Zertifikat Lehrkompetenz
Teaching probability: flipping coins and classrooms

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Summary: A mixed-methods approach was used for this case study. More precisely, the didactic strategies implemented in the course *Stochastics 2* in winter terms 2020/2021 and 2021/2022 can be summarized, in a few words, as following:

New educational normality:
clicker questions, adapted flipped classrooms, word clouds, interactive teaching, but ALWAYS blackboard (chalk or virtual) talk.

The mixed pedagogical methods that I have proposed combine what worked best in both virtual and non-virtual teaching, in my courses in *Stochastics 2* (Advanced Probability) for Bachelor Students in Mathematics, given in winter terms 20/21 and 21/22, respectively. The preliminary planning of the teaching strategies, that serve as a basis for the underlying study, started in September 2021. I taught this course one year before fully online on a virtual blackboard when I have prepared lecture notes for the students, and some of the methods that worked well online, will be improved and adapted to the classroom setting as well. The planning has been done under the optimistic scenario that after three semesters of fully online teaching, we will finally return to teaching in classroom. For me it was clear that

we cannot return to the same frontal teaching style we had before the pandemic,

and we have to keep using parts of the online technologies in classroom as well. Under these optimistic circumstances, the focus during my course was to engage the students more than ever by means of some visual mathematical experiments (coin flips, throwing dies, etc), to accommodate their needs (word clouds, slow down, do more applications, review parts that are harder to understand) as much as possible, at the risk of not covering the whole theory in class and transforming spontaneously a theoretical course into an exercise solving session

where I would have taken the student role of exercise-solver. This non-virtual teaching scenario lasted only a couple of weeks, and we were back to online teaching, earlier than expected, so the methods had to be again adapted to the new situation, but this time was different that in March 2020. Nevertheless, this couple of weeks helped to connect with the students and made them feel more comfortable in the upcoming online sessions and discussions.

The current work deals with a detailed analysis of the new educational normality that we are currently all facing, and it describes mixed implemented strategies in my courses that were successful, according to students' feedback and engagement, final exam grades, and intermediate quizzes. Briefly, I have combined several teaching methods (blackboard talk, flipped classroom, clicker questions, student-educator role exchange, word clouds, office hours on the campus benches) and **I have adapted the methods from week to week**, according to the new situation and according to the students suggestions and level of excitement/involvement. The main focus that I always had in mind during my classes was not to loose motivated students by spending too much time on trivial things on one hand, and on the other hand to keep also the less motivated ones active by addressing them directly with less involved questions.

Universal teaching strategy: One strategy and method that I kept and will always keep in my mathematics classes is **writing on the (chalk or virtual) blackboard**: this is the most important tool we have for introducing students to "doing mathematics". The chalk talk slows down the doing since

"chalk reveals itself word by word and so students focus on it word by word."

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1 Description of the course and the setup

The reference course (VO) for this work is **Stochastics 2**, a course for mathematics students in the 5th Semester of Bachelor, it is a compulsory course, and it investigates, in less technical terms, methods of advanced probability, or in even less technical terms *how to loose your*

money in a casino game or in any other game involving random experiments. It deals with the theory of randomness, and how to make randomness feasible from a mathematical point of view. This field of study has a wide range of applications in: insurance mathematics, evolutionary models in biology, physics, computer science.

I would like to emphasize that in a mathematics course, we don't expect students to learn theorems, proofs, and definitions by heart, what is really important is that they get acquainted with the rigorous way of thinking and deducing proofs and results based on what they have accumulated so far. We build a solid background of mathematical tools and ideas, rigorous thinking, on which new methods can be founded. It is then a great satisfaction for us as teachers, and for students as well, when we prove theorems together, step by step, or when we understand how to apply these tools in real-life models.

[In mathematics] the results are important . . . but the results aren't as important as the process that gets you to the results.

This process is what we expect from our students to go through. When this happens, then we can say that we have reached our goal as instructors. The methods and theorems together with their proofs we investigate during the course Stochastics require a high level of mathematical maturity, that one acquires during the first four semesters of studies, with a lot of commitment, motivation, and hard work in teams.

"Stochastik ist an sich ein mega nerviges Fach, aber die LV Leiterin hat es gut gemacht, auch für diese harten Umstände". *WS20/21, Stochastik 2*

Given that the students enrolled in my class in WS21/22 had only one "normal" semester out of the first five ones, all the other four being fully online, I was not sure if they have achieved this level of mathematical maturity and motivation for self-study. I have to admit that I was somehow positively surprised. Even if they were facing the difficulties of online studies of a hard study field (Mathematics is considered by many pupils already in school very hard) and some of them didn't even find in one semester classmates, most of them still found the right motivation to keep going on and being competitive, and indeed the outcome was excellent.

Comparing two consecutive years – positive and negative aspects. Being given the chance to teach the same course **Stochastics 2** two years in a row, in winter term 20/21 and exactly one year later, both times online (with a couple of sessions in classroom in the second year), I can draw several conclusions based only on this direct comparison. One visible fact is that the number of students in mathematics in the fifth semester decreased drastically. Whether this is a consequence of the pandemic, or it is only a random fact, is something we cannot conclusively answer, for the moment. A second comparison is between the students' behavior and engagement in these two consecutive years.

