On the Uniqueness of Solutions of First Kind Volterra Integral Equations

1 Volterra Equations of First Kind

Volterra equations of first kind are a special form of integral equations. In the linear case, they are of the form

$$g(x) = \int_a^x k(x,y)\, f(y)\,\mathsf{d}y,$$

where the function $g : [a,b] \to \mathbb{R}$ and the kernel function $k : [a,b] \times [a,b] \to \mathbb{R}$ are given and f is the function to be solved for. Under certain conditions on both the function g and the kernel function k, this integral equation has a unique solution f, which further satisfies certain regularity properties.

2 Aims of the thesis

One widely used assumption to obtain uniqueness of the solution function is that the kernel function does not vanish on the diagonal, i.e. $k(x,x) \neq 0$ for all $x \in [a,b]$. If this condition is violated, a theorem by Volterra and Pérès gives conditions under which a solution still uniquely exists. These quite unhandy requirements will be studied in the simplest cases. The theory will be illustrated by simple examples which can be solved using the theory of ordinary differential equations. Furthermore, routines for numerically solving the integral equations should be considered.

References

[1] S. Fenyö and H.-W. Stolle. *Theorie und Praxis der linearen Integralgleichungen. 3*, volume 76 of *Lehrbücher und Monographien aus dem Gebiete der Exakten Wissenschaften (LMW). Mathematische Reihe.* Birkhäuser Verlag, Basel, 1984.

[2] Peter Linz. *Analytical and numerical methods for Volterra equations*, volume 7 of *SIAM Studies in Applied Mathematics.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1985.