

## RECHNEN MIT ALGEBRAISCHEN ZAHLEN

```
> restart;
```

### Rechnen mit "Wurzel aus -1"

```
> f:=x^2+1;
```

$$f := x^2 + 1$$

```
> irreduc(f);
```

*true*

```
> alias(alpha=RootOf(Z^2+1));
```

$\alpha$

```
> alpha^2;
```

$\alpha^2$

```
> evala(alpha^2);
```

-1

```
> (3*alpha+4)^(-1);
```

$$\frac{1}{3\alpha + 4}$$

```
> evala(%);
```

$$\frac{4}{25} - \frac{3\alpha}{25}$$

```
> gcdex(f,3*x+4,x,u,v);
```

1

```
> v;
```

$$\frac{4}{25} - \frac{3x}{25}$$

```
> subs(x=alpha,v);
```

$$\frac{4}{25} - \frac{3\alpha}{25}$$

```
> (5*alpha+1)^(-3);
```

$$\frac{1}{(5\alpha + 1)^3}$$

```
> evala((5*alpha+1)^(-3));
```

$$-\frac{37}{8788} + \frac{55\alpha}{8788}$$

```
> factor(f,alpha);
```

$$(x + \alpha)(x - \alpha)$$

### Rechnen mit "Wurzel aus 5"

```

[ > restart;
[ > irreduc(x^2-5);
[                                     true
[ > alias(beta=RootOf(Z^2-5));
[                                     beta
[ > 1/(1-beta);
[                                     1
[                                     1 - beta
[ > evala(%);
[                                     - 1/4 - beta/4

```

**Rechnen mit einer nicht-rationalen Nullstelle von  $x^5-1$**

```

[ > restart;
[ > g:=x^5-1;
[                                     g := x^5 - 1
[ > irreduc(g);
[                                     false
[ > factor(g);
[                                     (x - 1) (x^4 + x^3 + x^2 + x + 1)
[ > h:=x^4+x^3+x^2+x+1;
[                                     h := x^4 + x^3 + x^2 + x + 1

```

```

[ > alias(gamma=RootOf(Z^4+Z^3+Z^2+Z+1));
[                                     gamma
[ > (3*gamma^3-2*gamma^2+gamma-8)^(-1);
[                                     1
[                                     3 gamma^3 - 2 gamma^2 + gamma - 8
[ > evala(%);
[                                     - 725/7681 - 39/7681 gamma + 276/7681 gamma^2 - 128/7681 gamma^3
[ > evalf(%);
[                                     -0.1115461760 + 0.02608697276 I
[ > factor(h,gamma);
[                                     (x - gamma^2) (x + 1 + gamma + gamma^2 + gamma^3) (x - gamma^3) (x - gamma)

```

Die Nullstellen von  $x^5-1$  sind daher:  $\gamma$ ,  $\gamma^2$ ,  $\gamma^3$  und  $\gamma^4 (= -1-\gamma-\gamma^2-\gamma^3)$ .

## Rechnen mit einer Nullstelle eines irreduziblen Polynoms vom Grad 8

> **restart;**

> **k:=x^8+3\*x^7-2\*x^5-10\*x^4+x^3-x^2+1;**

$$k := x^8 + 3x^7 - 2x^5 - 10x^4 + x^3 - x^2 + 1$$

> **irreduc(k);**

*true*

> **solve(k,x);**

RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 1),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 2),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 3),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 4),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 5),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 6),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 7),  
 RootOf(\_Z<sup>8</sup> + 3 \_Z<sup>7</sup> - 2 \_Z<sup>5</sup> - 10 \_Z<sup>4</sup> + \_Z<sup>3</sup> - \_Z<sup>2</sup> + 1, index = 8)

> **alias(delta=RootOf(k));**

$\delta$

> **(delta^5+6\*delta^4-7\*delta^3+5)^(-3);**

$$\frac{1}{(\delta^5 + 6\delta^4 - 7\delta^3 + 5)^3}$$

> **evala(%);**

$$\frac{28901457451767222417904957261}{4990699088006375160444186249875} + \frac{11260171840535574464432053517}{4990699088006375160444186249875} \delta$$

$$+ \frac{28531798434239121566782634663}{4990699088006375160444186249875} \delta^2 + \frac{86141981662615533476483633422}{4990699088006375160444186249875} \delta^3$$

$$+ \frac{14121000430397504857441663724}{4990699088006375160444186249875} \delta^4 - \frac{12214828163570422766567893244}{4990699088006375160444186249875} \delta^5$$

$$- \frac{29770701370492559768597426093}{4990699088006375160444186249875} \delta^6 - \frac{8596273642992979862774095588}{4990699088006375160444186249875} \delta^7$$

> **factor(k,delta);**

$$(-x + x^7 - 2x^4 - 10x^3 + x^2 + 3x^6 - \delta - 2\delta^4 - 10\delta^3 + \delta^7 + 3\delta^6 + \delta^2 + x^6\delta + 3x^5\delta + x^5\delta^2$$

$$+ 3x^4\delta^2 + x^4\delta^3 - 2x^3\delta + 3x^3\delta^3 + x^3\delta^4 - 10x^2\delta - 2x^2\delta^2 + 3x^2\delta^4 + x^2\delta^5 + x\delta - 10x\delta^2$$

$$- 2x\delta^3 + 3x\delta^5 + x\delta^6)(x - \delta)$$

In  $\mathbb{Q}[\delta]$  hat  $k$  daher nur eine Nullstelle, nämlich  $\delta$ .