- **Winter 20/21 (56 students enrolled, ca. 35 active)**: the students attending my class were in their second semester of online teaching, and they have had three semesters of frontal teaching in classroom. They have also attended another course of

mine before, and they were familiar with my teaching style. I have noticed that the posed questions were answered immediately, I was being interrupted if something was unclear or if I was too fast, and the office hours were very interactive. They have been working in teams in order to solve the exercises, and the forum of the course was used intensively. The grades of the final exam were similar to the previous years, so we couldn't see here big differences.

- **Winter 21/22 (23 students enrolled, ca. 10 active)**: this generation had only one semester of normal teaching, all the other three being fully online, and they are approaching the last semester of their studies. They have also attended a course of mine online before, but we didn't get the chance to know each other in person. Most of them were working alone in solving the exercises, they were harder to reach at the beginning even in classroom. Nobody wanted to sit in the front rows, posed questions didn't receive an answer right away. When insisting and giving them more time, the answers will eventually arrive. So, it was more than obvious that the approach has to be different: the main focus is to connect and interact with the students, to ask for feedback, and help them overcome the difficulties they have faced before. It was the perfect time to implement some additional concepts and help them in the process of attaining the mathematical maturity and independence. The big surprise was for me the outcome of the final exam: 50% of the students that took the final exam did very good (grade 1) and this is a huge improvement in comparison with the previous year. It is nevertheless hard to say if this is the outcome of the teaching strategies used in the current semester. Since the number of participants in this semester was less than half the number from the previous year, I also suspect that the students that have problems with online teaching and independent study either gave up or took a break until the situation will be more clear. The remaining ones are the best ones.

Conclusion to the comparison and some questions to reflect on

1. Why did the students in the winter term 21/22 outperform in the final exam, considering that at the beginning of the semester I have given them less credit since they were more affected by the pandemic? The level of difficulty of the final exam was comparable to the previous exams.

2. Were the used methods indeed more efficient? The accompanying exercise sessions for my course went also very good according to my colleagues, and the students got much better grades than a year before.

3. Even if the methods offered were very diverse and adapted from week to week, according to the word clouds filled in by the students, the interaction in online sessions was not optimal. They were not asking many questions, I had to work more than in the years before to transform them from passive listeners to active participants.

2 Presentation of the methods and strategies

Preliminaries. After three semesters of teaching courses fully online, the idea of returning in classroom was a challenge both for students and for me, as course instructor. Returning to the somehow old-fashioned frontal classes teaching method, in which the instructor writes and explains the theorems and proofs on the blackboard 90 minutes long, while the students write down the blackboard content without having much time to process, was not adequate anymore. Some of the harder classes would then turn into a monologue, and this is something we have to avoid. In the upcoming subsections, I will present in details the ratatouille of used strategies, that may be implemented both in classroom and in virtual mode, that are flexible, and allow students to take actively part in the lecture and to influence further development/methods implemented.

Available support for the course: the students were given lecture notes for the course. Since there is also plenty of material available on the web, videos on Youtube, books that accompany the course, pure presentation of the lecture notes on the blackboard is not the most efficient way to use the time available in classroom, and this time may be spend in an effective way for instance in processing the lecture notes together with the participants. Another effective way of using the time is to go with the students through some parts of the lecture notes that are difficult to understand, and to explain them in detail.

2.1 Basic instruction form: chalk (or virtual) blackboard

The typical image of a mathematician is a person standing in front of a blackboard using chalk to write strange symbols. Lecturing in mathematics at the blackboard is still the most common form of instruction for both undergraduate and graduate students in mathematics. This basic tool is also the most important one that guides my teaching of mathematical courses: **synchronous writing on the (chalk or virtual) blackboard.** Studies suggests that chalk talk, namely, writing out a mathematical narrative on the board while talking aloud, is the central pedagogical genre of the undergraduate mathematics lecture classroom. See for instance [2]. The research paper [8] explores how writing mathematics (on paper, blackboards, or even in the air) is indispensable for doing and thinking mathematics.

When comparing what writing on the board offers university teachers of mathematics with some other alternatives, for example, overhead transparencies or PowerPoint presentations, all of my students in Innsbruck and at Technical University of Graz, where I was teaching previously Mathematics courses for Electrical Engineering and Biomedical Engineering students, were adamant in their support of chalk talk as the principal instructional approach, although some acknowledged benefits from peripheral supports that other technologies may provide. Some excerpts from teaching evaluations concerning the blackboard teaching:

"Dass der Stoff in der VO direkt per Tablet vorgerechnet/ aufgeschrieben wurde, dadurch war die Geschwindigkeit gut dem Stoff angepasst". *WS20/21, VO Stochastik 2*

