```
> with(LinearAlgebra);
```

[*&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip*]

```
>
```

```
> w1 := <3,1,0,-1>;
  w2 := <1,2,2,-1>;
  w3 := <0,-2,1,4>;
```

$$w1 := \begin{bmatrix} 3 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$w2 := \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}$$

$$w3 := \begin{bmatrix} 0 \\ -2 \\ 1 \\ 4 \end{bmatrix}$$

Berechnung einer Basis, deren Vektoren paarweise zueinander orthogonal sind, von dem von w1,w2,w3 erzeugten Vektorraum:.

```
> U:=GramSchmidt([w1,w2,w3]);
```

$$U := \left[\begin{bmatrix} 3 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} \frac{-7}{11} \\ \frac{16}{11} \\ 2 \\ \frac{-5}{11} \end{bmatrix}, \begin{bmatrix} \frac{51}{37} \\ \frac{-32}{37} \\ \frac{67}{37} \\ \frac{121}{37} \end{bmatrix} \right]$$

Berechnung des Standardskalarprodukts von w1 und w2:

```
> DotProduct(w1,w2);
```

$$6$$

Berechnung der Gramschen Matrix des Standardskalarproduktes bezüglich der Basis (U[1], U[2], U[3]):

```
> for i from 1 to 3 do
  for j from 1 to 3 do
  A[i,j]:=DotProduct(U[i],U[j]);
> od; od;
> A:=Matrix(3,(i,j) -> A[i,j]);
```

$$A := \begin{bmatrix} 11 & 0 & 0 \\ 0 & \frac{74}{11} & 0 \\ 0 & 0 & \frac{615}{37} \end{bmatrix}$$

Berechnung einer ON-Basis von dem von w1,w2,w3 erzeugten Vektorraum:

```
> ONB:=GramSchmidt([w1,w2,w3], normalized);
```

$$ONB := \left[\begin{bmatrix} \frac{3\sqrt{11}}{11} \\ \frac{\sqrt{11}}{11} \\ 0 \\ -\frac{\sqrt{11}}{11} \end{bmatrix}, \begin{bmatrix} -\frac{7\sqrt{814}}{814} \\ \frac{8\sqrt{814}}{407} \\ \frac{\sqrt{814}}{37} \\ -\frac{5\sqrt{814}}{814} \end{bmatrix}, \begin{bmatrix} \frac{17\sqrt{22755}}{7585} \\ -\frac{32\sqrt{22755}}{22755} \\ \frac{67\sqrt{22755}}{22755} \\ \frac{121\sqrt{22755}}{22755} \end{bmatrix} \right]$$

Berechnung der Gramschen Matrix des Standardskalarprodukts bezüglich der berechneten ON-Basis:

```
> for i from 1 to 3 do
  for j from 1 to 3 do
  B[i,j]:=DotProduct(ONB[i],ONB[j]);
> od; od;
> B:=Matrix(3,(i,j) -> B[i,j]);
```

$$B := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Berechnung der Norm einer Spalte bezüglich dem Standardskalarprodukt:

```
> Norm(w1, Euclidean);
```

$$\sqrt{11}$$

"Normalisierung der Spalte w1:

```
> Normalize(w1, Euclidean);
```

$$\begin{bmatrix} \frac{3\sqrt{11}}{11} \\ \frac{\sqrt{11}}{11} \\ 0 \\ -\frac{\sqrt{11}}{11} \end{bmatrix}$$