Numerical computation of periodic orbits

Bachelor project

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This term project is concerned with the numerical computation of periodic solutions of autonomous systems of differential equations

$$y' = f(y).$$

Typical examples of problems possessing periodic solutions are the *Lorenz attractor*

$$\begin{aligned} y_1' &= a(y_2 - y_1) \\ y_2' &= y_1(b - y_3) - y_2 \\ y_3' &= y_1 y_2 - c y_3 \end{aligned}$$

and the *van der Pol oscillator*

$$y'' - \varepsilon(1 - y^2)y' + y = 0.$$

The goal of the project is twofold: to give a brief account on the theory, and to discuss and program an appropriate numerical method.

The employed method will be a Newton-type iteration applied to the abstract problem

$$\mathcal{F}\big(y(0), T\big) = y(T) - y(0) = 0,$$

where $T > 0$ is the sought-after period. This iteration allows one to compute T and $y(0)$ in a fast and reliable way.

References.

[1] E. Hairer and G. Wanner, Solving Ordinary Differential Equations II, Stiff and Differential-Algebraic Problems, 2nd rev. ed., Springer, New York, 1996.