1)

Definiert:

$$3x \neq \frac{\pi}{2}k, \quad k \in \mathbb{Z} \Longleftrightarrow x \neq \frac{\pi}{6} + k\pi$$
$$\frac{2}{x} \in [-1,1) \Longleftrightarrow x \in (2,\infty) \cup (-\infty,-2]$$
$$\sin x + \cos x \neq 0 \Longleftrightarrow x = -\frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$$

Daher

$$D = ((2,\infty) \cup (-\infty,-2]) \setminus \left(\{\frac{\pi}{6} + k\pi \,:\, k \in \mathbb{Z}\} \cup \{\frac{-\pi}{4} + k\pi \,:\, k \in \mathbb{Z}\} \right)$$

Ableitung:

$$\left(\frac{\tan 3x + 2^{ax^3}}{(\sin x + \cos x)\arccos\frac{2}{x}} \right)'$$
$$= \frac{\left(\frac{3}{\cos^2(3x)} + 3ax^2\log(2)2^{ax^3}\right)(\sin x + \cos x)\arccos\frac{2}{x} - \left((\cos x - \sin x)\arccos\frac{2}{x} + (\sin x + \cos x)\frac{2}{x^2}\frac{1}{\sqrt{1-\frac{4}{x^2}}}\right)\left(\tan 3x + 2^{ax^3}\right)}{(\sin x + \cos x)^2 \arccos^2\frac{2}{x}}$$
$$= \frac{3 + 3ax^2\log(2)\cos^2(3x)2^{ax^3}}{\cos^2(3x)(\sin x + \cos x)\arccos\frac{2}{x}} - \frac{(\cos x - \sin x)(\tan 3x + 2^{ax^3})}{(\sin 2x + 1)\arccos\frac{2}{x}} - \frac{2(\tan 3x + 2^{ax^3})}{|x|\sqrt{x^2-4}(\sin x + \cos x)\arccos^2\frac{2}{x}}$$

2)

a)

$$0 = \frac{d}{dx}\left(\cosh y + e^{x\sin x}e^{-y} - 2\right) = y'\sinh y + (x\cos x + \sin x - y')e^{x\sin x}e^{-y}$$
$$\Longleftrightarrow \qquad y' = \frac{x\cos x + \sin x}{\sinh y + e^{x\sin x}e^{-y}}$$
$$\Longrightarrow \qquad y'|_{x=\pi} = \frac{-\pi}{1} = -\pi$$

da $y|_{x=\pi} = f(\pi) = 0$.

b)

$$\cosh y + e^{x\sin x}e^{-y} = \frac{1}{2}\left(e^y + e^{-y}\right) + e^{x\sin x}e^{-y} = 2$$
$$\Longleftrightarrow \qquad e^y + (1 + 2e^{x\sin x})e^{-y} = 4$$
$$\Longleftrightarrow \qquad (e^y)^2 - 4e^y + (1 + 2e^{x\sin x}) = 0$$
$$\Longleftrightarrow \qquad e^y = 2 \pm \sqrt{3 - 2e^{x\sin x}}$$
$$\Longleftrightarrow \qquad y = \log(2 \pm \sqrt{3 - 2e^{x\sin x}})$$

Mit der Bedingng $f(\pi) = 0$ erhalten wir

$$\log(2-1) = 0, \qquad \log(2+1) \neq 0$$

Also

$$f(x) := y = \log(2 - \sqrt{3 - 2e^{x\sin x}})$$

Also

$$f'(\pi) = \left. \frac{(\sin x + x\cos x))e^{x\sin x}}{\left(2 - \sqrt{3 - 2e^{x\sin x}}\right)\sqrt{3 - 2e^{x\sin x}}} \right|_{x=\pi} = -\pi$$

3

a)

$$\cos 2x \;=\; \cos^2 x - \sin^2 x$$
$$1 = \cos 0 \;=\; \cos^2 x + \sin^2 x$$

Also

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\begin{aligned}
\int_0^{\pi/2} x\sin^2 x\,dx &= \frac{1}{2}\int_0^{\pi/2} x - x\cos 2x\,dx \\
&= \frac{1}{2}\left[\frac{x^2}{2}\Big|_0^{\pi/2} - \frac{x\sin 2x}{2}\Big|_0^{\pi/2} + \int_0^{\pi/2}\frac{\sin 2x}{2}\right] \\
&= \frac{1}{4}\left[\frac{\pi^2}{4} - \frac{\cos 2x}{2}\Big|_0^{\pi/2}\right] = \frac{\pi^2}{16}+\frac{1}{8}+\frac{1}{8} = \frac{\pi^2}{16}+\frac{1}{4}
\end{aligned}$$

b)

$$\begin{aligned}
\int_{-\log 3}^{0}\frac{e^{x/2}}{1+e^x}\,dx &= \left[\begin{array}{c} y = e^{x/2} \\ dy = \frac{1}{2}e^{x/2}dy \end{array}\right] = \int_{1/\sqrt{3}}^{1}\frac{2}{1+y^2}\,dy \\
&= 2\arctan(y)|_{1/\sqrt{3}}^{1} = 2\arctan(1) - 2\arctan(1/\sqrt{3}) \\
&= 2\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \frac{\pi}{6}
\end{aligned}$$

