central tendency and spread and its associated Fourier coefficients.[14]

Since driving behaviour on highways is highly linear, both positional space measurements as well as their Fourier coefficients are very smooth[15] in comparison to urban traffic environments. As such, most sections will match well with the expected distribution of measures of central tendency and spread as well as their respective DFT spectrum and the expectation values in mean (again depicted by the fitted lines). We conclude two important points from this observation: First of all, deviant and possibly hazardous driving behaviour on highways is easier to discover than in other environments, and secondly, it means we can economize on measured data, e.g. by limiting the DFT to even fewer coefficients. Since highways are the

[14] As basic property of Fourier analysis, the longitudinal acceleration behaviour must differ from the velocity coefficients only by a constant as the former is the time derivative of the latter. As such, plotting and analyzing both measurement parameters is redundant if they are derived from the same source. If, however, as is the case with some of the emerging hardware solutions, velocity is obtained from GPS measurement whereas acceleration data is measured by respective acceleration sensors, the comparison at hand provides either redundancy or plausibility checks of either device.

[15] "Smoothness" of the DFT spectrum of the acceleration behaviour is meant here as in the sense of small coefficients.

least risky road type anyway [19], it seems reasonable to limit, at the level of sections, costly pattern recognition algorithms to a minimum.

Interurban driving sections consist of quasi-periodic transitions of overland parts with (short) village transits characterized by two distinct modes of operation: In the village the maximum speed is lower than in-overland, but usually gives rise to less prominent acceleration values both, in longitudinal and lateral direction. As a consequence, in the overland parts, this kind of road type consists of many strong (longitudinal) acceleration and deceleration processes that are all of comparable scale. It is therefore prone to analysis of its discrete Fourier transform since the driving behaviour forced by the environment is as close to periodic as possible, which can be observed in section (c) of Fig. 5 (above). Consequently, the Fourier coefficients of this example trip vary greatly both in longitudinal as well as lateral direction; compared to the expected mean value [fitted line in section (c) of Fig. 5, below], overland sections show extremely diverse Fourier coefficient distributions. This indicates that at the level of individual frequency ranges a lot of information about the driving style can be acquired once a broad data base is obtained, e.g. reaction times as well as over- or understeering in roads following twisting terrain.

In conjunction with cluster analysis methods merging measures of central tendency and spread together with quantitative Fourier analysis of the respective frequency ranges, we can match the observed telematic driving profile with driving styles obtained from, e.g., Weidner et al. [25]. We consequently obtain scoreable results for both individual sections as well as full trips.

It is important to note however that the analysis of full trips is, although technically identically obtained, treating a different time scale than sections. Its analysis aims to compare trips of possibly vastly differing characteristics, whereas on the level of sections each observation (i.e., the observed stretch of a singular category, e.g., driving at a certain time with certain weather conditions on a certain road type) should be directly comparable. In practice, the level of contextual discrimination depends both on the contextual data available as well as the portfolio size in order to obtain significant results.

3.3 Insurance period scale

While the previous scales could in principle be attributed to ad-hoc data, analysis on the level of the insurance period, whenever available, aims to describe long-term behaviour that cannot be deduced from single trips. This highest aggregation level still can be of use in order to gain knowledge about the claims expectations, most notably by exploring variables that potentially change over the time of the insurance period. For example, on the level of maneuvers and sections, adding context information about weather or traffic conditions can be used to assess whether the driver conforms to the imminent environmental requirements, but not whether he is consistent over time in his behaviour. To this end we can examine, either on the level of the full insurance period or on aggregated time periods such as weeks, months or quarters, questions about the superordinate driving behaviour by aggregation. This level thus shares most similarity with regards to classical insurance tariff calculations, e.g. by comparing mileage or adherence to the overall

