Exercises 1.12.2014

8.1) Calculate and plot the region of A-stability for the following Runge–Kutta methods:

- explicit Euler method
- two explicit Runge–Kutta methods (one of order two and one of order four)
- trapezoidal method

8.2) Construct the following collocation methods:

- The Gauss-Legendre method of order four (use $c_0 = \frac{1}{2} - \frac{1}{6}\sqrt{3}$, $c_1 = \frac{1}{2} + \frac{1}{6}\sqrt{3}$)
- The Raudau-IIA method of order three (use $c_0 = 1/3$, $c_1 = 1$)

Determine their stability functions and plot the region of A-stability.

8.3) Assume that you have a tridiagonal system of linear equations, i.e.

$$A = \begin{bmatrix} b_1 & c_1 & & & & & \\ a_2 & b_2 & c_2 & & & & \\ & a_3 & b_3 & c_3 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & & & a_n & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_{n-1} \\ d_n \end{bmatrix}.$$

Perform Gaussian elimination on this system and count the number of arithmetic operations (i.e. additions, multiplications, divisions) necessary to obtain a solution.

8.4) Implement the Crank-Nicolson method with the standard centered finite difference discretization and the implicit Euler method (using the same space discretization) for

$$\partial_t u(t,x) = \nabla \cdot (a(x)\nabla u(t,x)) + s(t,x)$$

on the domain $[-1,1]$ with

$$a(x) = \alpha/(1+x^2), \qquad s(t,x) = \sin^2(t)\mathrm{e}^{-30x^2}$$

for different values of $\alpha > 0$. You can use the Matlab backslash operator or a similar library to solve the resulting linear system of equations. What are your conclusions?

References

Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems, E. Hairer, G. Wanner, 2010