"Die Vorlesung über Big Blue Button hat sehr gut geklappt und die sauberen "Tafel"-Mitschriften über OneNote sind äußerst praktische Unterlagen. So muss man selbst nicht schreiben und kann sich viel besser auf den Inhalt der Vorlesung konzentrieren". *WS20/21, VO Stochastik 2*

"Sehr gut die Sachen erklärt. Angenehmes Tempo. Zum mitschreiben angenehm". *WS20/21, VO Stochastik 2*

"Der Vortragsstil an der Tafel ist wirklich gut. Wenn auch teilweise in der Vorlesung die Finger krachten, hat man das Gefuühl, in dieser VO Vieles neues gelernt zu haben." *SS19, TU Graz, Mathematik B*

"Sehr nett, sympathisch. Ist bemüht. Schönes leserliches Tafelbild. Erklärt gut. Finde es super, dass sie auch Bsp vorrechnet. Das macht die VO etwas verständlicher. Schreibt quasi jeden Schritt dazu und erklärt sie uns kurz und aussagekräftig." *WS18/19, TU Graz, Mathematik A*

"Sehr schönes Tafelbild, bemüht, sympathisch, gute Kombination aus Theorie und praktischen Beispielen." *WS18/19, TU Graz, Mathematik A*

Only looking at the few students' opinions above, it is clear that the blackboard talk together with oral explanations are well established and efficient methods of teaching mathematics and reaching students directly, and keeping this method in mathematical courses is the natural way to go. The blackboard talk offers more spontaneity, it slows us down while in a computer presentation one can go much faster than the students can comprehend the mathematical stuff. Based on these studies, we keep in most of the lectures chalk talk, or graphical blackboard talk, together with new technologies.

Iconic character of blackboards in mathematics

Chalk board: in the classroom, we ask the class if some of the least recent writing on the board can be removed; we erase a section of what we have written, which we deem no longer necessary, for the continuation of the disciplinary narrative; we use colored chalk to highlight parts of the writing; we draw boxes around key elements in the lecture; we divide the board into sections by drawing vertical lines; and/or move sliding boards in a highly systematic way. All these visual methods are of great importance for the presentation of new mathematical concepts, the students appreciate having the full story on the blackboard.

Virtual board: in online setting, we have always only one "piece of blackboard" available, and going back and forth makes the presentation confusing. We don't have to erase sections, but we don't see them in front of our eyes in order to understand how all of them work together. The virtual blackboard works better if students are given lecture notes previously, so that they can make directly notes and comments on them.

Writing on a board or preparing presentations? While on a presentation, there is too much mathematical information to process, and the brain turns off at some point, because it cannot keep up with the speed of the presentation, in a board talk we are constrained to slow and explain in the same time with writing. A presentation is perfectly prepared, it contains no mistakes but a blackboard talk gives us teachers the opportunity to make mistakes, and in my experience, this makes students even more comfortable, because we transmit them that it's ok to experiment and make mistakes. And they may think: well, it's not as easy as it sounds, right? Even though the course started with board talks, I was planning to change the teaching mode during the semester, if the students expressed their wish in this direction. So I have asked the students several times about their opinion concerning blackboard talk or computer presentation. Which one do they prefer? On an anonymous word cloud, their answers speak a clear language: chalkboard is the teaching method mathematics students prefer.

During a presentation it is hard to take hand-written notes, because of the speed in changing slides. Nevertheless, there is strong evidence that students who take notes during instruction perform better than students who don't and that further improvement of performance depends on how students act on the notes taken. Research studies also confirm that although note taking during a lecture does not improve immediate performance when compared with not taking notes, after a time lapse of two weeks students who revised on their own notes performed significantly better than students who revised on lecturer produced notes. So this motivates me even more to keep the blackboard talk that allows students to take notes in an appropriate speed. See [9] for additional studies and information in this direction.

Conclusion. Writing on the board is a teaching method that will always have a special role in mathematics courses and in research talks as well, and this will remain the cornerstone of my teaching strategies. A chalk board alone is not sufficient for a good mathematical course, **chalk board management** is also very important. Emphasizing the role of writing on a board in mathematics is done best in the following comparative sentence:

Doing mathematics, for the most part, without writing is almost as trying to talk about music without playing it.