4)

$$\begin{aligned}
\int_{-2}^{-3/2}\frac{x}{\sqrt{-2x^2-8x-7}}\,dx &= \int_{3/2}^{2}\frac{-x}{\sqrt{-2x^2+8x-7}}\,dx \\
&= \int_{3/2}^{2}\frac{-x}{\sqrt{-(\sqrt{2}x-2\sqrt{2})^2+1}}\,dx = \left[\begin{array}{c} y=\sqrt{2}x-2\sqrt{2} \\ dy = \sqrt{2}\,dx \end{array}\right] \\
&= \frac{1}{\sqrt{2}}\int_{-1/\sqrt{2}}^{0}\frac{-y/\sqrt{2}-2}{\sqrt{1-y^2}}\,dy = \frac{1}{2}\int_{-1/\sqrt{2}}^{0}\frac{-y}{\sqrt{1-y^2}}\,dy - \frac{2}{\sqrt{2}}\arcsin(y)|_{-1/\sqrt{2}}^{0} = \left[\begin{array}{c} z=y^2 \\ dz = 2y\,dy \end{array}\right] \\
&= \frac{1}{4}\int_{1/2}^{0}\frac{-1}{\sqrt{1-z}}\,dy - \sqrt{2}\arcsin(y)|_{-1/\sqrt{2}}^{0} = \frac{1}{2}\sqrt{1-z}|_{1/2}^{0} - \sqrt{2}\arcsin(y)|_{-1/\sqrt{2}}^{0} \\
&= \frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{2}}{4}\pi = \frac{1}{2}-\frac{\sqrt{2}}{4}+\frac{\pi}{2}
\end{aligned}$$

5)

a) Klarerweise ist der Integrant fuer einen Intervall $[1,b]$ beschraenkt und integrierbar. Im Fall $\alpha \leq 0$ gilt

$$\int_1^\infty (2x^2+x)^{-\alpha}\,dx$$

was klarerweise divergent ist. Daher betrachten wir $\alpha > 0$.

$$\begin{aligned}
\left|\int_1^\infty\left(\frac{1}{2x^2+x}\right)^\alpha dx\right| &\leq \int_1^\infty\frac{1}{(2x^2+x)^\alpha}\,dx \leq \int_1^\infty\frac{1}{(2x^2)^\alpha}\,dx \\
&= \frac{1}{2^\alpha}\int_1^\infty\frac{1}{x^{2\alpha}}\,dx
\end{aligned}$$

Der letzte Term konvergiert genau dann wenn $2\alpha > 1 \iff \alpha > \frac{1}{2}$. In diesem Fall konvergiert unser Integral ebenso (Majorante). Es bleibt zu zeigen dass das Integral in $\alpha \in (0,\frac{1}{2}]$ divergert. Dazu betrachten wir

$$\int_1^\infty\left(\frac{1}{2x^2+x}\right)^\alpha dx \geq \int_1^\infty\left(\frac{1}{2x^2+x^2}\right)^\alpha dx = \frac{1}{3^\alpha}\int_1^\infty\frac{1}{x^{2\alpha}}\,dx \to \infty, \qquad \text{fuer } \alpha\in(0,\frac{1}{2}]$$

Also folgt nach dem Minorantenkriterium dass unser Integral fuer $\alpha \in (0,\frac{1}{2})$ divergiert.

b)

$$\begin{aligned}
\int_1^\infty\frac{1}{2x^2+x}\,dx &= \int_1^\infty\frac{1}{x}-\frac{2}{2x+1}\,dx = [\log x - \log(2x+1)]_1^\infty = \log\frac{x}{2x+1}\Big|_1^\infty \\
&= \log\lim_{x\to\infty}\frac{x}{2x+1} - \log\frac{1}{3} = \log 3 - \log 2 = \log\frac{3}{2}
\end{aligned}$$

6)

$$\begin{aligned}
y|_{x=\pi/2} &= 1 \\
y'|_{x=\pi/2} &= 3\sin(3x)e^{-\cos 3x}|_{x=\pi/2} = -3 \\
y''|_{x=\pi/2} &= \left[9\cos(3x)e^{-\cos 3x} + 9\sin^2(3x)e^{-\cos 3x}\right]_{x=\pi/2} = 9
\end{aligned}$$

Daher

$$T_2(x) = 1 - 3\left(x - \frac{\pi}{2}\right) + 9\left(x - \frac{\pi}{2}\right)^2$$

Kontrolle:

$$e^{-z} = \sum_{n=0}^{\infty} \frac{(-z)^k}{k!} = 1 - z + \frac{z^2}{2} + \mathcal{O}(z^3)$$

$$\begin{aligned}
\cos(3x) &= \cos(3\left(w + \frac{\pi}{2}\right)) = \sin 3w = \sum_{n=0}^{\infty} \frac{(-1)^n(3w)^{2n+1}}{(2n+1)!} = 3w + \mathcal{O}(w^3) \\
&= 3(x - \frac{\pi}{2}) + \mathcal{O}((x - \frac{\pi}{2})^3)
\end{aligned}$$

mit $x = w + \frac{\pi}{2}$. Daher

$$e^{-\cos 3x} = 1 - 3(x - \frac{\pi}{2})^2 + \frac{9}{2}\left(x - \frac{\pi}{2}\right)^2 + \mathcal{O}((x - \frac{\pi}{2})^3)$$