

# Convexity properties of the Urysohn universal metric space

Bachelor thesis topic

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In [3], P. S. Urysohn constructed a metric space  $\mathbb{U}$  with the following properties:

- (1)  $\mathbb{U}$  is complete and separable.
- (2)  $\mathbb{U}$  contains an isometric copy of every separable metric space.
- (3) Every isometry between two finite subsets of  $\mathbb{U}$  can be extended to an isometry on  $\mathbb{U}$ .

The properties (1)–(3) determine  $\mathbb{U}$  uniquely up to isometries. An detailed exposition of the Urysohn metric space can be found in Chapter 5 of [2].

In [1] convexity properties of the space  $\mathbb{U}$  are considered. In particular it is shown that  $\mathbb{U}$  is metrically convex and satisfies the finite ball intersection property.

The aim of this project is to understand and present these results in detail, i.e. providing all necessary background material and detailed proofs of the theorems.

## References

- [1] Asuman Güven Aksoy and Zair Ibragimov. Convexity of the Urysohn universal space. *J. Nonlinear Convex Anal.*, 17(6):1239–1247, 2016.
- [2] Vladimir Pestov. *Dynamics of infinite-dimensional groups*, volume 40 of *University Lecture Series*. American Mathematical Society, Providence, RI, 2006.
- [3] Pavel S. Urysohn. Sur un espace métrique universel. *Bull. Sci. Math.*, 51:43–64, 74–90, 1927.