Seien $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{Q}$.

$$\lambda_1 v_1' + \lambda_2 v_2' + \lambda_3 v_3' = 0 \Rightarrow$$

$$\lambda_1 v_1 + \lambda_2 (v_1 + v_2) + \lambda_3 (v_1 + v_3) = 0$$

$$\Rightarrow (\lambda_1 + \lambda_2 + \lambda_3) v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$$

$$\xRightarrow[(v_1, v_2, v_3) \text{ Basis}]{} \lambda_1 + \lambda_2 + \lambda_3 = 0, \lambda_2 = 0, \lambda_3 = 0$$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$. Also ist (v_1', v_2', v_3') unabh.

29) $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$$w_j = \begin{array}{cl} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ -A_{1q_j} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ -A_{rq_j} \end{pmatrix} & \begin{array}{l} \\ \\ \\ p_1\text{-te Zeile} \\ \\ \\ \\ q_j\text{-te Zeile} \\ \\ \\ \\ p_r\text{-te Zeile} \end{array} \end{array}$$

$w_1 = \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$ (w_1) ist Basis v. $L(A, 0)$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad w_1 = \begin{pmatrix} 1 \\ 0 \\ -0 \end{pmatrix} \quad w_2 = \begin{pmatrix} 0 \\ 1 \\ -0 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & \boxed{0} & 1 \end{pmatrix} \quad w_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \\ -0 \end{pmatrix}, \quad w_2 = \begin{pmatrix} -3 \\ 0 \\ 1 \\ -\boxed{0} \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 1 & -1 \end{pmatrix} \quad w_1 = \begin{pmatrix} 1 \\ -0 \\ 0 \\ 0 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$w_3 = \begin{pmatrix} 0 \\ +1 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 \end{pmatrix} \quad w_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad w_2 = \begin{pmatrix} +1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad w_3 = \begin{pmatrix} -0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad w_1 = \begin{pmatrix} 1 \\ -0 \\ -0 \\ 0 \\ 0 \\ -0 \end{pmatrix} \quad w_2 = \begin{pmatrix} 0 \\ -2 \\ -2 \\ 1 \\ 0 \\ -0 \end{pmatrix} \quad w_3 = \begin{pmatrix} 0 \\ -3 \\ -1 \\ 0 \\ 1 \\ -0 \end{pmatrix}$$

$$A=\begin{pmatrix}1&0&3\\0&1&2\\0&0&0\end{pmatrix},\quad b=\begin{pmatrix}1\\2\\1\end{pmatrix}\neq 0 \quad L(A,b)=\emptyset$$

$r=2 \qquad b_3\neq 0$

$$A=\begin{pmatrix}0&0&1\\0&0&0\end{pmatrix},\quad b=\begin{pmatrix}2\\1\end{pmatrix}\neq 0 \quad L(A,b)=\emptyset$$

$r=1 \qquad b_2\neq 0$

$$A=\begin{pmatrix}1&2&3&0\\0&0&0&1\end{pmatrix},\quad b=\begin{pmatrix}2\\1\end{pmatrix} \qquad z=\begin{array}{c}\begin{pmatrix}0\\\vdots\\0\\b_1\\0\\\vdots\\0\\b_r\end{pmatrix}\end{array}\begin{array}{l}\\\\\\p_1\text{-te Zeile}\\\\\\\\p_r\text{-te Zeile}\end{array}$$

$$z=\begin{pmatrix}2\\0\\0\\1\end{pmatrix}$$

$$A=\begin{pmatrix}0&1&1&-1\end{pmatrix},\quad b=(3) \qquad z=\begin{pmatrix}0\\3\\0\\0\end{pmatrix}$$

$$A=\begin{pmatrix}1&1&-1&0\end{pmatrix},\quad b=(3) \qquad z=\begin{pmatrix}3\\0\\0\\0\end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},\quad b = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix},\quad z = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

