Proseminar "Einführung in die Mathematik 1"

WS 2010/11

21. Oktober 2010

13) $M := \{-2, 3, 0, 2\}$. $\sum_{m \in M} a_m = \sum_{m \in M} (m^2 \cdot 3) := \sum_{i=1}^{4} a_{f(i)} = \sum_{i=1}^{4} (f(i)^2 \cdot 3)$ mit der Funktion $f: \{1, 2, 3, 4\} \to M, 1 \mapsto -2, 2 \mapsto 3, 3 \mapsto 0, 4 \mapsto 2.$

Also gilt: $\sum_{m \in M} (m^2 \cdot 3) = (-2)^2 \cdot 3 + 3^2 \cdot 3 + 0^2 \cdot 3 + 2^2 \cdot 3 = 3(4 + 9 + 0 + 4) = 3 \cdot 17 = 51.$

$\prod_{m \in M} a_m = \prod_{m \in M} ((m-2) \cdot 4) := \prod_{m \in M} a_{f(i)} = \prod_{i=1}^{4} ((f(i) - 2) \cdot 4)$ mit der Funktion

$f: \{1, 2, 3, 4\} \to M, 1 \mapsto -2, 2 \mapsto 3, 3 \mapsto 0, 4 \mapsto 2.$

Also gilt: $\prod_{m \in M} ((m-2) \cdot 4) = (-2-2) \cdot 4 \cdot (3-2) \cdot 4 \cdot (0-2) \cdot 4 \cdot (2-2) \cdot 4 = 0.$

14)

```
> J:=[5,3,6,-1];
```

$$J := [5, 3, 6, -1] \tag{1.1}$$

```
> f:= j->2*j^2-1;
```

$$f := j \to 2 j^2 - 1 \tag{1.2}$$

```
> g:= j->j^3+1;
```

$$g := j \to j^3 + 1 \tag{1.3}$$

```
> add(f(i),i=J);
```

$$138 \tag{1.4}$$

```
> mul(f(i),i=J);
```

$$59143 \tag{1.5}$$

```
> add(g(i),i=J);
```

$$371 \tag{1.6}$$

```
> mul(g(i),i=J);
```

$$0 \tag{1.7}$$

```
>
```

15) `A:=Matrix(2,3,[4,-2,0,3/7,-3,-5]);`

$$\begin{bmatrix} 4 & -2 & 0 \\ \frac{3}{7} & -3 & -5 \end{bmatrix} \quad \textbf{(1.8)}$$

B: =Matrix(2, 3, [1, 9, 0, -39/13, -45/14, 3]);

$$\begin{bmatrix} 1 & 9 & 0 \\ -3 & -\frac{45}{14} & 3 \end{bmatrix} \quad \textbf{(1.9)}$$

-3/13*A+5/7*B;

$$\begin{bmatrix} -\frac{19}{91} & \frac{627}{91} & 0 \\ -\frac{204}{91} & -\frac{2043}{1274} & \frac{300}{91} \end{bmatrix} \quad \textbf{(1.10)}$$

3/4*A-4/3*B;

$$\begin{bmatrix} \frac{5}{3} & -\frac{27}{2} & 0 \\ \frac{121}{28} & \frac{57}{28} & -\frac{31}{4} \end{bmatrix} \quad \textbf{(1.11)}$$

16) A: =Matrix(2, 3, [7, 5, 9, 6, 5, 5]);

$$\begin{bmatrix} 7 & 5 & 9 \\ 6 & 5 & 5 \end{bmatrix} \quad \textbf{(1.12)}$$

B: =1/60*A;

$$\begin{bmatrix} \frac{7}{60} & \frac{1}{12} & \frac{3}{20} \\ \frac{1}{10} & \frac{1}{12} & \frac{1}{12} \end{bmatrix} \quad \textbf{(1.13)}$$

map(evalf, B);

$$\begin{bmatrix} 0.1166666667 & 0.08333333333 & 0.1500000000 \\ 0.1000000000 & 0.08333333333 & 0.08333333333 \end{bmatrix} \quad \textbf{(1.14)}$$

