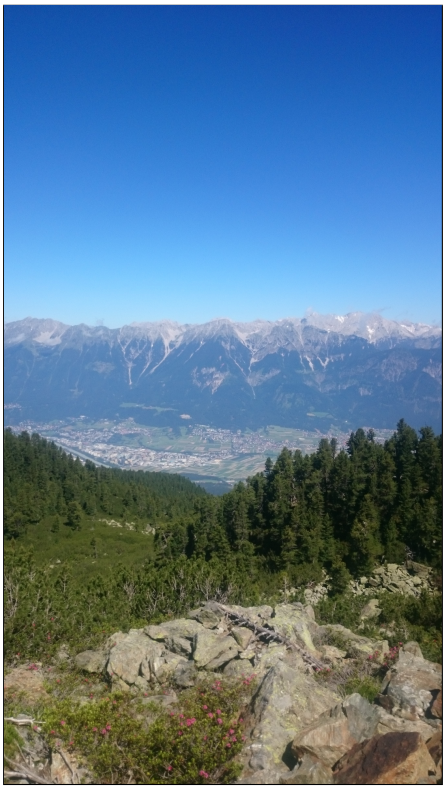
9th NAI Workshop on

Numerical Analysis of Evolution Equations



November 08–11, 2016

Innsbruck, Austria

Welcome

We wish you a warm welcome to Innsbruck, and we are looking forward to an interesting *9th NAI Workshop on Numerical Analysis of Evolution Equations*.

The intention of the workshop is to provide a platform for exchanging new ideas and results in the development of innovative integrators for evolution equations. The workshop covers both, theoretical and practical aspects, and wants to bring together numerical analysts working in the field as well as PhD students who intend to start in this area. The present workshop continues a series of conferences that were held in Innsbruck from 2004 to 2014.

We wish you a scientifically inspiring and enjoyable time in Innsbruck. If you have any questions, please do not hesitate to contact us.

Alexander Ostermann, Alfredo Bellen, Katharina Schratz, Lukas Einkemmer, and Peter Kandolf

Accommodation

The workshop will take place in Vill at the Bildungsinstitut Grillhof. The address is

Tiroler Bildungsinstitut - Grillhof
Grillhofweg 100
6080 Vill, Austria
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The workshop starts in the evening of November 08, 2016 with an informal dinner and will end on November 11, 2016 after lunch.

The conference fee (350 Euro before 25th September, 400 Euro from 26th September) includes the accommodation in a double room at Grillhof, full board (breakfast, lunch, dinner, coffee breaks), and the excursion. The surcharge for single accommodation is 50 Euro.

Scientific Program

Contributions

All communications will be given in plenary sessions. Each contribution is scheduled for **25 minutes** including a brief discussion.

The conference language is English.

Equipment

The seminary room is equipped with a data projector. A Windows computer with Adobe Acrobat Reader will be provided as well as a presenter. Talks can be transferred to the conference computer through USB sticks or CD/DVD. We recommend you not to use your own laptop. However, if you need to use your own laptop, be sure it provides a VGA or HDMI connection. In addition, the room is equipped with a blackboard and an overhead projector.

Schedule

Tuesday, 08 November 2016	
17.00 – 19.00	***Registration at Grillhof***
19.00	***Dinner***
Wednesday, 09 November 2016	
08.30 – 08.35	***Opening***
08.35 – 09.20	Mari Paz Calvo Word series: some applications in numerical integration
09.20 – 09.45	David Cohen Exponential integrators for nonlinear Schrödinger equations with white noise dispersion
09.45 – 10.10	Chiara Piazzola Solution of large-scale Lyapunov differential equations
10.10 – 10.35	***Coffee break***
10.35 – 11.00	Christian Stohrer Finite element heterogeneous multiscale method for time-dependent Maxwell's equations
11.00 – 11.25	Hermann Mena Splitting methods for stochastic partial differential equations
11.25 – 11.50	Martina Moccaldi Adapted numerical integration of advection-reaction-diffusion problems generating periodic wavefronts
11.50 – 12:15	Andreas Sturm Locally implicit time integration for linear Maxwell's equations
12.15 – 14.00	***Lunch break***
14.00 – 14.25	Markus Gasteiger ADI preconditioners for the solution of the steady-state Vlasov equation
14.25 – 14.50	Tobias Jahnke Limit dynamics of the dispersion-managed nonlinear Schrödinger equation
14.50 – 15.15	Marcel Mikl Adiabatic midpoint rule for the dispersion-managed nonlinear Schrödinger equation
15.15 – 15:40	Robert Altmann Splitting methods for constrained diffusion-reaction systems
15.40 – 16.10	***Coffee break***
16.10 – 16.35	Simone Buchholz Mind the gap - two approaches to highly oscillatory differential equations
16.35 – 17.00	Raffaele D'Ambrosio Stability issues for stochastic multistep methods

17.00 – 17.25	Luben Vulkov Numerical solution of degenerate ultraparabolic equations for pricing of Asian options
17.25 – 17.50	Jonas Köhler ADI splitting and the discontinuous Galerkin method
17.50 – 18.15	Gregor Staggl An extension of the Savage–Hutter equations for the modeling of gravity driven mass flows over arbitrary topography in one space dimension
18.30	***Dinner***
20.00	***Evening programme***