30) – $A = (a\ b)$, $\vec{b} = (c)$

– $A = (r\ s\ t)$, $\vec{b} = (u)$

A hat Stufenform $\Longleftrightarrow$ ($a = 1$ od. ($a = 0$ u. $b = 1$) od. ($a = b = 0$)) bzw. ($r = 1$ od. ($r = 0$ u. $s = 1$) od. ($r = s = 0$ u. $t = 1$) od. $r = s = t = 0$)

– $A = (1\ b)$, $\vec{b} = (c)$, $w_1 = \begin{pmatrix} -b \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} c \\ 0 \end{pmatrix}$

– $A = (1\ s\ t)$, $\vec{b} = (u)$, $w_1 = \begin{pmatrix} -s \\ 1 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} -t \\ 0 \\ 1 \end{pmatrix}$

$z = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}$

– $A = (0\ 1)$, $\vec{b} = (c)$ $w_1 = \begin{pmatrix} 1 \\ -0 \end{pmatrix}$ $z = \begin{pmatrix} 0 \\ c \end{pmatrix}$

– $A = (0\ 1\ t)$, $\vec{b} = (u)$ $w_1 = \begin{pmatrix} 1 \\ -0 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ -t \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ u \\ 0 \end{pmatrix}$

– $A = (0\ 0)$, $\vec{b} = (c)$, 1. Fall: $c \neq 0 \Rightarrow L(A, \vec{b}) = \emptyset$

2. Fall: $c = 0$.

$w_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

-- $A = (0\,0\,1)$, $\vec{b} = (u)$: $w_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$

- $A = (0\,0\,0)$, $\vec{b} = (u)$: 1. Fall: $u \neq 0 \Rightarrow L(A, b) = \emptyset$
2. Fall: $u = 0$, $w_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $w_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
$z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

A hat nicht Stufenform

1. Fall: $a \neq 0$. Multipliziere mit $\frac{1}{a}$:
- $A = \left(1 \ \frac{b}{a}\right)$, $\vec{b} = \left(\frac{c}{a}\right)$, $w_1 = \begin{pmatrix} -\frac{b}{a} \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} \frac{c}{a} \\ 0 \end{pmatrix}$

$r \neq 0$, Multipliziere mit $\frac{1}{r}$.
- $A = \left(1 \ \frac{s}{r} \ \frac{t}{r}\right)$, $\vec{b} = \left(\frac{u}{r}\right)$,
$w_1 = \begin{pmatrix} -\frac{s}{r} \\ 1 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} -\frac{t}{r} \\ 0 \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} \frac{u}{r} \\ 0 \\ 0 \end{pmatrix}$.

2. Fall: $a = 0$. Dann ist $b \neq 0$, sonst Stufenform
Multipliziere mit $\frac{1}{b}$:
- $A = (0\,1)$; $\vec{b} = \left(\frac{c}{b}\right)$, $w_1 = \begin{pmatrix} 1 \\ -0 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ \frac{c}{b} \end{pmatrix}$.

$r = 0$. 1. Unterfall: $s \neq 0$, Multipliziere mit $\frac{1}{s}$:
- $A = \left(0 \ 1 \ \frac{t}{s}\right)$, $\vec{b} = \left(\frac{u}{s}\right)$, $w_1 = \begin{pmatrix} 1 \\ -0 \\ 0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ -\frac{t}{s} \\ 1 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ \frac{u}{s} \\ 0 \end{pmatrix}$

2. Unterfall: $s = 0$. $\Rightarrow t \neq 0$, sonst Stufenform, Multipliziere mit $\frac{1}{t}$.

- $A = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} \frac{u}{t} \end{pmatrix}$, $w_1 = \begin{pmatrix} 1 \\ 0 \\ -0 \end{pmatrix}$, $w_2 = \begin{pmatrix} 0 \\ 1 \\ -0 \end{pmatrix}$, $z = \begin{pmatrix} 0 \\ 0 \\ \frac{u}{t} \end{pmatrix}$