universität innsbruck
Institut für Mathematik

Inn'formal Probability Seminar

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"Critical probabilities for positively associated, finite-range dependent percolation models"

Abstract:
Consider Bernoulli bond percolation on a d-regular tree. It is well-known that the critical probability above which an infinite cluster exists, is equal to $1/(d-1)$ in this case. This follows from a basic result in the theory of branching processes. In this talk, we are interested in the general class of bond percolation models which have a finite range of dependence and satisfy positive association (the FKG inequality). We will prove that Bernoulli bond percolation percolates worst among the models in this class, meaning that for any such bond percolation model with edge marginals strictly larger than $1/(d-1)$, an infinite cluster exists. This result supports the general idea that positive association is a favourable property for percolation to occur, and it extends to locally finite, infinite trees. Without positive association, Ballister and Bollóbas had proved in 2012 that substantially different percolation behaviour is possible.

We then move on to general graphs and present a new way of combining positive association with a finite range of dependence. This will allow us to study stochastic domination by Bernoulli bond percolation from above and below, and we will see and extension an earlier result of Liggett, Schonmann and Stacey. In the specific case of 1-independent, positively associated bond percolation models which naturally arise in renormalizations, it moreover allows us to obtain quantitative bounds on critical probabilities on the hypercubic lattice $\mathbb{Z}^d$ for $d = 2$ and as $d \to \infty$.

Wednesday | 14.06.2023 | 13:00
SR 609 | Civil Engineer Building