Eliminating even crossings on surfaces

Bachelor thesis topic

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Given a graph $G = (V, E)$ the classical theorem by Hanani [1] and Tutte [6] says that $G$ is planar if it can be drawn in the plane in such a way that every two non-adjacent edges cross an even number of times (including 0 times). This is the so-called ‘strong’ version of the theorem. The ‘weak’ version differs from it by imposing the condition of even number of crossings on all pairs of edges, not only the non-adjacent ones. The original proofs of the theorem used Kuratowski’s characterization of planarity, but there are more recent proofs, which are elegant and algorithmic. A particularly nice one was given by Pelsmajer, Schaefer, and Štefankovič [3].

Can the statement analogous to the Hanani–Tutte theorem be true for surfaces other than the plane? It is known that the weak version of the theorem indeed generalizes to all surfaces [4]. But a positive result about the strong version of the theorem is known only for the projective plane [5]. Recently, Fulek and Kynčl [2] have provided a counter-example to the extension of the strong Hanani–Tutte theorem for all orientable surfaces of orientable genus at least four. The remaining cases are open.

The goal of the thesis is to study the presented area, to understand the current state of the art and to provide its self-contained exposition.

Bibliography