2.2 Word clouds

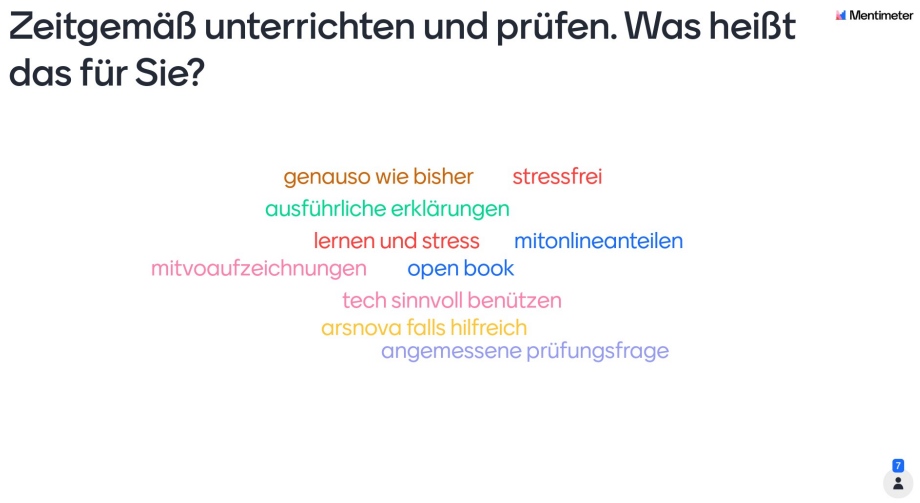
Word clouds or tag clouds can be very efficiently used as educational tools, for instance to convert a reflection assignment into a word cloud image. They cultivate major goals in higher education such as: critical thinking and peer interaction, see [5]. A word cloud takes the most frequently used words in a particular text and randomly displays them by size, based on their frequencies. Word clouds also display variation in color, typography, and composition, offering an aesthetically pleasing look. Studies [4, 3] reveal that that students analyzing a text in word clouds reported higher scores on critical thinking than students analyzing the same text in a linear form. While this tool is more powerful when being used

by a big number of participants, even in smaller sessions may have advantages, as it was the case in the current course. The word cloud had in the context of my course, a bigger relevance for planning upcoming sessions of my course.

What for and why? The main motivation I had for using word clouds was to involve students in taking decisions that concern the future planning of the course: these may be related with the teaching approach and speed, with the examination mode, with checking whether some significant parts of an important chapter have been understood or not. At some instances I have even asked them a general question only to connect and create a friendly atmosphere. The word clouds have the effect of giving students the feeling that they really have a saying in me planning future sessions of the course and taking their concerns seriously. While the plan and the theory to cover during this compulsory lecture are fixed in the curriculum, the approach to them is flexible, and it can be adapted on the way if the participants wish so. The outcome of these clouds was, in my opinion, that the students appreciated very much the fact that I didn't consider them spectators in my course, but adults that can decide together with me what is best to keep their motivation at a high level.

In the following, I will take a few examples of the questions I have asked in order to build the word clouds, and discuss how did the students' answers and comments influence the future planning and organization of the course. I will explain what exactly I did change, from week to week, in order to take their concerns seriously. The number of participants was low, about 10, but this still helped in keeping the students involved.

1. ***Contemporary teaching and examination: what does this mean for you?***



Some answers given: the same as it is, with online tools, with videos of the courses, detailed explanations, arsnova quizzes, use of technology, open book exam. See the picture above for the original answers.

Reaction to students' suggestions: the videos/online recordings from the previous yeas have been made available for the students. Online technologies have been used in the upcoming

courses (even in classroom) as following: regular Arsnova clicker questions in order to determine their level of understanding, breakout rooms for more interaction/discussion during the online courses, short beamer presentations at the beginning of the course in order to review the previous week theory, and one slide with what is needed for the current lecture. The exam question has been addressed in detail in an upcoming session.

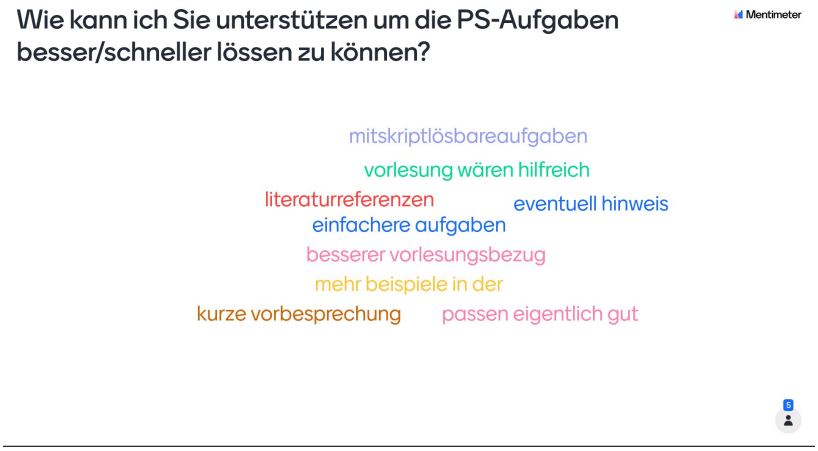
2. ***How would you describe stochastics?***
This question was mostly of a general type, and the main goal was to understand how is this course perceived by students, and if the level of difficulty is rather high or low. This influenced the future planning of the course in the way that I have split longer parts in subsections, longer proofs were given as independent study, and for a better understanding the main steps were discussed in class.

3. ***How would you describe an assignment/exam, where you performed great?***

Some answers given: open book, enough time to think, fair, not too easy but not to hard. *Reaction to students' suggestions*: For supporting students' suggestions, I came up with the idea of a semi-open book exam: they were allowed for the final exam to write by hand a A4-sheet with the theory from the course, and I didn't expect them to memorize facts, but just

to understand how to use the material taught in concrete applications. The level of difficulty was kept high, and the outcome of the final exam was very good: half of the students got a "very good", and 30% obtained "good" (2). Based on this outcome, I think the methods implemented were efficient and helped the students in the learning process.