Thursday, 10 November 2016

08.30 – 09.15	Martin J. Gander Space-time parallel methods based on domain decomposition
09.15 – 09.40	Lukas Einkemmer A comparison of boundary corrections for Strang splitting
09.40 – 10.05	Michaela Mehlin Muli-level local time-stepping methods of Runge–Kutta type for wave equations
10.05 – 10.30	Johannes Eilinghoff Fractional error estimates of splitting schemes for the nonlinear Schrödinger equation
10.30 – 10.55	***Coffee break***
10.55 – 11.20	Patrick Krämer Numerical methods for an efficient integration of the Maxwell–Dirac system
11.20 – 11.45	Antti Koskela Krylov approximation of polynomially perturbed linear ODEs
11.45 – 12.10	Naomi Auer Magnus integrators on graphic processing units
12.10 – 12.35	Robin Flohr A splitting approach for freezing waves
12.35 – 12.50	Miglena N. Koleva Two-grid method for solving non-linear models in mathematical finance
12.50 – 14.00	***Lunch break***
14.00 – 18.30	***Excursion to Rattenberg and Kristallglas Kisslinger*** *http://www.kisslinger-kristall.com/*
18.30	***Conference Dinner, Bierstindl, Innsbruck***

Friday, 11 November 2016	
08.30 – 08.55	David Hipp Numerical analysis of wave equations with dynamic boundary conditions
08.55 – 09.20	Martina Prugger A Riemann solver free numerical method for two-dimensional conservation laws
09.20 – 09.45	Othmar Koch Error analysis of splitting methods for parabolic problems under Dirichlet boundary conditions
09.45 – 10.10	Francesca Scarabel Numerical bifurcation analysis of nonlinear delay equations through pseudospectral discretization
10.10 – 10.35	***Coffee break***
10.35 – 11.00	Davide Liessi Approximating the stability of linear periodic delay models by pseudospectral methods
11.00 – 11.25	Stefano Maset Conditioning and relative error propagation in linear autonomous ordinary differential equations
11.25 – 11.50	Koondanibha Mitra A linear domain decomposition method for unsaturated flow in porous media
11.50 – 12.15	Peter Kandolf The action of trigonometric and hyperbolic matrix functions
12.15 – 12.40	Winfried Auzinger Similarity to contraction: the companion matrix case
12.40 – 12.45	***Closing***
12.50	***Lunch***

Abstracts of Talks

In alphabetical order

Splitting Methods for Constrained Diffusion-Reaction Systems

Robert Altmann * (TU Berlin, Germany)
Alexander Ostermann (University of Innsbruck)

We consider nonlinear diffusion-reaction systems which have an additional constraint such as having a prescribed integral mean. With the help of Lie and Strang splitting we would like to treat the nonlinearity separately. This means that the time integration is reduced to the solution of a linear constrained system and a nonlinear ODE.
However, Strang splitting suffers from order reduction which limits its efficiency. This is caused by the fact that the nonlinear subsystem produces inconsistent initial values for the constrained subsystem. In this talk we show that the incorporation of an additional correction term resolves this problem without increasing the computational costs.
The results of this talk are based on the arXiv preprint [1].

References

[1] Altmann, R., Ostermann, A., 2016. Splitting methods for constrained diffusion-reaction systems. arXiv:1607.07683.

Magnus integrators on graphic processing units

Naomi Auer (University of Innsbruck, Austria)

Given a first order, linear differential equation $Y'(t) = A(t)Y(t)$ with initial value $Y(0) = Y_0$, the solution can be expressed in the exponential form $Y(t) = \exp(\Omega(t))Y_0$ following the idea of Wilhelm Magnus. Magnus integrators are time integrators that make use of this representation of the solution by truncating $\Omega(t) = \sum_{k=1}^{\infty} \Omega_k(t)$ after a certain number of terms. The computation of the matrix exponential can be realized in different ways, we chose Leja interpolation.
The aim is a comparison of an implementation for the CPU and an implementation for the GPU. NVIDIA graphic processing units (GPUs) provide a massively parallel architecture. The CUDA programming model extends C/C++ (or other well established programming languages) by a relatively small number of keywords, which allow to run parts of the code on the GPU instead of the CPU. The GPU is used for highly parallelizable program sequences while non-parallelizable code sequences are usually executed on the CPU.

Similarity to contraction: the companion matrix case

Winfried Auzinger (Technische Universität Wien, Austria)

We consider a family of companion matrices

$$C = \begin{pmatrix} 0 & 1 \\ -c_0 & -c_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\zeta_1\,\zeta_2 & \zeta_1+\zeta_2 \end{pmatrix} \in \mathbb{C}^{2\times 2}\,, \quad \zeta_1,\zeta_2 \in \mathbb{C}\,,$$

with characteristic polynomial $\zeta^2 + c_1\zeta + c_0 = (\zeta-\zeta_1)(\zeta-\zeta_2)$. Under the asymptotic stability condition

$$|\zeta_1| \le 1, \quad |\zeta_2| \le 1, \quad \text{and} \quad |\zeta| < 1 \ \text{ if } \ \zeta_1 = \zeta_2 = \zeta\,, \tag{1}$$

the matrix C is power-bounded, i.e., there exists a constant $K = K(\zeta_1,\zeta_2)$ such that $\|C^\nu\|_2 \le K$ for all $\nu \in \mathbb{N}$. Since C is not normal, in general we have $K > 1$ and, in particular $K \gg 1$ near the unstable limit, i.e., for $\zeta_1 \approx \zeta_2$ with $|\zeta_1| \approx |\zeta_2| \approx 1$.
We study the question of finding – for arbitrary $\zeta_1,\zeta_2 \in \mathbb{C}$ satisfying the stability condition (1) – a 'natural' similarity transformation

$$C = X\,T\,X^{-1}\,,$$

depending on ζ_1,ζ_2, such that

$$\|T\|_2 \le 1\,. \tag{2}$$

For this purpose we make an ansatz $X = X(\delta)$, $T = T(\delta)$ based on the LQ-decomposition of the Vandermonde matrix associated with C, with a scaling parameter δ which needs to be determined. We present the idea of our construction and the technique for the proof of (2). The optimal choice for δ such that (2) is indeed valid turns out to be

$$\delta = \sqrt{\tfrac{1}{2}(1-|\zeta_1|^2)(1-|\zeta_2|^2) + \tfrac{1}{4}|\zeta_1-\zeta_2|^2} > 0\,,$$

which may be called the 'distance to instability' related to the spectrum of C; it exactly vanishes in the unstable limit.
Further topics discussed include

- analogous results for the case that C satisfies a stability condition with respect to the left complex half plane;
- the 3×3 case – here only tentative numerical results are available;
- some numerical considerations, in particular concerning order reductions, for the case where C represents a stiff second-order ODE.