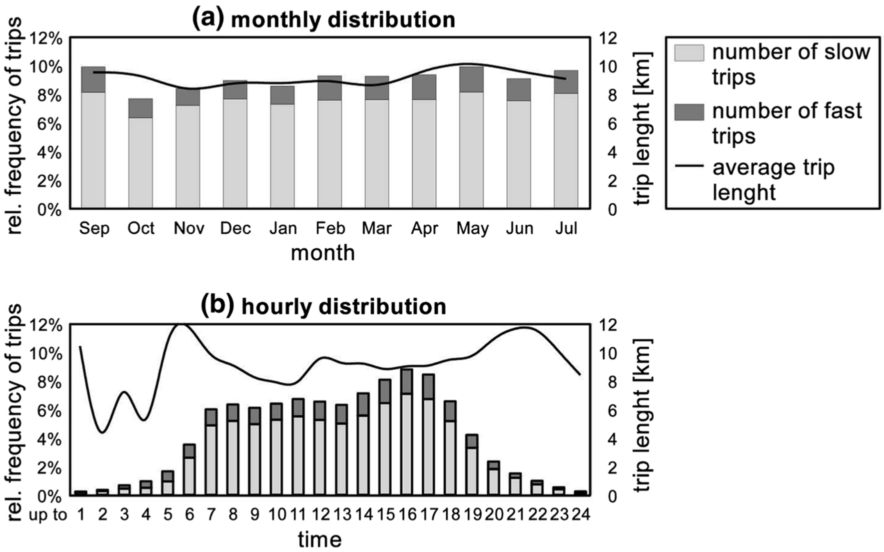

Fig. 6 Vehicle trip distribution of the telematic portfolio per month and according to time of day

regio class with the actual position of the car. Furthermore, we can compare driving styles with respect to various indirect measures, such as whether the driver undertakes regular commute trips and their respective influence on the total claims expectations.

To highlight these mentioned statistical measures, we present results from a telematic portfolio of a German car insurer that covers 199,388 trips with a total length of 1,836,123 km from several hundred vehicles for a period of 11 months (see [24], for more details). Figure 6 shows the percentage distribution of the recorded trips and the average trip length per month respectively according to time of day; a differentiation is made between the share of trips attributed to either slow or speedy driving behaviour.[16]

There are several observations to be drawn from the data. First of all, in the winter months (October to February), both total number of trips as well as average trip length see a significant decrease, the former being most notable in October due to German fall holidays occurring in this month. The effect of school holidays resulting in longer, albeit fewer trips can also be observed prominently in April and June; the examination of the August school break most prone to holiday traffic jams is unfortunately not part of the test interval, but we nevertheless conjecture the effect to be amplified for that month. Moreover, we can observe that the faster

[16] The main point of investigation in Weidner et al. [24] was to determine representative driving styles differentiating the velocity and longitudinal acceleration behaviour for each trip. In their research, they find two groups of driving styles depending on the velocity process—with the same or similar acceleration and deceleration value distribution—those with lower (i.e. up to 50 km/h in the mean) and those with higher velocity values (i.e. more than 50 km/h in the mean), whereas no differentiation regarding road type was made.

driving style shows a significantly reduced share during the winter months, allowing us to conclude that on average the telematic portfolio drivers adapted to the more dangerous road conditions and less visibility in the winter term.

Using the time of day as the primary differentiation parameter, we observe that during the night time there are considerably less trips, whereas the rush hour of this portfolio can be located between 7 and 9 o'clock in the morning as well as 3 and 6 o'clock in the afternoon. During working hours the amount of trips is relatively constant, although the average trip length is not: Being maximal at the 5-6am and 8-9pm intervals due to above-average-length commutes, average trip length is minimal during the night[17] and shows a local minimum at 11am, then rising considerably at lunch time. Regarding the driving styles it is worth noting that although the ratio of the slow and fast trips is approximately constant during the day time, it almost balances during night hours because of the extensive reduction of traffic during this time.

Taken together, these results indicate individual behavioural patterns depending on the season and time of day. For example, it would seem likely that an exploration of the expectancy claims comparing policy holders going on vacation with those staying at home would yield differentiation potential for large enough portfolios. In order to exhaust other differentiation opportunities, the relative ratio between slow and fast trips per driver can be addressed, possibly incorporating additional contextual parameters such as weather or, more explicitly, temperature. Of course in larger portfolios, one can also perform the derivation based on more detailed driving styles to gain more precise data about the respective distributions. Other questions of interest that can be examined without adding contextual information include for example detailed lighting models incorporating data about the relative angle of the sun most important at twilight conditions as well as the winter term merged with directional data predicting the amount of direct or indirect glare, more explicit distributions for the road type (either by merging with GPS-based map data or by accessing the road type by pattern matching) or changes in the driving style distribution of an individual driver-vehicle unit over time to account for changed driving habits or abilities.