4. *How can I support you in solving the exercises?*
I would like to mention that the exercise sessions are completely separate from the course, and I was not in charge of the exercise sheets. Anyway, together with my colleagues we have chosen the problems/exercises together, so that they match the theoretical part presented in the course.



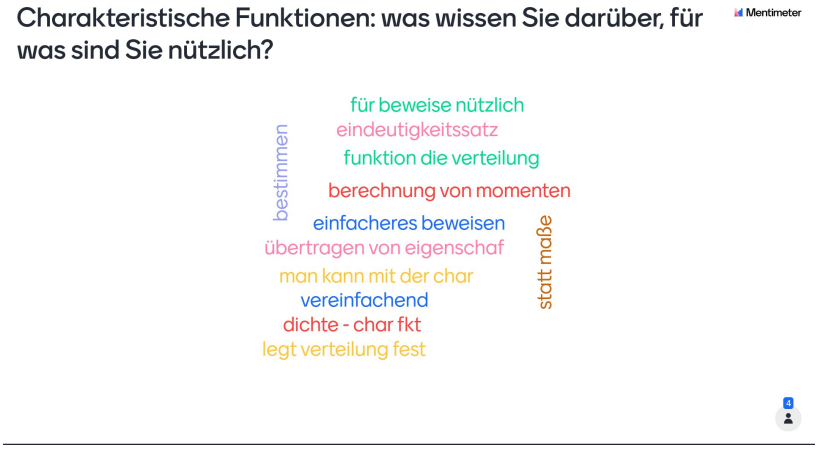
Some answers given: more problems/exercises in the course, additional office hours for discussing solutions, more bibliography.

Reaction to students' suggestions: While in the theoretical part of the course several applications and problems were discussed, the time to cover step-by-step solutions was not enough. What have I done to address this concern was the following:

- one of the two weekly courses started with a short discussion of the exercise sheets, and solution hints for the more involved exercises.
- I have offered them on a regularly basis exercise solving office hours, in which I will solve spontaneously in detail complex problems, so that they see that this process is also for me one that requires concentration, making errors, and trying different solution ways.
- The week before the final exam I have introduced several review/learning sessions, in which we go together through the whole course and we discuss carefully the delicate parts.

Even though the exercise and the course were two separate entities, after making this word cloud, the exercise session became somehow a part of the course in which the students noticed that even if they have another lecturer for the exercise session, they can discuss with me the difficulties as well.

5. ***Characteristic functions: what do you know about them, for what do we need them?***



This word cloud was realized after we have completed an important chapter of the course, which has many connections with the previous parts of the lecture. The idea was to see if the students got the right meaning and importance of such objects.

Conclusion. Word clouds are educational tools that I have used for the first time in my classes, and I plan to use them in future classes as well for several reasons. First of all, they represent ways to meaningfully engage students in an asynchronous environment and to involve them in the future planning of the course. If one takes into consideration the answers and modifies the structure of the course accordingly, this requires more preparation and higher flexibility from week to week. It works both in classroom and online, and if the questions are appropriately chosen, it may help us recognize difficulties and problems at early stages. Due to the anonymity of the answers, the students dare to say more than if asked directly. This tool can also be seen as a continuous feedback during the semester. If overused, it may get annoying, but if used in reasonable amounts, it is a powerful source of constructive critique.

2.3 Clicker questions

Clicker questions or quizzes are also among the variety of interactive educational tools that I have employed during the lecture in order to get an immediate feedback, direct responses and to activate students during the lecture. These quizzes have been implemented using ARSnova, an open source innovative and efficient audience-response-system and have the advantage of solving the fear of asking irrelevant questions or providing wrong answers in front of a large audience. Since I didn't want to abuse the use of new technologies, I was alternately using word clouds for opinions and suggestion on general topics, on my approach to the lecture, on aspects of the course where the students may interfere. On the other hand, I was using clicker questions to facilitate intellectual engagement but I have to emphasize

that clickers in themselves are not a solution to anything. Like a chalkboard, they can only serve to extend the capabilities of the instructor. When designing the clicker questions and preparing the corresponding material for lecturing, I found [1] to be a very good guide in this direction. I have learned that clickers become useful when I have a clear idea as to what I want to achieve with them.

The goals I intended to achieve by using clicker questions in the lecture were the following:

- to improve student engagement and student-student interaction.
- to check if the taught material has been understood.
- to rapidly collect an answer to a question from every student.
- when a significant fraction chooses an incorrect answer, we would go together over the question and the answers.

Several studies, see [6], reveal that the students in classes using well-implemented clicker questions are more engaged and they ask more numerous and deeper questions and this was the main motivation for implementing such a tool in my class. In addition, it was a powerful method for opening a discussion with students on topics harder to digest. Since the course Stochastics 2 is a follow up of the course Stochastics 1, that the students attended 18 month before, this big intermediate break is favorable for forgetting a lot of material that is needed in the current lecture. In order to refresh the notions and review parts of the course Stochastics 1, I wanted to have a clear idea of how much do they still remember, which parts have been completely forgotten, and where should I start as a lecturer with the review part, in order to bring them all to the same starting line. For getting this information in an efficient way, a clicker questions system seemed to be the perfect solution to be used in the very first lecture of the semester. The students have been encouraged to work in groups, and this indeed stimulated a lively discussion. I will go very briefly into some clicker questions used during the lecture, and explain how the answers stimulated further student-instructor interaction.