References

[1] Auzinger, W., 2016. A note on similarity to contraction for stable 2×2 companion matrices. Ukr. Mat. Zh. 68 (3), 400–407.

Mind the gap - two approaches to highly oscillatory differential equations

Simone Buchholz * (Karlsruhe Institute of Technology, Germany)
Ludwig Gauckler (Free University Berlin, Germany)
Volker Grimm (Karlsruhe Institute of Technology, Germany)
Marlis Hochbruck (Karlsruhe Institute of Technology, Germany)
Tobias Jahnke (Karlsruhe Institute of Technology, Germany)

In this talk a particular class of linear, highly oscillatory problems

$$q''(t) = -\Omega^2 q(t) + Gq(t), \qquad t > 0, \qquad q(0) = q_0, \quad q'(0) = q_0', \tag{1}$$

is considered, where Ω is a symmetric positive semi-definite matrix of arbitrary large norm (e.g. representing the spatial discretization of a differential operator), and where $\|G\|$ is moderate. For such problems, trigonometric integrators have been constructed and analyzed under a finite-energy condition, see e. g. [2]. These methods involve filter functions which are chosen in such a way that the oscillatory parts of the local error do not sum up in the global error. To prove this in the error analysis is a delicate matter and usually excludes standard techniques like Taylor expansion or a Lady Windermere's fan argument.
It is known that applying a symmetric trigonometric integrator to (1) is equivalent to applying Strang splitting to an *averaged* version of the first-order formulation of (1). Most of the analysis of splitting methods for different classes of pdes is based on a technique established in [3], where the local error is interpreted as a quadrature error. The error bounds then rely on commutator bounds. Unfortunately, this technique does not generalize to highly oscillatory problems in an obvious way.
In this talk we build a bridge between these two worlds by proving error bounds for trigonometric integrator considered as a splitting method. The novelty of our analysis in [1] is not the error bound itself, but the fact that it is proven by techniques which, to the best of our knowledge, have so far not been considered in the context of trigonometric integrators.
We gratefully acknowledge financial support by the Deutsche Forschungsgemeinschaft (DFG) through RTG 1294, CRC 1114, CRC 1173, and project GA 2073/2-1.

References

[1] Buchholz, S., Gauckler, L., Grimm, V., Hochbruck, M., Jahnke, T., 2016. Closing the gap between trigonometric integrators and splitting methods for highly oscillatory differential equations. Preprint (*https://www.waves.kit.edu/preprints.php*).

[2] Grimm, V., Hochbruck, M., 2006. Error analysis of exponential integrators for oscillatory second-order differential equations. J. Phys. A., 5495-5507.

[3] Jahnke, T., Lubich, C., 2000. Error bounds for exponential operator splittings. BIT (40), 735-744.

Word series: some applications in numerical integration

Mari Paz Calvo (Universidad de Valladolid, Spain)

Word series have been recently introduced as an alternative to the well known B-series, that have played a very important role in the analysis of numerical integrators. Word series are formal series that are parameterized by words of an alphabet. Although they possess a narrower scope of application, they are more compact and easier to handle than the corresponding B-series parameterized by rooted trees with coloured nodes. In the talk I shall provide some general background on word series and present some applications related to the numerical integration of differential systems.

Exponential integrators for nonlinear Schrödinger equations with white noise dispersion

David Cohen * (University of Innsbruck/Umeå University, Austria/Sweden)
Guillaume Dujardin (Inria Lille Nord-Europe)

We consider the numerical integration in time of the nonlinear Schrödinger equation with power law nonlinearity and random dispersion. We introduce a new explicit exponential integrator for this purpose that integrates the noisy part of the equation exactly. We prove that this numerical scheme is of mean-square order 1 and we draw consequences of this fact. We compare our exponential integrator with several other numerical methods from the literature. We finally propose a second exponential integrator, which is symmetric and, in contrast to the first one, preserves the L^2-norm of the solution.

Stability issues for stochastic multistep methods

Evelyn Buckwar (Johannes Kepler University of Linz)
Raffaele D'Ambrosio * (University of Salerno, Italy)
Martina Moccaldi (University of Salerno)
Beatrice Paternoster (University of Salerno)

The aim of this talk is the analysis of various stability issues for numerical methods designed to solve stochastic differential equations. We first aim to consider a nonlinear system of Itô stochastic differential equation (SDE)

$$dX(t) = f(X(t))dt + g(X(t))dW(t), \quad t > 0. \tag{1}$$

Under suitable regularity conditions, exponential mean-square stability holds, i.e. any two solutions $X(t)$ and $Y(t)$ of (1) with $\mathbb{E}|X_0|^2 < \infty$ and $\mathbb{E}|Y_0|^2 < \infty$ satisfy

$$\mathbb{E}|X(t) - Y(t)|^2 \leq \mathbb{E}|X_0 - Y_0|^2 e^{\alpha t}, \tag{2}$$

with $\alpha < 0$. We aim to investigate the numerical counterpart of (2) when trajectories are generated by stochastic linear multistep methods, in order to provide stepsize restrictions ensuring analogous exponential mean-square stability properties also numerically [1, 4]. We next consider the following second order stochastic differential equation

$$\ddot{x} = f(x) - \eta s^2(x)\dot{x} + \varepsilon s(x)\xi(y), \tag{3}$$

describing the position of a particle subject to the deterministic forcing $f(x)$ and a random forcing $\xi(t)$ of amplitude ε. The dynamics exhibits damped oscillations, with damping parameter η. We aim to analyze asymptotic mean-square stability properties for indirect stochastic multistep methods, applied to the system equivalent to (3)

$$\begin{cases} dX(t)) &= V(t)dt, \\ dV(t) &= -\eta s^2(X(t))V(t)dt + f(X(t))dt + \varepsilon s(X(t))dW(t), \end{cases}$$

with special emphasis to understanding the ability of such methods in retaining long-term invariance laws [2, 3].

