```
> restart; with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

```
> w1 := <1,2,-2>;
```

$$w1 := \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

```
> w2 := <2,2,-1>;
```

$$w2 := \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

```
> w3 := <-2,1,2>;
```

$$w3 := \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Berechnung einer orthogonalen Familie (A[1], A[2], A[3]), die denselben Vektorraum wie w1, w2, w3 erzeugt:

```
> A:= GramSchmidt([w1,w2,w3]);
```

$$A := \left[\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} \frac{10}{9} \\ \frac{2}{9} \\ \frac{7}{9} \end{bmatrix}, \begin{bmatrix} \frac{-22}{17} \\ \frac{33}{17} \\ \frac{22}{17} \end{bmatrix} \right]$$

```
> DotProduct(A[2],A[3]);
```
(Skalarprodukt der ersten zwei Komponenten von A)

$$0$$

```
> DotProduct(A[1],A[1]);
```

$$9$$

A[1] ist nicht normiert!

```
> v1:=Normalize(A[1],Euclidean);
```

$$v1 := \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{-2}{3} \end{bmatrix}$$

```
> v2:=Normalize(A[2], Euclidean);
```

$$v2 := \begin{bmatrix} \frac{10\sqrt{17}}{51} \\ \frac{2\sqrt{17}}{51} \\ \frac{7\sqrt{17}}{51} \end{bmatrix}$$

```
> v3:=Normalize(A[3], Euclidean);
```

$$v3 := \begin{bmatrix} -\frac{2\sqrt{17}}{17} \\ \frac{3\sqrt{17}}{17} \\ \frac{2\sqrt{17}}{17} \end{bmatrix}$$

(v1,v2,v3) ist eine ON-Basis.

Eine ON-Basis hätte auch direkt berechnet werden können:

```
> B:=GramSchmidt({w1,w2,w3},normalized);
```

$$B := \left\{ \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{-2}{3} \end{bmatrix}, \begin{bmatrix} \frac{10\sqrt{17}}{51} \\ \frac{2\sqrt{17}}{51} \\ \frac{7\sqrt{17}}{51} \end{bmatrix}, \begin{bmatrix} -\frac{2\sqrt{17}}{17} \\ \frac{3\sqrt{17}}{17} \\ \frac{2\sqrt{17}}{17} \end{bmatrix} \right\}$$

```
> C:=<B[1]|B[2]|B[3]>;
```

$$C := \begin{bmatrix} \frac{1}{3} & \frac{10\sqrt{17}}{51} & -\frac{2\sqrt{17}}{17} \\ \frac{2}{3} & \frac{2\sqrt{17}}{51} & \frac{3\sqrt{17}}{17} \\ \frac{-2}{3} & \frac{7\sqrt{17}}{51} & \frac{2\sqrt{17}}{17} \end{bmatrix}$$

```
> Determinant(C); evalf(Trace(C));
```

$$-1$$

$$0.9800950001$$

Die Basis (B[1], B[2], B[3]) ist also verschieden orientiert wie die Standardbasis (e1,e2,e3).

```
> CrossProduct(B[1],B[2]);
```

$$\begin{bmatrix} \frac{2\sqrt{17}}{17} \\ -\frac{3\sqrt{17}}{17} \\ -\frac{2\sqrt{17}}{17} \end{bmatrix}$$

Das Kreuzprodukt von B[1] und B[2] ist - B[3].

```
> C.Transpose(C);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Die Matrix C ist also orthogonal und hat Determinante 1, also beschreibt sie eine Spiegelung oder Drehspiegelung.

```
> NullSpace(C-IdentityMatrix(3));
```

Berechnet eine Basis der Drehachse,

also des Eigenraum zum Eigenwert 1 von C.

$$\{\ \}$$

Die Basis ist die leere Menge, also hat C nur 0 als Fixpunkt, daher beschreibt diese Matrix eine Drehspiegelung.