The first basic question (Maßtheorie, see the first Figure below for the outcome) concerns a basic object in stochastics, σ-algebras, and the right answer was the first and the third. Since almost all students gave the right answer, it was no need to go into deeper explanations.

On the other hand, for the second question (Konvergenz, see the second picture below for the outcome) which concerns different types of convergence for sequences of random variables, while all four answers are true, the students didn't get it quite right. This was a completely new topic introduced in the week before, and I wanted to check their level of understanding, and if there are further explanations needed. And indeed this material required more explanations and applications.

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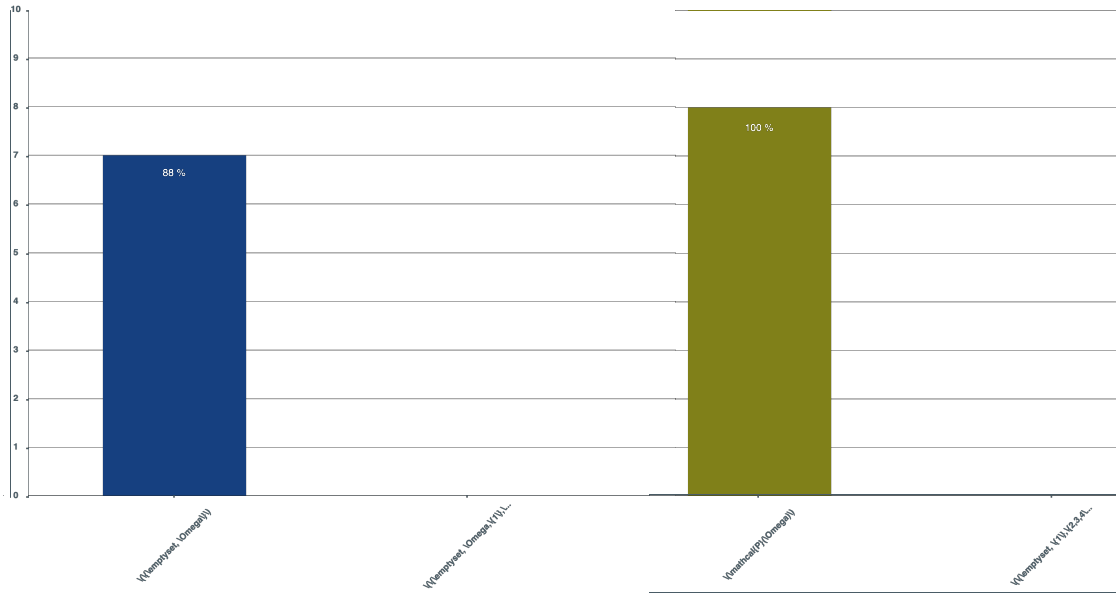
Masstheorie

8 Answers Export

Es sei $\Omega = \{1, 2, 3, 4\}$. Welche der folgenden Mengensysteme sind σ-Algebren auf der Menge Ω?



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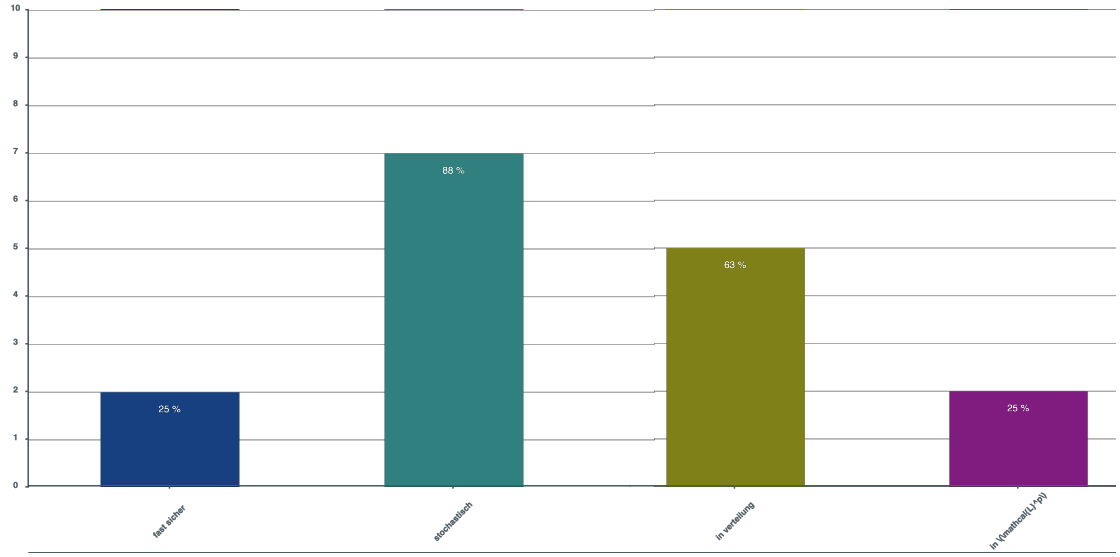
Konvergenz