References

[1] E. Buckwar, R. D'Ambrosio, Exponential mean-square stability of linear multistep methods, submitted.

[2] K. Burrage, G. Lythe, Numerical methods for second-order stochastic differential equations, SIAM J. Sci. Comput. 29(1), 245–264 (2007).

[3] R. D'Ambrosio, M. Moccaldi, B. Paternoster, Long-term preservation of invariance laws by stochastic multistep methods, in preparation.

[4] D.J. Higham, P.E. Kloeden, Numerical Methods for Nonlinear Stochastic Differential Equations with Jumps, Numer. Math., 101, 101–119 (2005).

Fractional error estimates of splitting schemes for the nonlinear Schrödinger equation

Johannes Eilinghoff * (Karlsruhe Institute of Technology, Germany)
Roland Schnaubelt (Karlsruhe Institute of Technology)
Katharina Schratz (Karlsruhe Institute of Technology)

We investigate the Lie and the Strang splitting for the cubic nonlinear Schrdinger equation on the full space and the torus in up to three spatial dimensions. We prove that the Strang splitting converges in L^2 with order $1 + \theta$ for initial values in $H^{2+2\theta}$ with $\theta \in [0, 1)$ and that the Lie splitting converges with order one for initial values in H^2.

References

[1] Eilinghoff, J., Schnaubelt, R., Schratz, K., 2016. Fractional error estimates of splitting schemes for the nonlinear Schrödinger equation. JMAA 442, 740–760.

A comparison of boundary corrections for Strang splitting

Lukas Einkemmer * (University of Innsbruck, Austria)
Alexander Ostermann (University of Innsbruck)

It is well known that splitting methods suffer from order reduction in the presence of non-trivial boundary conditions. This usually implies that Strang splitting is no more accurate than Lie splitting. To remedy this issue corrections have been developed that allow Strang splitting to retain second order accuracy.
The method proposed in [1] uses a modified (but time dependent) boundary condition. A convergence analysis of this method in the liner case has been performed in [2]. We will refer to this method as time dependent boundary correction (TDBC).
More recently, an alternative approach has been proposed in [3] (we refer to this method by the abbreviation CEC). This correction does not require a modification of the boundary condition but requires the computation of a global correction. The convergence of this scheme for diffusion-reaction problems has been shown in [3] (for Dirichlet boundary conditions) and in [4] (for Neumann, Robin, and mixed boundary conditions).
In this talk we demonstrate that the methods used to proof convergence for the CEC method can be extended to perform a mathematically rigorous convergence analysis for the TDBC correction in the nonlinear case.
In addition, we present the results of numerical simulations for diffusion-reaction, advection-reaction, and dispersion-reaction equations in order to evaluate the relative performance of these two corrections. Furthermore, we introduce an extension of both methods to obtain order three locally and evaluate under what circumstances this is beneficial. We find that while the performance is comparable for diffusion-reaction problems, for advection-reaction problems the CEC method can outperform the TDBC method by more than an order of magnitude. In addition, whether performing the third order correction is benifical is highly problem dependent.
This talk is based on [5]

References

[1] R.J. LeVeque, J. Oliger, 1983, Numerical methods based on additive splittings for hyperbolic partial differential equations. Math. Comp. 40(162), 469-497.

[2] I. Alonso-Mallo, B. Cano, N. Reguera, 2016, Avoiding order reduction when integrating linear initial boundary value problems with exponential splitting methods. Private copy.

[3] L. Einkemmer, A. Ostermann, 2015, Overcoming order reduction in diffusion-reaction splitting. Part 1: Dirichlet boundary conditions. SIAM J. Sci. Comput. 37(3), A1577-A1592.

[4] L. Einkemmer, A. Ostermann, 2016, Overcoming order reduction in diffusion-reaction splitting. Part 2: oblique boundary conditions. To appear in SIAM J. Sci. Comput.

[5] L. Einkemmer, A. Ostermann, 2016, A comparison of boundary correction methods for Strang splitting. arXiv preprint, arXiv:1609.05505.

A Splitting Approach for Freezing Waves

Robin Flohr * (Karlsruhe Institute of Technology, Germany)
Jens Rottmann–Matthes (Karlsruhe Institute of Technology)

We propose a numerical method which is able to approximate viscous profiles by a direct long-time forward simulation. A difficulty with long-time simulations is the fact that traveling solutions leave the computational domain. To handle this problem one should go in a suitable co-moving frame. Since the speed of the (numerical) steady state is often unknown, we use the method of freezing [2] to calculate the right co-moving frame on the fly. As a test example we consider the (viscous) Burgers equation, which leads to the PDAE

$$\begin{cases} v_t = v_{xx} - (\frac{1}{2}v^2)_x + \mu v_x, \\ 0 = \Psi(v, \mu), \\ \gamma_t = \mu(t). \end{cases}$$

The variable μ represents the unknown speed of the profile and the algebraic constraint $\Psi(v, \mu)$, which is called phase condition, is needed for the well-posedness and fixes the frame. To this problem we use the operator splitting approach. The benefit of splitting methods in this context lies in the possibility to solve hyperbolic and parabolic parts with different numerical algorithms. Our numerical methods use the central schemes from Kurganov and Tadmor [1] to solve the hyperbolic problem. In combination with Lie- and Strang-splitting, we constructed full-discrete schemes to solve hyperbolic-parabolic problems. First promising numerical experiments for the Burgers equation show that we can expect linear and quadratic convergence respectively of a numerical steady state to an analytically given traveling wave.

