Bachelor project:
Integrating low-rank components into weighted KSVD for dictionary based inpainting

Supervisor: Karin Schnass

Inpainting is the procedure of filling in missing information in a signal, as for instance missing pixels in a grayscale image, Fig. 1.

Dictionary based inpainting relies on the concept that every image patch of size $s_1 \times s_2$ can be sparsely approximated in a flat patch-dictionary. If we vectorise both the patches, $y_n \in \mathbb{R}^d$ for $d = s_1 \cdot s_2$, and the unit norm atoms, $\phi_k \in R^d$, and collect the atoms in the dictionary $\Phi = (\phi_1, \ldots, \phi_K) \in \mathbb{R}^{d \times K}$, beingly sparsely approximated means that up to a small error each patch y can be represented as linear combination of a small (sparse) number of dictionary atoms,

$$y \approx \sum_{k \in I} \phi_k x_k = \Phi_I x_I + \eta \quad \text{where} |I| = S \ll d, \tag{1}$$

The constraint that the dictionary should be flat or in other words that the energy of the atoms should be evenly distributed across the coordinates, $\phi_k^2(j) \approx 1/d$, ensures that the dictionary is robust to erasures. Even if several coordinates are missing we can still distinguish the atoms on the available coordinates and more importantly even if several coordinates of a patch are missing we can still identify which atoms are needed to sparsely represent the patch.

Dictionary based inpainting then takes the following form. Denote by M the projection onto the subset of coordinates that are not erased. Since any patch y is sparse in the dictionary Φ, any damaged patch My is sparse in the damaged dictionary $M\Phi$.

$$y \approx \Phi_I x_I \quad \Rightarrow \quad My \approx M\Phi_I x_I, \tag{2}$$

and we can reconstruct the original patch by sparsely approximating My in $M\Phi$ with Orthogonal Matching Pursuit (OMP) to get coefficients $\tilde{x}_I \approx x_I$ and then setting $\tilde{y} = \Phi \tilde{x}_I$. To find a good dictionary for a given image one can learn the dictionary directly from the corrupted data using a specialised dictionary learning algorithm such as 'weighted K singular value decompositions' (wKSVD) or 'iterative thresholding and K residual means for masked data' (ITKrMM). However, one problem is that these dictionaries tend to be ill conditioned if the data contains a low-rank component, that is a subspace that contains most of the signal

Figure 1: Damaged picture and reconstruction

energy. To get a well-conditioned dictionary one has to remove this low rank component before the learning, as is possible for ITKrMM for low-rank components of arbitrary sizes, and for wKSVD in case of a one dimensional low rank component. The goal of this project is to integrate the existence of a low-rank component of arbitrary size into the wKSVD algorithm and to compare the inpainting performance of wKSVD vs. ITKrMM for various low-rank component sizes.

Tasks:

- Familiarise yourself with dictionary based inpainting and dictionary learning from erased data, [1, 2].
- Generalise the wKSVD algorithm for erased data to account for low-rank components of arbitrary sizes.
- Compare the inpainting performance of wKSVD to ITKrMM dictionaries with various low-rank component sizes.

⋆ Compare the inpainting performance of wKSVD to ITKrMM dictionaries for several low-rank component sizes, sparsity levels (in the learning), images and corruption types.

References

[1] J. Mairal, M. Elad, and G. Sapiro. Sparse representation for color image restoration. *IEEE Transactions on Image Processing*, 17(1):53–69, 2008.

[2] V. Naumova and K. Schnass. Dictionary learning from incomplete data. *arXiv:1701.03655*, 2017.