Masters / PhD thesis

Optimal track design for high-speed railways: Efficient finite elements for wave propagation

Trains running at or above critical speeds can lead to large time-dependent deformations of the track, due to resonances. Consequences include train derailment, formation of cracks in the track, and high levels of ground vibration in the neighbouring environment due to wave propagation. The mathematical modelling of tracks for high-speed railways plays an important role in the planning of HS2 and motivates the development of new numerical methods for high-frequency wave propagation. Standard approaches based on highly refined mesh grids and very small time steps require infeasible computational costs [5, 6].

The proposed project studies new finite element methods for the wave equation

$$\partial_t^2 p(t, \mathbf{x}) - \Delta_{\mathbf{x}} p(t, \mathbf{x}) = 0$$

and the equations of elastodynamics. The objective is to develop fast reliable numerical schemes, based on the including physical information into the approximation space or on automatically refined meshes, which allow to predict and optimize the dynamic effects induced by high-speed trains.

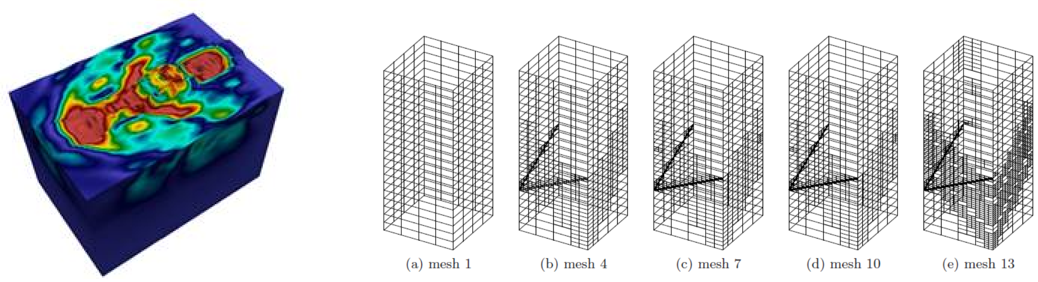
Depending on the interests of the student, the project may involve

a) efficient **generalised finite element methods** for wave propagation [3, 4],

b) **numerical analysis** of finite element methods for wave equations [1, 2],

c) **modelling of elastic wave propagation** at high frequencies [5, 6].

Since the late 1990s **generalized finite elements** (a) based on plane-wave approximations have been shown to significantly reduce the computational cost for the



(a) (b)

Figure 1: (a) Ground Mach wave due to moving train at critical speed. (b) Adaptive space-time meshes to resolve shock waves.

numerical approximation of wave emission and scattering problems [3, 4]. Instead of resolving the oscillations of the solution on a fine mesh, oscillatory basis functions allow to rapidly approximate the solution on coarse spatial meshes. The extension of such numerical methods to the time-dependent wave equation is in its infancy. It would provide a way for efficient simulations on coarse meshes and with large time steps. First numerical results indicate the fast convergence of the proposed approach as the number of enrichment functions is increased.

The methods are a key ingredient to **optimize the track design** by minimizing quantities of interest like the radiated energy or displacement. To do so, **continuum mechanical models** for the track and ground need to be developed [5, 6].

Towards **numerical analysis** (b), we currently focus on efficient methods based on clever mesh refinements. For geometric singularities (e.g. waves scattered at cracks), graded meshes which are appropriately refined towards the singularity can be proven to give optimal convergence [1]. However, typically the location of cracks is not known and theorems (sharp computable error estimates) give rise to mesh refinements [2]. Doing this in space and time, letting the numerical method track the wave crest, produces both nice geometries and provably accurate solutions.

Industry partners

Institute for High Speed Rail and System Integration (Leeds) and partners.

References

[1] H. Gimperlein, F. Meyer, C. Özdemir, D. Stark, E. P. Stephan, *Boundary elements with mesh refinements for the wave equation*, Numer. Math. 139 (2018), 867-912.

[2] H. Gimperlein, C. Özdemir, D. Stark, E. P. Stephan, *A residual a posteriori estimate for the time-domain BEM*, Numer. Math. 146 (2020), 239-280.

[3] O. Laghrouche, P. Bettess, E. Perrey-Debain, J. Trevelyan, *Wave interpolation finite elements for Helmholtz problems with jumps in the wave speed*, Computer Methods in Applied Mechanics and Engineering 194 (2005), 367-381.

[4] E. Perrey-Debain, J. Trevelyan, P. Bettess, *On wave boundary elements for radiation and scattering problems with piecewise constant impedance*, IEEE Transactions on Antennas and Propagation 53 (2005), 876-879.

[5] A. El Kacimi, P. K. Woodward, O. Laghrouche, G. Medero, *Time domain 3D finite element modelling of train-induced vibration at high speed*, Computers & Structures 118 (2013), 66-73.

[6] S. B. Mezher, D. P. Connolly, P. K. Woodward, O. Laghrouche, J. Pombo, P. A. Costa, *Railway critical velocity – Analytical prediction and analysis*, Transportation Geotechnics 6 (2016), 84-96.

Prerequisites

Interest in computational modelling. Programming skills can be helpful.

Contact

Univ.-Prof. Dr. Heiko Gimperlein (heiko.gimperlein@uibk.ac.at)