References

[1] A. Kurganov and E. Tadmor, 2000. New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection-Diffusion Equations. J. Comput. Phys, 241–282.

[2] Beyn, W.-J., Otten, D., Rottmann-Matthes, J., 2014. Stability and computation of dynamic patterns in PDEs. Current challenges in stability issues for numerical differential equations, 89–172.

Space-Time Parallel Methods Based on Domain Decomposition

Martin J. Gander (University of Geneva, Switzerland)

Domain decomposition methods like the classical Schwarz method and the Dirichlet-Neumann and Neumann-Neumann methods have historically been developed for steady partial differential equations. All these methods have however also a natural waveform relaxation extension to time dependent partial differential equations. These waveform relaxation variants are still based on a spatial decomposition of the physical domain into subdomains, but then time dependent problems are solved in the subdomains, and information is exchanged on the space-time interfaces between subdomains in an iteration that converges in space-time to the underlying solution of the evolution problem. While domain decomposition methods for steady problems converge linearly, the waveform relaxation variants exhibit superlinear convergence for parabolic problems, and often become direct solvers for hyperbolic problems. After a simple introduction to the main classes of domain decompositions for steady problems, I will give in my talk an overview over the development of their waveform relaxation variants as it happened over the past 20 years.

ADI Preconditioners for the Solution of the Steady-State Vlasov equation

Lukas Einkemmer (University of Innsbruck)
Markus Gasteiger * (University of Innsbruck, Austria)
Alexander Ostermann (University of Innsbruck)

The concern of the current work is to find numerical solutions to the steady-state inhomogeneous Vlasov equation. This problem has countless applications in the kinetic simulation of non-thermal plasmas.
For such problems the direct application of iterative methods (such as Krylov based methods like GMRES or relaxation schemes) is computationally expensive. This is due to the fact that a large number of iterations is required for any sufficiently fine space discretization.
Our approach relies on a preconditioner that is based on an ADI type splitting method. This preconditioner is then applied to both GMRES and Richardson iteration. We explain the specific construction and implementation of such preconditioning methods and present the observed computational gains. We also discuss the advantages and disadvantages for specific problems.

Numerical analysis of wave equations with dynamic boundary conditions

David Hipp * (Karlsruhe Institute of Technology, Germany)
Marlis Hochbruck (Karlsruhe Institute of Technology)

In this talk we consider the wave equation in a bounded domain $\Omega \subset \mathbb{R}^d$ with dynamic boundary conditions on $\Gamma = \partial\Omega$. In contrast to standard boundary conditions of Dirichlet, Neumann or Robin type, dynamic boundary conditions model the momentum of the wave on the boundary, cf. [1], and therefore often better describe the physical reality. While a variety of dynamic boundary conditions are well understood analytically, to the best of our knowledge, the numerical approximation of such problems has not been studied so far.
The two guiding examples of dynamic boundary conditions on which we focus in this talk are the acoustic boundary condition, cf. [2],

$$\begin{aligned} \delta_t &= \partial_\nu u && \text{on } \Gamma, \\ \delta_{tt} + d\delta_t + k^2\delta &= -\rho u_t && \text{on } \Gamma, \end{aligned}$$

and the kinetic boundary condition, cf. [3],

$$u_{tt} = c_\Gamma \Delta_\Gamma u - \partial_\nu u \qquad \text{on } \Gamma.$$

We begin by showing how these examples can be dealt within a unified framework. This framework allows us to construct suitable function spaces and operators in an abstract way such that the well-posedness of the corresponding evolution equations can be shown by means of semigroup theory.
The second part of the talk addresses the numerical treatment for the case $\Omega \subset \mathbb{R}^2$. We describe a finite element method for the spatial discretization in the bulk Ω and on the surface Γ and prove error bounds. As an exemplary time stepping schemes we consider the implicit midpoint rule. Following the method of lines we apply the scheme to the spatial semidiscretization. Energy techniques can then be used to prove full order of convergence in space and time in the energy norm. These theoretical results are illustrated by numerical tests.

References

[1] Goldstein, G. R., 2006. Derivation and physical interpretation of general boundary conditions. Advances in Differential Equations 11 (4), 457–480.

[2] Beale, J. T., 1976. Spectral properties of an acoustic boundary condition. Indiana University Mathematics Journal 25 (9), 895–917.

[3] Vitillaro, E., 2013. Strong solutions for the wave equation with a kinetic boundary condition. Recent Trends in Nonlinear Partial Differential Equations I: Evolution Problems 594, 306-316.

