

# Shall we play a game?

## Topological and abstract games

Bachelor thesis topic

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A topological game is a game with two players playing on a topological space with total information. An example of such a game is the *Choquet game* on a nonempty topological space  $X$ . Here two players, let's call them Eve and Odd, are playing nonempty open subsets of  $X$

|     |       |       |     |
|-----|-------|-------|-----|
| Eve | $U_0$ | $U_1$ | ... |
| Odd | $V_0$ | $V_1$ | ... |

where  $U_0 \supset V_0 \supset U_1 \supset V_1 \supset \dots$ . Odd wins if the intersection of all  $V_i$  is empty, i.e. if

$$\bigcap_{i=0}^{\infty} V_i = \emptyset.$$

The aim of this thesis is to study some examples of topological or abstract games with applications to analysis. These games include the *Choquet game* and the *Banach-Mazur game* for the characterisation of Baire spaces, Polish spaces and comeagre sets, see Section I.8 of [1].

Moreover, the abstract Banach-Mazur game for the construction of the Gurarii space given in [2] should be included.

### References

- [1] Alexander S. Kechris. *Classical descriptive set theory*, volume 156 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1995.
- [2] Wiesław Kubiś. Game-theoretic characterization of the Gurarii space. *Arch. Math. (Basel)*, 110(1):53–59, 2018.