3.4 Multiscale considerations

To capture insights from all investigated scales, a weighted averaging must be performed. Since the format of Definition 5 is chosen such that at all times any individual bi-comparative score function represents the statistical risk associated with the respective behaviour pattern, the overall score can be obtained by multiplication of the scores of all scales. For weighting factors between different scale levels, it is crucial that every characteristic of a trip is included in the entire

[17] It is worth noting that naturally, the available amount of data for the night time is two orders of magnitudes smaller than at day time, such that all conclusions must be thoroughly checked against both portfolio size and total number of trips, especially if conclusions are drawn with respect to large personal injury and material damage occurring at night time. Consequently, for a risk-adequate scoring it is necessary to compare the distribution of the individual ratio of driving styles (per hour) with the portfolio or segment of choice, weighted with the risk associated with each driving style and time of day.

driving style combination in order to capture the risk information extracted from the respective scale. This means that these factors are best chosen as shares normalized with respect to their effective influence on the entire risk across the whole ensemble.

Keep in mind that the shares of specific driving styles within the same level need also to be selected in such a way that they properly reflect the ratios of the driving styles across all sequences collected per vehicle within the defined time horizon. One possible approach would be to define a weighting based on mileage or driven time.

4 Conclusion and Outlook

In this research we present an approach for evaluation of car insurance telematic data performed over weighted convex driving style combinations. Therefore, we subdivide telematic driving profiles into four scale levels, differing in their period length and time granularity, and differentiate driving behaviour with respect to the chosen scale. Our empirical data leads to the conclusion that, taking into account pattern recognition and Fourier analysis, we can represent driving situations accurately, demonstrate differentiation potential and, given comprehensive risk experience, can score driving behaviour for riskindividual pricing.

This implies that, in practice, risk-classification can be performed as detailed as the respective technological implementation allows. It is important to note that at the current stage of development, there is no canonical way to assess the true, actuarial risk of any identified pattern—but without collecting the data and formatting it to suit advanced analyzes a priori, no analysis can be performed once the risk experience can be assessed in posteriority. If however, the information is reduced before identifying the most important observables, the precision of any analysis will inevitably be poor.

Although widely considered as transitional solution to bridging the gap toward autonomous driving, one can already see the massive impact of telematics technologies when compared to the approximate traditional tariff criteria of non-telematic insurance pricing. Not only can semantically rigorous replacement properties such as the actual garage usage of the policy owner be thoroughly tracked, but also consequently be used to verify and enhance the understanding of not only the telematic portfolio, but also, when cross-referenced with standard portfolios, expand on the sociodemographic knowledge about all policy owners.

Appendix: Measures of central tendency and dispersion

See Tables 1, 2, 3, 4, 5 and 6.

Table 1 Features concerning turn right maneuver of Fig. 4a (16 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	10.00	5.75	8.00	7.50
Distance [m]	42.64	31.60	37.78	36.04
1st order of motion				
Min [km/h]	12.00	15.00	14.00	12.00
Max [km/h]	19.00	24.00	20.00	26.00
Median [km/h]	16.00	20.00	17.00	15.00
Iqr [km/h]	2.00	5.00	2.50	9.00
2nd order of motion				
Min [m/s^2]	0.03	0.02	0.05	0.05
Max [m/s^2]	0.96	2.47	0.39	2.45
Median [m/s^2]	0.45	0.22	0.20	0.86
Iqr [m/s^2]	0.55	1.10	0.11	1.09

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.* deceleration, i.e. all measured negative values in longitudinal direction, *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