Limit dynamics of the dispersion-managed nonlinear Schrödinger equation

Tobias Jahnke * (Karlsruhe Institute of Technology, Germany)
Marcel Mikl (Karlsruhe Institute of Technology)

Dispersion-managed optical fibers are modelled by the nonlinear Schrödinger equation

$$\partial_t u(t,x) = \tfrac{i}{\varepsilon}\gamma\left(\tfrac{t}{\varepsilon}\right)\partial_x^2 u(t,x) + i|u(t,x)|^2 u(t,x)\,, \quad x \in \mathbb{T}, \quad t > 0 \tag{1}$$

on the one-dimensional torus $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ with a small parameter $0 < \varepsilon \ll 1$ and a discontinuous, rapidly changing coefficient $\gamma(t/\varepsilon)$. Approximating the solution numerically is a challenging task because typical solutions oscillate in time with frequency $\sim \varepsilon^{-1}$ which imposes severe step-size restrictions for traditional methods.
In this talk, we present a transformation of (1) to an equivalent problem which, in contrast to (1), converges to a well-defined limit equation when $\varepsilon \to 0$. The limit equation can then be solved with standard methods because its solution does not depend on the critical parameter ε, and after transforming back, this yields an approximation to $u(t,x)$. Our main result states that the accuracy of this procedure is $O(\varepsilon + \tau^p)$ or, in special cases, $O(\varepsilon^2 + \tau^p)$, where τ and p are the step-size and the order, respectively, of the method used for the limit equation.
This is the first of two talks about the dispersion-managed nonlinear Schrödinger equation. In the second talk, Marcel Mikl will present a tailor-made numerical method for (1).

The action of trigonometric and hyperbolic matrix functions

Nickolas J. Higham (University of Manchester)
Peter Kandolf * (University of Innsbruck, Austria)

The action of trigonometric and hyperbolic matrix functions finds various applications, e.g. in the field of differential equations or network analysis. We present an algorithm that can compute $\cos(A)V$ and $\sin(A)V$ as well as $\cosh(A)V$ and $\sinh(A)V$ simultaneously. It is based on a procedure for computing the action of the matrix exponential $\exp(A)V$. The characteristics of the new approach can be exploited to improve performance in the exponential case as well.

References

[1] Al-Mohy, A.H., Higham, N.J., 2011. Computing the action of the matrix exponential, with an application to exponential integrators. SIAM J. Sci. Comput. 33 (2), 488-511.

[2] Higham, N. J.,Kandolf, P., 2016. Computing the Action of Trigonometric and Hyperbolic Matrix Functions, submitted.

Error Analysis of Splitting Methods for Parabolic Problems under Dirichlet Boundary Conditions

Winfried Auzinger (Technische Universität Wien)
Harald Hofstätter (Technische Universität Wien)
Othmar Koch * (Universität Wien, Austria)
Mechthild Thalhammer (Universität Innsbruck)

We analyze the convergence of splitting methods for a reaction-diffusion equation under homogeneous Dirichlet boundary conditions of the form

$$\partial_t\, u(x,t) = \Delta u(x,t) + F(u(x,t)), \quad (x,t) \in \Omega \times (0,T), \tag{1a}$$

$$u(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,T], \tag{1b}$$

$$u(x,0) = u_0(x), \quad x \in \overline{\Omega}, \tag{1c}$$

for an unknown function $u\colon \overline{\Omega} \times [0,T] \to \mathbb{R}$; Ω is a bounded domain with smooth boundary.
We prove that while the local error shows an order reduction, the full convergence order is restored for the global error of the Lie–Trotter splitting. The local convergence result deviates for consistent initial conditions in the domain of A ($u_0(x) \in \mathcal{D}(A)$) and inconsistent initial conditions ($u_0(x) \notin \mathcal{D}(A)$):

Theorem:

1. If $u_0 \in H^1(\Omega)$, the Lie–Trotter approximation satisfies the local error bound (for any $\varepsilon > 0$)
$$\|\mathcal{L}(t,u_0)\| \le C\,\|u_0\|_{H^1}\, t^{\frac{5}{4}-\varepsilon}.$$

2. To observe the full order of the local error, we have to assume $u_0 \in D(A) = H^2(\Omega) \cap H_0^1(\Omega)$. Then the Lie–Trotter approximation satisfies the local error bound (C depends on $\|u_0\|_{H^2}^2$)
$$\|\mathcal{L}(t,u_0)\| \le C\,t^2.$$

The full convergence order of the global error can be established nonetheless:

Theorem: Let $u_0 \in C(\overline{\Omega}) \cap H^1(\Omega)$ (inconsistent initial conditions permitted). Then

$$\|u_n - u(t_n)\|_{L^2} \le C\,\Delta t^{1-\varepsilon}, \quad n = 1,\dots,N.$$

Even for the second order Strang splitting, the full convergence order holds in both situations.

ADI splitting and the discontinuous Galerkin method

Marlis Hochbruck (Karlsruhe Institute of Technology)
Jonas Köhler * (Karlsruhe Institute of Technology, Germany)

We consider the alternating direction implicit (ADI) method for the time-integration of Maxwell's equations with linear, isotropic material properties on a cuboid. This method was introduced by Namiki in [2] and Zhen, Chen, Zhang in [3], where it is combined with a spatial discretization by finite differences on the Yee grid. Its main advantage is unconditional stability, while only being of linear complexity, which is obtained by splitting the differential operator in a suitable way and using the implicit Peaceman–Rachford scheme to propagate in time. This splitting is designed in such a way, that the flows in the different spatial directions decouple and the linear systems corresponding to the subproblems exhibit tridiagonal form. For the abstract Cauchy problem, an error analysis of the method was given by Hochbruck, Jahnke and Schnaubelt in [1].
In this talk, we combine the ADI method with a discontinuous Galerkin discretization of the spatial domain, which allows for more flexibility and requires weaker regularity assumptions in the error analysis. However, the desired efficiency is not so apparent anymore for this case. Nevertheless, for a regular mesh consisting of cuboids, we show how the special structure of the grid can be exploited to preserve the linear complexity. This result is also validated by numerical experiments.