8 Answers Export

Sei $(X_n)_{n\in\mathbb{N}}$ eine Folge unabhängiger Zufallsvariablen mit $X_n \in \{0, 1\}$, definiert durch: $\mathbb{P}(X_n = 0) = 1 - \frac{1}{2^n}$ und $\mathbb{P}(X_n = 1) = \frac{1}{2^n}$. Für eine geignete Zufallsvariable X, wie konvergiert $(X_n)_{n\in\mathbb{N}}$ gegen X?



Statistics Vote

2.4 Flipping coins and classrooms

The lecture has been taught live (in classroom or online), by having as a basic strategy the (chalk or virtual) board talk, because this is in my opinion the way to stay connected and interact directly with the students. Nevertheless, several elements from *Flipped or Blended Classroom* have been introduced as alternatives for students that could not attend the live lectures, and for those ones that choose their own learning speed. Flipped classroom is one of the methods that is adapting best to this "new educational normality" and turns the traditional learning environment upside-down, but digital (virtual) teaching is not automatically *Flipped Classroom* or the other way round. See [7] and the references therein for a lecture-oriented flipped classroom approach.

In addition to the lecture notes available to the students from the beginning of the semester, in order to support the students that were working or not living in Innsbruck due to health conditions, the following alternatives have been offered as well:

- During the first weeks when the teaching was in classroom, the recordings in Big Blue Button of the course made one year before (WS 20/21) have been made available.
- Starting with the month November 2021, when we have resumed to online teaching, I have kept the live teaching mode on a graphical board, and I have recorded the lectures and made them available in OLAT. So the students had the option of watching the recordings whenever they want.

The idea of preparing well-polished videos in advance and making them available to students, and meeting online once-twice a week with students in order to discuss the content of the videos was not very appreciated by the students in my class in the pandemic situation. They felt more the need of interacting live with the instructor, of interrupting spontaneously the instructor in order to pose a question and get the answer right away. After three-four semesters of online teaching, most of the attendants mentioned that watching videos of lectures does not stimulate their attention very much, they get tired and loose focus very fast, and they still need to sit down and take notes in order to understand the content of the course. The basic principle of flipped classroom, in which students come into contact with new content before class, while the in-class time is used for activities that help students make sense of it, was nevertheless used in various ways during the course: for instance by having access to the recordings and to the lecture notes. The in-class time has been extensively used for activities that promote an understanding of the content of the lectures: the use of clicker questions with subsequent peer-discussions in breakout rooms, (group) work on quizzes and tasks, examining possible applications and problems, but also teacher-driven discussions of difficult material. Some students evaluations/comments that support my choices of teaching strategies are given below.

"Virtuelle Lehre ist natürlich nicht ideal, aber ich finde man hat das bestmöglichste aus der Situation gemacht". *WS21/22, VO Stochastik 2*

" Das Skript ist sehr gut und ausführlich geschrieben, sodass, auch wenn man öfters bei Vorlesungen verhindert war, den Stoff sich gut selbst erarbeiten konnte."
WS21/22, VO Stochastik 2

" StudentInnen werden angeregt, Verbesserungsvorschläge vorzubringen, um die Veranstaltung so gut wie möglich umzusetzen."
WS21/22, VO Stochastik 2

" Es fand immer wieder Rückfrage mit den Studierenden statt, ob die verwendeten Methoden für die virtuelle Lehre so passen, oder ob wir (z.B. von anderen LV) Verbesserungsvorschläge haben. Die Kommunikationsbasis um die virtuelle Lehre interaktiver zu machen war gegeben (auch wenn wir als Studierende nicht immer darauf eingegangen sind...)"
WS20/21, VO Stochastik 2

"Eine der besten Umsetzungen heuer."
WS20/21, VO Stochastik 2

"Am Anfang wurde immer gefragt wie es uns geht; es wurde oft nachgefragt, ob die verwendeten Methoden für die virtuelle Lehre passen – hat alles persönlicher gemacht"
WS20/21, VO Stochastik 2

Flipping coins and throwing dies. In an advanced probability class, some theoretical parts can be best understood when performing basic random experiments such as: flipping coins and throwing dies, and observing/writing down their outcomes after several repetitions. Before proving laws of large numbers and central limit theorems, it is very instructive for students to see how do we get there. So I would go to classroom with a small bag of dies/coins, and let the students perform some live experiments. While this works much better in a classroom with real persons, it can be in the meantime done online, since there are many programs that simulate random experiments. So when asking students if they would prefer the lecture in flipped classroom mode, an answer that deserves mentioning was:

"let us flip coins and not classrooms."