C: =Matrix(6, 6, [[0, a, b, c, d, e], [a, 0, 10, 12, 15, 11], [b, 10, 0, 9, 14, 10], [c, 12, 9, 0, 12, 7], [d, 15, 14, 12, 0, 8], [e, 11, 10, 7, 8, 0]]);

$$\begin{bmatrix} 0 & a & b & c & d & e \\ a & 0 & 10 & 12 & 15 & 11 \\ b & 10 & 0 & 9 & 14 & 10 \\ c & 12 & 9 & 0 & 12 & 7 \\ d & 15 & 14 & 12 & 0 & 8 \\ e & 11 & 10 & 7 & 8 & 0 \end{bmatrix} \quad \textbf{(1.15)}$$

17) A: =Matrix([2/5, 2, -2/3, -4]);

$$\begin{bmatrix} \frac{2}{5} & 2 & -\frac{2}{3} & -4 \end{bmatrix} \tag{1.16}$$

```
B:=Matrix(2,2,[-3,4,3/4,11]);
```

$$\begin{bmatrix} -3 & 4 \\ \frac{3}{4} & 11 \end{bmatrix} \tag{1.17}$$

```
C:=Matrix([2,7/8]);
```

$$\begin{bmatrix} 2 & \frac{7}{8} \end{bmatrix} \tag{1.18}$$

```
F:=Matrix([[2,3,-1,-3],[2/13,3/7,5,63/6]]);
```

$$\begin{bmatrix} 2 & 3 & -1 & -3 \\ \frac{2}{13} & \frac{3}{7} & 5 & \frac{21}{2} \end{bmatrix} \tag{1.19}$$

```
G:=Matrix(<3,4,2,-3>);
```

$$\begin{bmatrix} 3 \\ 4 \\ 2 \\ -3 \end{bmatrix} \tag{1.20}$$

```
with(LinearAlgebra):
A.G;
```

$$\begin{bmatrix} \frac{298}{15} \end{bmatrix} \tag{1.21}$$

```
B.F;
```

$$\begin{bmatrix} -\frac{70}{13} & -\frac{51}{7} & 23 & 51 \\ \frac{83}{26} & \frac{195}{28} & \frac{217}{4} & \frac{453}{4} \end{bmatrix} \tag{1.22}$$

```
C.B;
```

$$\begin{bmatrix} -\frac{171}{32} & \frac{141}{8} \end{bmatrix} \tag{1.23}$$

```
C.F;
```

$$\begin{bmatrix} \frac{215}{52} & \frac{51}{8} & \frac{19}{8} & \frac{51}{16} \end{bmatrix} \tag{1.24}$$

```
F.G;
```

$$\begin{bmatrix} 25 \\ -\frac{3517}{182} \end{bmatrix} \tag{1.25}$$

```
G.A;
```

$$\begin{bmatrix} \frac{6}{5} & 6 & -2 & -12 \\ \frac{8}{5} & 8 & -\frac{8}{3} & -16 \\ \frac{4}{5} & 4 & -\frac{4}{3} & -8 \\ -\frac{6}{5} & -6 & 2 & 12 \end{bmatrix} \tag{1.26}$$

G. C;

$$\begin{bmatrix} 6 & \frac{21}{8} \\ 8 & \frac{7}{2} \\ 4 & \frac{7}{4} \\ -6 & -\frac{21}{8} \end{bmatrix} \tag{1.27}$$

18) $A_{ij} := i - 3j,\ B_{ij} := 2i + j,\ 1 \le i, j \le n$. $C := A \cdot B$ und D $:= B \cdot A$. Dann ist für alle ganzen Zahlen k, l mit $1 \le k, l \le n$:

$$C_{kl} = \sum_{j=1}^{n} A_{kj} B_{jl} = \sum_{j=1}^{n} (k - 3j)(2j + l) = \frac{2kn(n+1)}{2} - \frac{6n(n+1)(2n+1)}{6} + kln - \frac{3ln(n+1)}{2} = n(n+1)\left(k - 2n - 1 - \frac{3l}{2}\right) + kln,$$

$$\mathrm{D}_{kl} = \sum_{j=1}^{n} B_{kj} A_{jl} = \sum_{j=1}^{n} (2k + j)(j - 3l) = \frac{2kn(n+1)}{2} + \frac{n(n+1)(2n+1)}{6} - 6kln - \frac{3ln(n+1)}{2} = n(n+1)\left(k + \frac{(2n+1)}{6} - \frac{3l}{2}\right) - 6kln,$$

>
>