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Two-grid method for solving non-linear models in mathematical finance

Miglena N. Koleva (Ruse University, Bulgaria)

In this work we construct and investigate efficient finite difference approximation for solving a class of non-linear modifications of Black-Scholes equation, in which the volatility is assumed to be a function of the underlying asset, time and Greek Gamma (second spatial derivative) of the option. The constructed numerical method is focused on the Delta equation, i.e. the unknown solution is Greek Delta - the first spatial derivative of the option value. Monotonicity and stability of the numerical scheme are proved. One-grid and two-grid Picard and Newton methods for solving the non-linear algebraic system of equations are studied. Numerical results are presented and discussed.

Krylov approximation of polynomially perturbed linear ODEs

Antti Koskela * (KTH Royal Institute of Technology, Sweden)
Elias Jarlebring (KTH Royal Institute of Technology)
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We propose an efficient Krylov subspace method to compute for several values of ε and t the solution of the initial value problem

$$u'(t,\varepsilon) = \big(\sum_{\ell=0}^{N} \varepsilon^\ell A_\ell\big) u(t,\varepsilon), \quad u(0) = u_0,$$

where $A_0, A_1, \ldots, A_N \in \mathbb{C}^{n\times n}$ and $u_0 \in \mathbb{C}^n$. The approach is based on the Taylor expansion of the exact solution $u(t,\varepsilon)$ with respect to ε. The derivation of the Krylov subspace method and its convergence analysis follow from a theorem given in [2], which shows that the coefficient vectors of this expansion are given by the matrix exponential of a block Toeplitz matrix. We further specialize the algorithm and its error analysis for the case where the matrices $A_1, \ldots, A_N$ are of low-rank structure.

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Numerical Methods for an efficient integration of the Maxwell-Dirac System

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Solving the Maxwell-Dirac (MD) equation in the non-relativistic limit regime is numerically very delicate as the solution becomes highly oscillatory in time. In order to resolve the oscillations, standard integration schemes require severe time step restrictions.
The aim of the talk is to present a uniformly accurate numerical method for the solution which yields good results for the highly-oscillatory non-relativistic limit regime as well as for the purely relativistic regime.
In case of the non-relativistic limit regime, a recent idea lies in the asymptotic expansion of the solution, which allows us to break down the numerical task to solving a non-oscillatory Schrödinger-Poisson system (SP). The latter can be carried out very efficiently without any additional time step restriction, e.g. by applying classical splitting methods (cf. [1]). In the recent works [2, 3] this numerical approach has been successfully applied to other Klein-Gordon type equations. However, as this approach is based on the asymptotic expansion of the exact solution it only yields a good approximation in the non-relativistic limit regime.
In my talk I present the idea of solving a first order system in the so-called twisted variables instead, which gives a good approximation in both the relativistic and the non-relativistic regime. For numerical results in case of the Klein-Gordon equation, see [4].
Furthermore, I give a comparison of both methods numerically and highlight their advantages and disadvantages.

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Approximating the stability of linear periodic delay models by pseudospectral methods

Dimitri Breda (University of Udine, Italy)
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Realistic models of structured populations are often based on delay equations. Due to the complexity of such models, their dynamics cannot usually be studied analitically and must be approximated numerically. A method based on pseudospectral collocation for approximating the eigenvalues of evolution operators of linear delay differential equations has been recently developed in [1, 2]. The method can be applied in particular to the monodromy operator of linearized problems to study the local asymptotic stability of periodic solutions. In this talk we present an extension of that method to coupled renewal equations and delay differential equations. The extended method has been applied in [3] with promising results.

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Conditioning and relative error propagation in linear autonomous ordinary differential equations

Stefano Maset (Università di Trieste, Italy)

In this talk, we deal with the relative error propagation in the solution of linear autonomous ordinary differential equations

$$\begin{cases} y'(t) = Ay(t), \ t \geq 0, \\ y(0) = y_0 \end{cases}$$

with respect to perturbations in the initial value y_0. We also consider equations with a constant forcing term

$$\begin{cases} y'(t) = Ay(t) + b, \ t \geq 0, \\ y(0) = y_0 \end{cases}$$

which have a nonzero equilibrium point.

Muli-level Local Time-Stepping Methods of Runge-Kutta type for Wave Equations

Martin Almquist (Uppsala University)
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Wave type phenomena are common in many fields of science, such as seismology, acoustics and electromagnetics. The propagation of waves is often modeled by partial differential equations (PDEs), for which it is important to have accurate and efficient numerical solvers. In the presence of small geometric features or re-entrant corners in the spatial domain, locally refined meshes around the obstacles permit accurate simulations without introducing too many spatial unknowns and are thus computationally efficient.
Local mesh refinement, however, significantly decreases the performance of explicit time-stepping methods, as the smallest mesh elements dictate the size of the time-step in the entire domain. To circumvent this problem we propose multi-level local time-stepping (MLTS) schemes. In the presence of meshes with several levels of refinement, the MLTS methods allow for an appropriate time-step on each of the levels. The methods are based on explicit Runge–Kutta (RK) schemes and an extension of the work presented in [1], where only one level of refinement was considered. They retain the explicitness and the one-step nature of the underlying RK method and thus require no starting procedure and facilitate adaptivity in time. We show that the novel schemes keep the accuracy of the underlying RK scheme and present numerical results that illustrate the versatility of the approach.

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Splitting Methods for Stochastic Partial Differential Equations

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We analyse the convergence of the exponential Lie splitting applied to inhomogeneous second-order stochastic parabolic equations with additive/multiplicative noise and Dirichlet boundary conditions. The equations are considered in the white noise framework, where every stochastic squared integrable processes can be represented in terms of a orthogonal polynomial basis which is the core of the polynomial chaos methodology. After applying polynomial chaos to the equation, we obtain a system of infinitely many deterministic partial differential equations in terms of the coefficients of the solution. Each coefficient is computed by Lie splitting following the ideas in [1, 2]. We compute finely many coefficients and discuss the convergence of the solution. Our approach can be extended in the same setting to equations of hyperbolic type.

