# Modified Time Reversal in Photoacoustic Tomography

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## 1 Background

Photoacoustic tomography (PAT) is a novel tomographic imaging paradigm that combines the benefits of ultrasound imaging and optical imaging [4]. When a semi-transparent sample is illuminated with a short pule of electromagnetic energy, then parts of the optical energy become absorbed inside the sample which in turn induces an acoustic pressure wave (see Figure 1.1). The induced acoustic pressure waves are measured outside of the investigated object and used to recover an image of the interior.

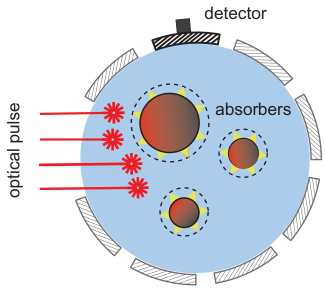


Figure 1.1: Illustration of PAT. Electromagnetic pulses are sent into the investigated object and induce acoustic waves. The acoustic waves are measured with ultrasound detectors on the boundary of the object and are used to recover an image of the object.

Let $d \in \{2,3\}$ denote the spatial dimension and suppose that $\Omega \subseteq \mathbb{R}^d$ is some domain containing the investigated object. The induced acoustic pressure $p \colon \mathbb{R}^d \times (0,\infty) \to \mathbb{R}$ then satisfies the wave equation

$$\begin{cases} \partial_t^2 u(x,t) - c(x)^2 \Delta u(x,t) = 0 & \text{for } (x,t) \in \mathbb{R}^d \times (0,T), \\ u\,(x,0) = h(x) & \text{for } x \in \mathbb{R}^d, \\ \partial_t u\,(x,0) = 0 & \text{for } x \in \mathbb{R}^d\,. \end{cases} \tag{1.1}$$

Here $h$ denotes the initial pressure distribution that caries diagnostic information about the investigated object, $\Delta$ is the Laplacian in the spatial variable, and $c(x)$ is the spatially varying speed of sound. The reconstruction problem in PAT with variable sound speed consists in reconstructing the initial datum $h$ from the restriction of $u$ to the observation surface,

$$\mathbf{\Lambda} h \coloneqq u|_{\partial\Omega \times [0,T]} \colon \partial\Omega \times [0,T] \to \mathbb{R}\,. \tag{1.2}$$

Here it is assumed that $c > 0$ is smooth, and that $1 - c$ and $h$ are supported in $\Omega$.

## 2 Reconstruction by time reversal

The time reversal is a simple numerical method to obtain an approximation to the pressure distribution $h$, see [1, 2]. In this thesis, we consider a modification of the time reversal technique proposed in [3].

Given data $g\colon \partial\Omega \times (0,T) \to \mathbb{R}$ consider the solution of the backward wave equation

$$\begin{cases} \partial_t^2 w(x,t) - c(x)^2 \Delta w(x,t) = 0 & \text{for } (x,t) \in \Omega \times (0,T), \\ w(x,t) = g(x,t) & \text{for } (x,t) \in \partial\Omega \times (0,T), \\ w(x,T) = \phi(x) & \text{for } x \in \mathbb{R}^d, \\ \partial_t w(x,T) = 0\,, & \text{for } x \in \mathbb{R}^d\,. \end{cases} \tag{2.1}$$

Here $\phi\colon \Omega \to \mathbb{R}$ is the solution of the elliptic boundary value problem $c(x)^2 \Delta\phi(x) = 0$ for $x \in \Omega$ with boundary conditions $\phi|_{\partial\Omega} = g(\,\cdot\,, T)$. Denote by $\mathbf{\Lambda}^\sharp g \coloneqq w(x,0)$.

In [3], it is shown that $\mathbf{\Lambda}^\sharp \mathbf{\Lambda} f = f - \mathbf{K} f$, where $\mathbf{K}$ is compact on a suitable Hilbert space $H_D(\Omega)$, and $\|\mathbf{K}\|_{H_D(\Omega)} < 1$. In this sense, $\mathbf{\Lambda}^\sharp \mathbf{\Lambda} f$ can be considered as an approximate reconstruction of $f$. Moreover, $f$ can be exactly recovered via the Neumann series $f = \sum_{m=0}^{\infty} \mathbf{K}^m (\mathbf{\Lambda}^\sharp \mathbf{\Lambda} f)$.

## 3 Aims of the Bachelor thesis

The aim of this thesis is to summarize the main results of [3, Section 2] concerning the modified time reversal technique (2.1). Further, the wave equation (1.1) and the modified time reversal technique (2.1) will be numerically implemented in Matlab, and illustrated by numerical results.

## References


[1] P. Burgholzer, G. J. Matt, M. Haltmeier, and G. Paltauf. Exact and approximate imaging methods for photoacoustic tomography using an arbitrary detection surface. *Phys. Rev. E*, 75(4):046706, 2007.

[2] Y. Hristova, P. Kuchment, and L. Nguyen. Reconstruction and time reversal in thermoacoustic tomography in acoustically homogeneous and inhomogeneous media. *Inverse Probl.*, 24(5):055006 (25pp), 2008.

[3] P. Stefanov and G. Uhlmann. Thermoacoustic tomography with variable sound speed. *Inverse Probl.*, 25(7):075011, 16, 2009.

[4] M. Xu and L. V. Wang. Photoacoustic imaging in biomedicine. *Rev. Sci. Instrum.*, 77(4):041101 (22pp), 2006.