Improving Photoacoustic Inversion on Parabolas

Supervisors:
Sergiy Pereverzyev Jr. (`sergiy.pereverzyev@uibk.ac.at`)
Markus Haltmeier (`markus.haltmeier@uibk.ac.at`)

Applied Mathematics Group
`https://applied-math.uibk.ac.at`

1 Background

Photoacoustic tomography (PAT) is a novel tomographic imaging paradigm that combines the benefits of ultrasound imaging and optical tomography [8]. When a semitransparent sample is illuminated with a short pulse of electromagnetic energy, then parts of the optical energy become absorbed which in turn induces an acoustic pressure wave inside the sample (see Figure 1.1). The induced acoustic pressure waves are measured outside of the investigated object and used to recover an image of the interior.
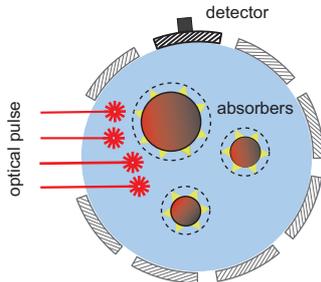

Figure 1.1: ILLUSTRATION OF PAT. Electromagnetic pulses are send into the investigated object and induce an acoustic pressure wave. The acoustic waves are measured with ultrasound detectors outside of the object and used to recover an image of the interior.

2 Photoacoustic inversion on a parabola

For some given constant $c > 0$, denote by $\Omega = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_2 > cx_1^2\}$ a parabolic domain, and write $C_c^\infty(\Omega)$ for the set of all smooth functions $f \colon \mathbb{R}^2 \to \mathbb{R}$ that have a compact support in Ω. For $f \in C_c^\infty(\Omega)$, let $\mathbf{U}f \colon \mathbb{R}^2 \times (0, \infty) \to \mathbb{R}$ denote the solution of the following initial value problem for the wave equation,

$$\begin{cases} (\partial_t^2 - \Delta)\, u(x,t) = 0 & \text{for } (x,t) \in \mathbb{R}^2 \times (0, \infty), \\ u(x,0) = f(x) & \text{for } x \in \mathbb{R}^2, \\ (\partial_t u)(x,0) = 0 & \text{for } x \in \mathbb{R}^2. \end{cases} \tag{2.1}$$

Here ∂_t denotes differentiation with respect to the temporal variable t, and Δ is the Laplacian with respect to the spatial variable x. Photoacoustic inversion on Ω consists in recovering the unknown $f \in C_c^\infty(\Omega)$ from the given boundary data $(\mathbf{U}f)(z,t) \coloneqq u(z,t)$ for $(z,t) \in \partial\Omega \times (0, \infty)$.

Photoacoustic inversion can be approached by various solution techniques. Among these techniques, the derivation of explicit inversion formulas of the back-projection type is particularly appealing. Such formulas are only known for special domains including half-planes [1], discs [7, 3] or ellipses [5]. Recently, in [6], we showed that for any parabola the following inversion formula holds:

$$f(x) = \frac{1}{\pi} \int_{\partial\Omega} \langle \nu_z, x - z \rangle \int_{|z-x|}^{\infty} \frac{(\partial_t t^{-1} \mathbf{U} f)(z, t)}{\sqrt{t^2 - |z - x|^2}} \, dt ds(z) \quad \text{for } x \in \Omega. \tag{2.2}$$

3 Aims of the Bachelor thesis

In the practical implementation of (2.2), the integration domain has to be replaced by a curve $\Gamma \subseteq \partial\Omega$ of finite length. This can be achieved by replacing the data $u(z, t)$ with zero outside Γ. However, according to [4, 2], such a hard cutoff is a major source of reconstruction artifacts. Thus, it is suggested to use a smooth cutoff of the extended data.

The goal of this thesis is to discuss and numerically implement the improvements proposed in [4, 2] for evaluating (2.2) in MATLAB. In particular, it should be clarified whether smooth extensions of the data lead to a reduction of artifacts. An existing MATLAB code for the numerical realization of the wave equation (2.1) and the inversion formula (2.2) (using a hard cutoff) will be provided by the advisors.

References

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