Bachelor project:
Free the Bird - Inpainting

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Inpainting is the procedure of filling in missing information in a signal, as for instance missing pixels in an image, Fig. 1.

Figure 1: Alcatraz bird and almost free bird

Dictionary based inpainting relies on the concept that every image patch of size $s_1 \times s_2$ can be sparsely approximated in a flat patch-dictionary. If we vectorise both the patches, $y_n \in \mathbb{R}^d$ for $d = s_1 \cdot s_2$, and the unit norm atoms, $\phi_k \in R^d$, and collect the atoms in the dictionary $\Phi = (\phi_1, \ldots, \phi_K) \in \mathbb{R}^{d \times K}$, beingly sparsely approximated means that up to a small error each patch y can be represented as linear combination of a small (sparse) number of dictionary atoms,

$$y \approx \sum_{k \in I} \phi_k x_k = \Phi_I x_I \quad \text{where} \quad |I| = S \ll d. \tag{1}$$

The constraint that the dictionary should be flat or in other words that the energy of the atoms should be evenly distributed across the coordinates, $\phi_k^2(j) \approx 1/d$, ensures that the dictionary is robust to erasures. Even if several coordinates are missing we can still distinguish the atoms on the available coordinates and more importantly even if several coordinates of a patch are missing we can still identify which atoms are needed to sparsely represent the patch.
Dictionary based inpainting then takes the following form. Denote by M the projection onto the subset of coordinates that are not erased. Since any patch y is sparse in the dictionary Φ, any damaged patch My is sparse in the damaged dictionary $M\Phi$.

$$y \approx \Phi_I x_I \quad \Rightarrow \quad My \approx M\Phi_I x_I, \tag{2}$$

and we can reconstruct the original patch by sparsely approximating My in $M\Phi$ to get coefficients $\tilde{x}_I \approx x_I$ and then setting $\tilde{y} = \Phi \tilde{x}_I$. The problem is that the more pixels are lost the more difficult is to find a good sparse approximation because the dictionary becomes more coherent, ie. different atoms start to resemble each other. To inpaint a picture with big gaps one should therefore use an iterative procedure, where in each iteration the missing pixels from patches with only a few missing pixels are inpainted. The goal of this project is to develop this iterative inpainting procedure and to free the bird.

Tasks:

- Familiarise yourself with dictionary based inpainting and sparse approximation, in particular HTP and OMP, [2, 1, 3].
- Based on [2, 1, 3], develop a theorem characterising how many pixels can be safely inpainted if the patches are S-sparse in a flat, incoherent dictionary.
- Implement an iterative inpainting procedure as described above and inpaint the bird.
- ⋆ Compare the inpainting results for the bird for various dictionaries, patch sizes and maximally missing pixels/per patch, which are inpainted in one inpainting iteration. Free other animals.

References

[1] E. Höck. Hard thresholding pursuit for sparse approximation. BSc thesis, University of Innsbruck, 2016.

[2] V. Naumova and K. Schnass. Dictionary learning from incomplete data. *arXiv:1701.03655*, 2017.

[3] J.A. Tropp. Greed is good: Algorithmic results for sparse approximation. *IEEE Transactions on Information Theory*, 50(10):2231–2242, October 2004.