

# When one parallel is not enough: Hyperbolic Geometry

Bachelor thesis topic

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The parallel postulate of Euclidean geometry states that in the plane given a line  $\ell$  and a point  $p$  outside of this line there is a unique line  $q$  which is passing through  $q$  and parallel to  $p$ .

For a long time the question of whether this postulate is a consequence of the other axioms of Euclidean geometry has been considered. A negative answer to this question was given independently by a number of mathematicians including C. F. Gauß, N. I. Lobachevsky and J. Bolyai. Both Lobachevsky and Bolyai presented hyperbolic geometry as a counterexample to the hypothesis that the parallel postulate follows from the other axioms since in hyperbolic geometry these axioms are satisfied while the parallel postulate fails.

It turned out that hyperbolic geometry has a number of interesting applications and it has been studied in detail.

The aim of this thesis is give an introduction to hyperbolic geometry, to discuss a number of different models and to describe hyperbolic motions.

## References

- [1] Marcel Berger. *Geometry. II*. Universitext. Springer-Verlag, Berlin, 1987. Translated from the French by M. Cole and S. Levy.