Table 2 Features concerning turn right maneuver of Fig. 4b (16 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	9.50	4.75	7.00	7.00
Distance [m]	27.30	28.68	34.18	19.93
1st order of motion				
Min [km/h]	3.00	14.00	11.00	3.00
Max [km/h]	23.00	27.00	27.00	27.00
Median [km/h]	9.00	24.00	16.00	9.00
Iqr [km/h]	10.93	7.00	6.00	10.00
2nd order of motion				
Min [m/s^2]	0.21	0.28	0.15	0.10
Max [m/s^2]	1.38	1.95	0.64	1.37
Median [m/s^2]	0.75	1.31	0.29	0.44
Iqr [m/s^2]	0.47	0.34	0.20	0.66

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.*, deceleration, i.e. all measured negative values in longitudinal direction, *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

Table 3 Features concerning turn right maneuver of Fig. 4c (28 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	9.00	6.75	12.00	4.50
Distance [m]	39.38	43.47	68.75	22.57
1st order of motion				
Min [km/h]	0.00	0.00	0.00	8.00
Max [km/h]	31.00	35.00	35.00	34.00
Median [km/h]	15.00	27.00	28.00	16.50
Iqr [km/h]	16.25	14.00	32.00	9.00
2nd order of motion				
Min [m/s^2]	0.05	0.05	0.05	0.15
Max [m/s^2]	2.01	2.94	1.08	1.67
Median [m/s^2]	0.69	1.77	0.44	0.69
Iqr [m/s^2]	0.78	1.72	0.31	0.61

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.* deceleration, i.e. all measured negative values in longitudinal direction, *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

Table 4 Features concerning city section of Fig. 5a (329 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	124.50	148.00	112.50	134.75
Distance [m]	710.31	944.52	769.05	790.13
1st order of motion				
Min [km/h]	0.00	0.00	0.00	0.00
Max [km/h]	49.00	48.00	48.00	49.00
Median [km/h]	20.00	26.00	27.00	24.00
Iqr [km/h]	15.00	15.00	20.00	14.00
2nd order of motion				
Min [m/s^2]	0.03	0.02	0.05	0.05
Max [m/s^2]	2.45	3.43	3.48	2.45
Median [m/s^2]	0.72	0.26	0.25	0.29
Iqr [m/s^2]	0.66	0.59	0.39	0.25

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.* deceleration, i.e. all measured negative values in longitudinal direction, *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

Table 5 Features concerning highway section of Fig. 5b (304 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	83.25	58.25	116.50	32.50
Distance [m]	2345.11	1834.56	3451.97	982.93
1st order of motion				
Min [km/h]	52.00	62.00	52.00	67.00
Max [km/h]	130.00	131.00	131.00	131.00
Median [km/h]	102.00	113.00	103.00	103.00
Iqr [km/h]	26.50	24.00	24.00	24.00
2nd order of motion				
Min [m/s^2]	0.05	0.05	0.05	0.05
Max [m/s^2]	1.37	1.32	1.18	1.08
Median [m/s^2]	0.34	0.25	0.34	0.25
Iqr [m/s^2]	0.44	0.34	0.39	0.34

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.* deceleration, i.e. all measured negative values in longitudinal direction, *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

Table 6 Features concerning overland section of Fig. 5c (498 s in total)

	Longitudinal axis		Lateral axis	
	acc.	dec.	Left steer.	Right steer.
0th order of motion				
Duration [s]	159.75	173.00	303.50	35.75
Distance [m]	2115.44	2585.15	4420.53	398.95
1st order of motion				
Min [km/h]	0.00	0.00	0.00	0.00
Max [km/h]	90.00	92.00	92.00	88.00
Median [km/h]	47.00	54.00	49.00	38.00
Iqr [km/h]	28.00	38.00	35.00	26.00
2nd order of motion				
Min [m/s^2]	0.02	0.03	0.05	0.05
Max [m/s^2]	2.11	2.31	3.68	2.60
Median [m/s^2]	0.49	0.44	0.39	0.29
Iqr [m/s^2]	0.49	0.69	0.29	0.74

acc. acceleration, i.e. all measured positive values in longitudinal direction, *dec.* deceleration, i.e. all measured negative values in longitudinal direction; *left steer.* left steering, i.e. all measured values in left lateral direction, *right steer.* right steering, i.e. all measured values in right lateral direction

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