Bachelor project:
Masked Hard Thresholding Pursuit for dictionary based inpainting

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Inpainting is the procedure of filling in missing information in a signal, as for instance missing pixels in a grayscale image, Fig. 1.

Dictionary based inpainting relies on the concept that every image patch of size $s_1 \times s_2$ can be sparsely approximated in a flat patch-dictionary. If we vectorise both the patches, $y_n \in \mathbb{R}^d$ for $d = s_1 \cdot s_2$, and the unit norm atoms, $\phi_k \in R^d$, and collect the atoms in the dictionary $\Phi = (\phi_1, \ldots, \phi_K) \in \mathbb{R}^{d \times K}$, beingly sparsely approximated means that up to a small error each patch y can be represented as linear combination of a small (sparse) number of dictionary atoms,

$$y \approx \sum_{k \in I} \phi_k x_k = \Phi_I x_I + \eta \quad \text{where} |I| = S \ll d, \tag{1}$$

The constraint that the dictionary should be flat or in other words that the energy of the atoms should be evenly distributed across the coordinates, $\phi_k^2(j) \approx 1/d$, ensures that the dictionary is robust to erasures. Even if several coordinates are missing we can still distinguish the atoms on the available coordinates and more importantly even if several coordinates of a patch are missing we can still identify which atoms are needed to sparsely represent the patch.

Dictionary based inpainting then takes the following form. Denote by M the projection onto the subset of coordinates that are not erased. Since any patch y is sparse in the dictionary Φ, any damaged patch My is sparse in the damaged dictionary $M\Phi$.

$$y \approx \Phi_I x_I \quad \Rightarrow \quad My \approx M\Phi_I x_I, \tag{2}$$

and we can reconstruct the original patch by sparsely approximating My in $M\Phi$ to get coefficients $\tilde{x}_I \approx x_I$ and then setting $\tilde{y} = \Phi \tilde{x}_I$. To find a good sparse approximation for a fixed sparsity level one usually uses a non-exact but faster approximation routine such as thresholding or Orthogonal Matching Pursuit (OMP). The goal of this project is to adapt the Hard Thresholding Pursuit algorithm, which is more accurate than thresholding and faster than OMP, for sparsely approximating masked signals in their respective masked dictionary and to investigate its performance for inpainting.

Figure 1: Damaged picture and reconstruction

Tasks:

- Familiarise yourself with dictionary based inpainting, HTP and OMP, [2, 3, 1].
- Implement a masked version of HTP corresponding to masked OMP as provided (OMPm.m).
- Compare the performance of HTPm to OMPm for inpainting for several sparsity levels.

⋆ Compare the performance of HTPm to OMPm for inpainting for several images, corruption types, sparsity levels and dictionaries.

References

[1] E. Höck. Hard thresholding pursuit for sparse approximation. BSc thesis, University of Innsbruck, 2016.

[2] J. Mairal, M. Elad, and G. Sapiro. Sparse representation for color image restoration. *IEEE Transactions on Image Processing*, 17(1):53–69, 2008.

[3] V. Naumova and K. Schnass. Dictionary learning from incomplete data. *arXiv:1701.03655*, 2017.