Conclusion. A lecture-oriented adaptation of the flipped classroom approach into my course has been implemented in various ways (word clouds, clicker questions, lecture notes, recordings), but to conclude both mine and students' opinion is that

mathematics is best learned by doing, and not by watching.

This if for a mathematician very good news, and sends the message that, even if we can use technology in many ways to support university courses, they cannot replace real persons and in-class live teaching, be it on chalkboard with oral presentations or students presentations, etc. This is what both students and instructors need most: live interaction, social contact, body language and gestures that can influence so much the understanding of the taught material. Creation of audiovisual material can support a course, but it cannot replace the student-lecturer relationship.

2.5 Mixed-methods approach: conclusion and personal reflections

The underlying case study is based on a mixture of several educational tools, both online and in-class, that were implemented in my courses Stochastics 2 taught in two consecutive winter terms, in 2020/2021 and 2021/2022, mostly virtually, with a few exceptions in-class. While in winter 2020/2021 I didn't use many online educational tools because I was myself in a process of learning and embracing the idea of interactive teaching from another point of view, I could say that in the meantime I am aware of the availability of such tools, and how they can support us in improving our teaching. The current work, documented with student evaluations, reviews, peer-group observations, confirms that the implemented methods were efficient, they created the desired interaction with the students in such extraordinary times. Another evidence that confirms the efficiency of the methods is the big number of requests for Bachelor thesis supervisions after the current course.

Personal reflections. For my future teaching, I plan to keep and to improve the methods in my courses, and to adapt them from year to year. I would like to conclude this work with a brief summary of the strategies I have used, for the reader that wants to skip the details above.

- **Blackboard talk:** in a mathematicians life, writing on the blackboard and thinking aloud, will always be on the top list of my educational tools, because we are not teaching facts, but a way of doing things. In fact, using blackboard "mathematics" becomes visible as a process, not just as a final product.
- **Word clouds**: have been used in my teaching for the first time, and I found them effective in interacting with students in several ways. When I felt that I lost somehow connection and interaction, and I had to change something in the near future in order to reach the students better, but I wasn't sure what, I would ask a question, and let the students' answers displayed in a word cloud. According to their answers, I have planned/changed the speed/reviewed difficult material in the upcoming sessions. The students felt that their voices were heard, and for me as instructor, this tool served as a constant feedback, that I plan to keep in future teaching.
- **Clicker questions:** this is another tool that facilitated the interaction teacher-student and gave room to several interesting discussions. I have learned that having good questions is an important element for having a successful class using clickers. Ideally are questions that students interpret properly and see as interesting and challenging. Using clickers extensively may reduce the amount of content one can cover during class, but used in a moderate way can increase the activity of students, their interest in the material and in stimulating discussions. I indent to keep this e-tool in my classes, and to prepare a catalogue with interesting questions I can use in the future.
- **Lecture-oriented adaptation of flipped classroom**: several elements of flipped or blended classroom have been used in my teaching, but the strategy of having polished videos or presentations to watch in advance is not among the preferred ones for my

students. Having lecture notes they can read in advance was very useful, recordings from previous years have been made available but in my opinion not so successful, and the argument was that the concentration level decreases and the process of doing mathematics cannot be replaced by the process of watching. This is a good argument for me to believe at this stage that I don't plan at the moment to have my lectures in fully blended mode.

References

[1] Clicker resource guide: An instructors guide to the effective use of personal response systems (clickers) in teaching. *Carl Wieman Science Education Initiative (CWSEI) and The Science Education Initiative at the University of Colorado (CU-SEI)*, 2017.

[2] N. Artemeva and J. Fox. The writing's on the board: The global and the local in teaching undergraduate mathematics through chalk talk. *Written Communication*, 28:345–379, 2011.

[3] F. Calle-Alonso, V. Botón-Fernández, J. Sánchez-Gómez, M. Vega-Rodríguez, C. Pérez, and D. de la Mata. Word clouds as a learning analytic tool for the cooperative e-learning platform neurok. *Proceedings of the 10th International Conference on Computer Supported Education*, pages 508–513, 2018.

[4] M. Cavender and K. Medico-Letwinsky. Beyond reflection: Using word clouds to support metacognitive processes in preservice teacher mathematics education. *Journal of Practitioner Research*, 3:Article 2, 2018.

[5] A. Denoyelles and B. Reyes-Foster. Using word clouds in online discussions to support critical thinking and engagement. *Online Learning*, 19, 2015.

[6] D. Douglas and E. Mazur. *Clickers In the Classroom: How to Enhance Science Teaching Using Classroom Response Systems*. San Francisco: Pearson Education, 2005.

[7] F. Feudel and L. Fehlinger. Using a lecture-oriented flipped classroom in a proof-oriented advanced mathematics course. *International Journal of Mathematical Education in Science and Technology*, 0(0):1–28, 2021.

[8] C. Greiffenhagen. The materiality of mathematics: presenting mathematics at the blackboard. *Br J Sociol.*, 65:502–528, 2014.

[9] P. Iannone and D. Miller. Guided notes for university mathematics and their impact on students' note-taking behaviour. *Educ Stud Math*, 101:387–404, 2019.