Syllabus: Numerical methods for partial differential equations

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VO2+PS2 (winter term 2014)

In this lecture we will introduce a number of partial differential equations (PDEs) that are important in applications. Based on these equations numerical methods are introduced, explained, and analyzed which enable the efficient numerical solution of the PDE under consideration. We consider methods for both space and time discretization. Special emphasize is devoted to the the selection of a suitable numerical method for a given class of PDEs. A more detailed outline of the lecture and the methods covered can be found below.

07.10 Transport equation, finite differences space discretization (stability & consistency, uniform boundedness principle, Lax equivalence theorem)

14.10 Discontinuous solutions for transport problems (method of characteristics, upwind scheme, modified equation), Burgers' equation (shock waves)

21.10 Burgers' equation (weak solutions, entropy condition, conservative schemes, Godunov's method, Godunov's theorem)

28.10 Poisson's equation (weak solution, Finite element method with hat functions in one dimension)

04.11 Poisson's equation (finite element method, weak solution, Lax-Milgram theorem, conforming elements, L^2 optimality)

11.11 Poisson's equation (finite element method of order 1 on triangles in 2D, boundary conditions)

18.11 Heat equation (Fourier series, smoothing property, semigroups)

25.11 Heat equation with non-constant diffusivity (Runge–Kutta finite difference method, CFL condition, implicit methods, collocation methods, Radau methods, Backward differentiation formula)

02.12 Stability of numerical schemes (stability region, A-stability, error analysis for implicit Euler)

09.12 Semilinear problems in chemical kinetics (variations of constants formula, exponential Euler & Rosenbrock methods)

15.12 Overview of the activities of the work group (instead of the PS)

16.12 Vlasov–Poisson equation (splitting time discretization, semi-Lagrange space discretization)

13.01 Semilinear problems including the Laplacian (Fast Fourier transform, splitting time discretization)

20.01 Elements of GPU programming in C++

27.01 Maxwell's equation (physical motivation, theory, staggered grid space discretization)

03.02 Maxwell's equation (staggered grid space discretization, Gauss RK methods)

The examination of the lecture will be conducted orally (at the end of the term). Lecture notes will be permitted at the oral examination. At the end of each lecture additional literature in the form of papers and textbooks will be provided.

The exercises (PS2) will include a mix of blackboard presentations, programming exercises, and an extended project (most of which have to be prepared outside of the classroom). The extended project must be presented in January and can be conducted in teams of up to three students. For the programming exercises and the extended project the choice of programming language is left to the students.

The grading of the exercises is based on the number of exercises solved (including both programming and theoretical), the quality of the blackboard presentations, and on the extended project. There are no written examinations in the exercise sessions.

The language of instruction is English for the lecture and German for the exercises.

Extended projects:

- Implement a numerical approximation of the heat equation $u_t = \nabla \cdot (a(x)\nabla u)$ using the libmesh library (in 2D). Explain how the implementation relates to the finite element method as discussed in the lecture. **Claudia Schmuck & Lisa Schlosser & Lena Vogrin (19.01)**
- In the lecture (16.12) the splitting approach was introduced for the Vlasov–Poisson equation. Investigate weather splitting is a viable option for diffusion-reaction equations (1d), the Schroedinger equation (1d), the Navier–Stokes equation (3d), the vorticity equation (2d), heat equation (2d). Implement two cases one where you expect that splitting will perform well and one were you expect that splitting will not be competitive. Perform a numerical comparison with a standard time integrator. **Lena-Maria Pfurtscheller & Barbara Schretter (26.01)**
- Solve the spatial semi-discretization of a one and two-dimensional parabolic partial differential equation with the RADAU5 algorithm (the corresponding interface for C, Fortran, or Matlab can be used). Explain the difference in implementation if this numerical method is applied to a linear and a semi-linear problem, respectively. **Julia Lechner & Johannes Schwaighofer & Hellweger Valentin (19.01)**
- Implement the backward Euler and Crank–Nicolson method for a 1d semilinear parabolic problem. The idea of this exercise is to implement all the details by yourself (numerical method, Newton's method, linear solver). **Andreas Kofler & Naomi Auer (26.01)**
- Diffusion Monte Carlo. **Alexander Werlberger & Alexander Mischi (19.01)**
- Whitney and mixed finite element. **Arzbacher Stefan (12.01)**
- The piecewise parabolic method (PPM) for discontinuous solutions – theory and numerics. **Katharina Gangl & Matthias Köchl (26.01)**
- Numerics of the Gierer-Meinhardt model. **Lena Tschiderer & Marie-Christine Pali & Beatrix Huber (12.01)**
- Exponential integrators. **Erlsbacher (12.01)**