Exact Nonlinear Response Theory in the Driven Lattice Lorentz Gas

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One of the principal strategies to probe material properties is to apply an external stimulus and monitor the system's response. Within statistical physics the fundamental link between the *deterministic response* of a system and the correlation functions of *intrinsic fluctuations* is provided by the celebrated fluctuation-dissipation theorem (FDT). The framework applies whenever the unperturbed system is in thermal equilibrium and the forces are sufficiently small such that the response is linear.

It came as a big surprise that the equilibrium correlation functions yielding the transport coefficients via Green-Kubo relations display power-law tails rather than an exponential decay. First found in computer simulations of the velocity autocorrelation function (VACF) for hard spheres [1] these persistent correlations have been derived rigorously for dilute gases by systematically going beyond the Boltzmann equation. Then repeated collisions with the same particle yield a non-analytic dependence of the diffusion coefficients on density, frequency, and wavenumber. A similar exact low-density expansion for the related Lorentz model, where a tracer ballistically explores a random array of fixed scatterers, again reveals long-time tails in the VACF [2,3,4]. The emergence of persistent memory is believed to be a generic feature of complex transport even beyond the exactly solvable limiting cases and constitutes one of the pillars of our current understanding of dynamics.

As a consequence of the FDT these slow power laws manifest themselves in the response with respect to driving of infinitesimal magnitude. However, for strong driving the stationary state deviates strongly from the Gibbs-Boltzmann ensemble and no universal framework has been elaborated and no general exact results are available for systems far from equilibrium.

We determine the nonlinear time-dependent response of a tracer on a lattice with randomly distributed hard obstacles as a force is switched on [5]. The calculation is exact to first order in the obstacle density [6] and holds for arbitrarily large forces. In particular, we show that the nonlinear drift velocity in the stationary state becomes non-analytic in the driving force. Furthermore we demonstrate that the stationary velocity is approached exponentially fast for any finite value of the force, in striking contrast to the power-law relaxation predicted within linear response. We discuss the range of validity of our analytic results by comparison to stochastic simulations.

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