> **m:=x^7-2\*x^4-10\*x^3+x^2-x+3\*x^6-delta-2\*delta^4-10\*delta^3+delta**

```

^7+3*delta^6+delta^2+x^6*delta+3*x^5*delta+x^5*delta^2+3*x^4*del
ta^2+x^4*delta^3-2*x^3*delta+3*x^3*delta^3+x^3*delta^4-10*x^2*de
lta-2*x^2*delta^2+3*x^2*delta^4+x^2*delta^5+x*delta-10*x*delta^2
-2*x*delta^3+3*x*delta^5+x*delta^6;

```

```

m := -x + x^7 - 2x^4 - 10x^3 + x^2 + 3x^6 - delta - 2delta^4 - 10delta^3 + delta^7 + 3delta^6 + delta^2 + x^6 delta + 3x^5 delta + x^5 delta^2
+ 3x^4 delta^2 + x^4 delta^3 - 2x^3 delta + 3x^3 delta^3 + x^3 delta^4 - 10x^2 delta - 2x^2 delta^2 + 3x^2 delta^4 + x^2 delta^5 + x delta - 10x delta^2
- 2x delta^3 + 3x delta^5 + x delta^6

```

```

> irreduc(m,delta);

```

*true*

```

> alias(epsilon=RootOf(m));

```

$\delta, \epsilon$

```

> factor(m,[delta,epsilon]);

```

```

(-1 + x + 3x^5 - 2x^3 - 10x^2 + x^6 + delta + 3delta^5 - 2delta^3 + delta^6 - 10delta^2 + x^5 delta + x^4 delta^2 + x^3 delta^3 - 2x^2 delta
+ x^2 delta^4 - 10x delta - 2x delta^2 + x delta^5 - 10epsilon^2 + 3epsilon delta^4 + 3epsilon^2 delta^3 + 3epsilon^3 delta^2 + 3epsilon^4 delta + x^4 epsilon^2 + x^4 epsilon delta - 2epsilon^3
+ x^3 epsilon^3 + epsilon delta^5 + epsilon^2 delta^4 + epsilon^3 delta^3 + epsilon^4 delta^2 + epsilon^5 delta + x^2 epsilon delta^3 + x^2 epsilon^2 delta^2 + x^2 epsilon^3 delta - 2x epsilon delta + x epsilon delta^4
+ x epsilon^2 delta^3 + x epsilon^3 delta^2 + x epsilon^4 delta + epsilon^6 - 10epsilon delta + epsilon - 2epsilon delta^2 - 2epsilon^2 delta + x^3 epsilon delta^2 + x^3 epsilon^2 delta + 3x^4 epsilon + 3x^4 delta
+ 3x^3 delta^2 + 3x^2 delta^3 + 3x delta^4 - 10x epsilon - 2x epsilon^2 + x epsilon^5 - 2x^2 epsilon + x^2 epsilon^4 + 3epsilon^5 + x^5 epsilon + 3x^2 epsilon delta^2
+ 3x^2 epsilon^2 delta + 3x epsilon delta^3 + 3x epsilon^2 delta^2 + 3x epsilon^3 delta + 3x^3 epsilon delta + 3x^2 epsilon^3 + 3x epsilon^4 + 3x^3 epsilon^2)(x - epsilon)

```

```

> sort(%);

```

```

(x^6 + 3x^5 + delta x^5 + epsilon x^5 + 3epsilon x^4 + delta^2 x^4 + 3delta x^4 + epsilon^2 x^4 + epsilon delta x^4 - 2x^3 + 3delta^2 x^3 + epsilon^2 delta x^3 + delta^3 x^3
+ epsilon^3 x^3 + 3epsilon^2 x^3 + 3epsilon delta x^3 + epsilon delta^2 x^3 - 2delta x^2 - 10x^2 + epsilon^3 delta x^2 - 2epsilon x^2 + epsilon^4 x^2 + 3epsilon^3 x^2
+ 3epsilon delta^2 x^2 + 3epsilon^2 delta x^2 + epsilon^2 delta^2 x^2 + delta^4 x^2 + epsilon delta^3 x^2 + 3delta^3 x^2 + 3epsilon^2 delta^2 x + x - 2delta^2 x - 2epsilon delta x
+ epsilon^5 x + delta^5 x + 3epsilon delta^3 x - 10epsilon x + epsilon delta^4 x - 10delta x + epsilon^4 delta x + epsilon^3 delta^2 x - 2epsilon^2 x + 3epsilon^4 x + epsilon^2 delta^3 x
+ 3epsilon^3 delta x + 3delta^4 x + epsilon^2 delta^4 + epsilon^3 delta^3 + epsilon^5 delta - 2epsilon^2 delta + delta + 3delta^5 + epsilon^4 delta^2 + delta^6 - 10delta^2 - 10epsilon delta - 2delta^3
- 1 + 3epsilon^5 - 10epsilon^2 + epsilon - 2epsilon delta^2 + 3epsilon^3 delta^2 + 3epsilon^4 delta + 3epsilon delta^4 + 3epsilon^2 delta^3 - 2epsilon^3 + epsilon^6 + epsilon delta^5)(x - epsilon)

```

```

> evala(1/epsilon);

```

```

epsilon^6 delta + 3epsilon^5 delta + epsilon^5 delta^2 + epsilon^4 delta^3 + 3epsilon^4 delta^2 - 2epsilon^3 delta + 3epsilon^3 delta^3 + epsilon^3 delta^4 - 10epsilon^2 delta + 3epsilon^2 delta^4 + epsilon^2 delta^5
- 2epsilon^2 delta^2 - 2epsilon delta^3 + 3epsilon delta^5 - 10epsilon delta^2 + epsilon delta^6 + epsilon delta - delta + 3delta^6 - 2delta^4 - 10delta^3 + delta^7 + delta^2

```