Summer School 2022: Geometric dispersive PDEs

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Date: September 25 - 30, 2022

Place: University Center Obergurgl, Gaisbergweg 5, 6456 Obergurgl, Austria

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Organizers

- Tobias Lamm (Karlsruhe)
- Birgit Schörkhuber (Innsbruck)
- Tobias Weth (Frankfurt)

Main lectures

Roland Donninger (University of Vienna)
Self-similar blowup in wave equations

Sebastian Herr (University of Bielefeld)
The Cauchy problem for the wave maps equation in critical spaces

Mihaela Ifrim (University of Wisconsin-Madison)
Nonlinear evolutions: the interplay between long time solutions and low regularity well-posedness

Enno Lenzmann (University of Basel)
An invitation to completely integrable dispersive PDEs with criticality ***(cancelled)***

Speakers and abstracts

Roland Donninger (University of Vienna)
Self-similar blowup in wave equations

Many nonlinear wave equations have the property that their solutions develop singularities in finite time. In many cases, this phenomenon is evidenced by the existence of self-similar solutions. In order to understand the role of self-similar solutions in generic evolutions, it is necessary to study their stability. In the last decade there was tremendous progress in this direction thanks to an interplay of techniques from semigroup theory, spectral theory, harmonic analysis, and classical PDE analysis. The lecture will give an introduction to the necessary tools and present some of the key results that were obtained recently.

Sebastian Herr (University of Bielefeld)
The Cauchy problem for the wave maps equation in critical spaces

We will discuss the initial value problem for the wave maps equation. Wave maps are critical points of a Lagrangian for maps from Minkowski space-time into a Riemannian manifold. The resulting Euler-Lagrange equation is a nonlinear wave equation which exhibits null-structure. In these four lectures we will focus on this nonlinear wave equation and explain how this can be solved globally in time for small initial data in scaling-invariant spaces. We will draw a connection to the classical Fourier restriction problem in harmonic analysis. On the technical level, we will also discuss certain atomic function spaces which proved to be useful for critical problems.

Mihaela Ifrim (University of Wisconsin-Madison)
Nonlinear evolutions: the interplay between long time solutions and low regularity well-posedness

The goal of my lectures is to present a comprehensive introduction into the role played by tools from the study of enhanced lifespan of solutions in the study of low regularity well-posedness theory for fully nonlinear dispersive partial differential equations. We will start with a discussion of the local well-posedness problem for quasilinear equations as it presents a number of difficulties some of which are universal and others of which are more problem specific. A common standard for what well-posedness should mean has existed for a long time, going back to Hadamard. The next step is to introduce the quasilinear modified energy method for some model problems. The last two lectures will focus on showing how the first two lectures lead to the current low regularity results for the 2d gravity water waves and for the time-like minimal surface equations. This will include a quick and schematic overview of the paradifferential calculus which is needed in proving these results.

Irfan Glogić (University of Vienna)
Stable blowup for the supercritical hyperbolic Yang–Mills equations

We consider the Yang-Mills equations in (1+d)-dimensional Minkowski spacetime. Under equivariance symmetry, this model reduces to a semilinear radial wave equation, which admits in all supercritical dimensions, i.e., for d \geq 5, a closed form self-similar blowup solution, as discovered by Bizo\'n and Biernat. The significance of this discovery lies in the fact that this solution is conjectured to be the universal attractor for large equivariant data evolutions. As the first step in attacking this conjecture, in this talk we show that the Bizo\'n-Bernat blowup profile is nonlinearly stable in all odd dimensions d \geq 5. Our approach is based on similarity variables and non-self-adjoint spectral techniques.

Maciej Maliborski (University of Vienna)
Characteristic approach to the soliton resolution

I will present a toy model for studying the soliton resolution phenomenon. The soliton resolution conjecture states that global-in-time generic solutions of nonlinear dispersive wave equations resolve for late times into a superposition of decoupled nonlinear bound states (solitons) and radiation. Our main objective is to illustrate the advantages of employing outgoing null (or asymptotically null) foliations in analyzing the relaxation processes due to the dispersal of energy by radiation. Based on results from joint work with Piotr Bizoń and Bradley Cownden.

Program

Sunday, September 25, 2022

17.30 Get-Together
18:00 Dinner

Monday, September 26, 2022

07.30 – 08.30	Breakfast
09.00 – 10.00	***S. Herr***
10.00 – 10.30	Coffee Break
10.30 – 11.30	***S. Herr***
12.00 – 13.00	Lunch
13.15	***HIKE***
18.00 – 19.00	Dinner
19.30	**Meet and Greet**

Tuesday, September 27, 2022

07.30 – 08.30	Breakfast
09.00 – 10.00	***M. Maliboski***
10.00 – 10.30	Coffee Break
10.30 – 11.30	***I.Glogić***
12.00 – 13.00	Lunch
15.00 – 16.00	***M. Ifrim***
16.10 – 16.30	Coffee Break
16.30 – 17.30	***M. Ifrim***
18.00 – 19.00	Dinner

Wednesday, September 28, 2022

07.30 – 08.30	Breakfast
09.00 – 10.00	***R. Donninger***
10.00 – 10.30	Coffee Break
10.30 – 11.30	***R. Donninger***
12.00 – 13.00	Lunch
15.00 – 16.00	***S. Herr***
16.10 – 16.30	Coffee Break
16.30 – 17.30	***S. Herr***
18.00 – 19.00	Dinner

Thursday, September 29, 2022

07.30 – 08.30	Breakfast
09.00 – 10.00	***R. Donninger***
10.00 – 10.30	Coffee Break
10.30 – 11.30	***R. Donninger***
12.00 – 13.00	Lunch
15.00 – 16.00	***M. Ifrim***
16.10 – 16.30	Coffee Break
16.30 – 17.30	***M. Ifrim***
18.00 – 19.00	Dinner

Friday, September 30, 2022

07.30 – 08.30	Breakfast
09.00 – 09.30	***D. Yu***
09.30 – 10.00	***K. Marsden***
10.00 – 10.30	Coffee Break
10.30 – 11.00	***M. Ostermann***
11.00 – 11.30	***M. McNulty***
12.00 – 13.00	Lunch

END

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