Bachelor thesis: Adaptive step size control for ordinary differential equation solvers

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Prerequisites: Good knowledge of undergraduate numerical analysis, interest in numerical analysis

Language: English or German

Topic: To solve an ordinary differential equation

$$y'(t) = F(y)$$

we can use a one-step method (such as a Runge-Kutta method) or a multi-step method (such as an Adams–Bashforth method). In a naive implementation the step size is held fixed during the entire simulation. However, this can lead to an inefficient numerical scheme as, for a given tolerance, the optimal step size can vary significantly in the course of a simulation.

The goal of this bachelor thesis is

- Understand how the error of a Runge–Kutta scheme can be estimated and how an approximation to the optimal step size can be computed from that information.
- Learn what happens and how to recover from a failure of a given step size estimator (step rejection).
- Perform an (assisted) literature search on the stability constraints of adaptive multistep methods. Compare and constrast adaptive step size control for Runge–Kutta and multi-step methods.
- Implement the step size control mechanism discussed above and illustrate the theoretical results in the context of simple test problems.