Wir berechnen eine ON-Basis der Drehachse, eine ON-Basis der Drehebene / Spiegelungsebene und den Cosinus des Drehwinkels:

```
> NullSpace(C+IdentityMatrix(3));
```
Die Drehachse einer Drehspiegelung ist deren Eigenraum zum Eigenwert -1.

$$\left\{\left[\begin{array}{c} -\frac{17+3\sqrt{17}}{2(\sqrt{17}-17)} \\ \frac{4\sqrt{17}}{\sqrt{17}-17} \\ 1 \end{array}\right]\right\}$$

```
> z1:=op(%);
```
z1 ist eine Basis der Drehachse.

$$z1 := \left[\begin{array}{c} -\frac{17+3\sqrt{17}}{2(\sqrt{17}-17)} \\ \frac{4\sqrt{17}}{\sqrt{17}-17} \\ 1 \end{array}\right]$$

```
> z:=Normalize(z1, Euclidean);
```
z ist eine ON-Basis der Drehachse.

$$z := \left[\begin{array}{c} -\frac{17+3\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)} \\ \frac{8\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)} \\ \frac{2}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}} \end{array}\right]$$

```
> DotProduct(z,z);
```

$$\frac{(17+3\sqrt{17})^2}{\left(4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}\right)(\sqrt{17}-17)^2}$$

$$+\frac{1088}{\left(4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}\right)(\sqrt{17}-17)^2}+\frac{4}{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}$$

```
> evala(%);
```

$$1$$

```
> z_orthogonal:=NullSpace(Matrix(1,3,[z[1],z[2],z[3]]));
```

$$z\_orthogonal := \left\{ \begin{bmatrix} \frac{2(\sqrt{17}-17)}{17+3\sqrt{17}} \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{8\sqrt{17}}{17+3\sqrt{17}} \\ 1 \\ 0 \end{bmatrix} \right\}$$

```
> x:=Normalize(z_orthogonal[1], Euclidean); y:=CrossProduct(z,x);
```

x und y sind eine ON-Basis der Drehebene.

$$x := \begin{bmatrix} \frac{2(\sqrt{17}-17)}{\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})} \\ 0 \\ \frac{1}{\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}} \end{bmatrix}$$

$y :=$

$$\left[\frac{8\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}} \right.$$

$$\left. \frac{4(\sqrt{17}-17)}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})} \right.$$

$$+\frac{17+3\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}}\Bigg]$$

$$\Bigg[-\frac{16\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})}\Bigg]$$

```
> c:= (1/2)*(Trace(C)+1);
```

c ist der Cosinus des Drehwinkels

$$c := \frac{2}{3}+\frac{4\sqrt{17}}{51}$$

```
> evalf(c);
```

$$0.9900475001$$

```
> T:=<x|y|z>;
```

$T :=$

$$\Bigg[\frac{2(\sqrt{17}-17)}{\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})},$$

$$\frac{8\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}},$$

$$-\frac{17+3\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)}\Bigg]$$

$$\Bigg[0,\ \frac{4(\sqrt{17}-17)}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})}$$

$$+\frac{17+3\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}},$$

$$\left.\frac{8\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,(\sqrt{17}-17)}\right]$$

$$\left[\frac{1}{\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}},\right.$$

$$-\frac{16\sqrt{17}}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}\,\sqrt{1+\frac{4(\sqrt{17}-17)^2}{(17+3\sqrt{17})^2}}\,(17+3\sqrt{17})},$$

$$\left.\frac{2}{\sqrt{4+\frac{(17+3\sqrt{17})^2}{(\sqrt{17}-17)^2}+\frac{1088}{(\sqrt{17}-17)^2}}}\right]$$

```
> Determinant(T);
```

$$-\frac{136\,(-15489+2009\sqrt{17})}{(\sqrt{17}-17)^2\,(-81+\sqrt{17})^2}$$

```
> evala(%);
```

$$1$$

```
> evala(T.Transpose(T));
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> E:=evala(Transpose(T).C.T);
```

Matrix der Drehspiegelung bezüglich der Basis (x,y,z).

$$E := \begin{bmatrix} \frac{2}{3}+\frac{4\sqrt{17}}{51}, & -\frac{\sqrt{2754-34\sqrt{17}}}{136}+\frac{\sqrt{2754-34\sqrt{17}}\,\sqrt{17}}{408}, & 0 \\ \frac{\sqrt{2754-34\sqrt{17}}}{136}-\frac{\sqrt{2754-34\sqrt{17}}\,\sqrt{17}}{408}, & \frac{2}{3}+\frac{4\sqrt{17}}{51}, & 0 \\ 0, & 0, & -1 \end{bmatrix}$$

```
> evalf(E);
```

$$\begin{bmatrix} 0.9900475001 & 0.1407336054 & 0. \\ -0.1407336054 & 0.9900475001 & 0. \\ 0. & 0. & -1. \end{bmatrix}$$

```
> 2*Pi - arccos(0.3281852514);
```

Drehwinkel (im Bogenmaß) der Drehung, wenn die Drehebene durch (x,y) orientiert ist.

$$2\,\pi - 1.236414549$$

```
> evalf(%);
```

$$5.046770759$$

```
>
```