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Adiabatic Midpoint Rule for the dispersion-managed nonlinear Schrödinger Equation

Tobias Jahnke (Karlsruhe Institute of Technology)
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Modeling a dispersion-managed optical fiber leads to a nonlinear Schrödinger equation containing a rapidly changing piecewise constant coefficient function. The occurring oscillations of the solution impose severe step-size restrictions for traditional time-integrators. In this talk, we present and analyze a tailor-made numerical method for this equation which attains the desired accuracy with a significantly larger step-size than traditional methods.
The construction of this method is based on a favorable transformation to an equivalent problem, which is presented in the talk of Tobias Jahnke. To fully understand the behavior of the method, we have to deviate from the classical concept "stability + consistency = convergence". Instead, we utilize an error recursion formula which allows to exploit cancellation effects in the error terms. This results in an improved error behavior for specific step-sizes.

A Linear Domain Decomposition Method for Unsaturated Flow in Porous Media

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The Richards equation is a commonly used model for unsaturated flow through porous media. Using the Darcy law in the mass balance equation, and bringing the resulting to a dimensionless form gives [1]

$$\partial_t S = \nabla \cdot (k(S)\nabla p) + f(S, \vec{x}) \tag{1}$$

The nonlinear function $k(S)$ (Relative permeability) is determined based on experiments. Two unknowns are involved: S, the saturation and p, the pressure of the fluid. Commonly, it is assumed that these are related by a nonlinear relationship determined, again, based on experiments [1]

$$-p = P_c(S) \text{ for } 0 < S < 1 \tag{2}$$

The exact form of these nonlinearities are depending on the medium. Most realistic problems are involving heterogeneous media. Example in this sense are layered or fractured oil and gas reservoirs. In this case, the functions above are changing their depending on the location inside the medium. In particular, if the medium consists of adjacent homogeneous blocks, this poses difficulties for both mathematical analysis and numerical simulation since at the interfaces separating the homogeneous sub-domains, the model parameters become discontinuous.
In this work we consider a medium consisting of two homogeneous blocks and propose a non-overlapping domain decomposition scheme that couples the models in the two subdomains. The scheme involves linear iterations [2] [3]. After analyzing its convergence, we show some numerical results confirming the theoretical findings and discuss different implementation aspects.

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Adapted numerical integration of advection-reaction-diffusion problems generating periodic wavefronts

Raffaele D'Ambrosio (University of Salerno)
Martina Moccaldi * (University of Salerno, Italy)
Beatrice Paternoster (University of Salerno)

This talk deals with an adapted numerical treatment of advection-reaction-diffusion problems having periodic waves as fundamental solutions. Such problems are widely studied in applications, especially involving chemical processes: for instance, they are employed to describe the phenomenon of morphogenesis as explained in [3]. The adaptation in space has been carried out through a trigonometrically fitted method of lines. Indeed, classical finite difference methods could determine an extreme reduction in step-size to accurately follow the prescribed oscillations because they are constructed in order to be exact (within round-off error) on polynomials up to a certain degree. In this work, we employ adapted finite differences which are constructed in order to be exact (within round-off error) on functions other than polynomials, following the well-known *exponential fitting* strategy. These functions belong to a finite dimensional space (called fitting space) and are chosen according to the qualitative behaviour of the exact solution. However, the coefficients of the resulting method are no longer constant but rely on unknown parameters linked to the solution. Thus, an accurate estimate of these parameters is a crucial challenge in order to exploit the benefits of the exponential fitting. We approach this aspect by proposing a selection technique which suitably treats the truncation error. Once an advection-reaction-diffusion problem is discretized in space, the vector field of the resulting system of ordinary differential equations can be split into different terms, a stiff and a non-linear one. For this reason, we employ an implicit-explicit (IMEX) scheme which implicitly integrates only stiff constituents and explicitly treats the others, gaining benefits both in terms of stability and efficiency.
The effectiveness of this problem-oriented approach has been proved by a rigorous analysis of accuracy and stability properties and some numerical tests.

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Solution of large-scale Lyapunov differential equations

Hermann Mena (Yachay Tech University)
Alexander Ostermann (University of Innsbruck)
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Lyapunov differential equations are the key ingredient for the simulation of systems governed by stochastic partial differential equations. In this talk we are interested in large-scale problems and we propose to perform model reduction by low-rank approximation. In particular we present the dynamical low-rank approximation [1]. That is, a differential equations based approach for the approximation of the solution in low-rank factorized form. The strength of this strategy is that the time integration is performed only on the low-rank factors of the solution.

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A Riemann solver free numerical method for two-dimensional conservation laws

Alexander Kurganov (Tulane University)
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Tong Wu (NC State University)

The modeling of fluids usually results in a number of partial differential equations that relate the change of local properties (such as density, velocity, temperature, ...) in time to the corresponding change in space. A typical set of equations that result from such models are so-called conservation laws. Since the solution of such partial differential equations can develop discontinuities (i.e., shocks), solving them is an interesting problem in the field of numerical analysis. Due to Godunov's theorem, linear and monotone schemes are at most first order accurate and thus special care has to be taken to propagate shock waves without diminishing the performance of the scheme.
The most commonly used numerical methods that deal with such type of problems are based on solving 1D generalized Riemann problems. However, solving a Riemann problem comes with a large computational cost. Furthermore, since there is no analytic solution for the Riemann problem in multiple dimensions, splitting methods have to be used.
In this talk, we present a high resolution central-upwind-scheme in two space dimensions on a staggered grid in space. This scheme is able to evolve the solution without solving a Riemann problem.

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Numerical bifurcation analysis of nonlinear delay equations through pseudospectral discretization

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Mats Gyllenberg (University of Helsinki)
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Rossana Vermiglio (University of Udine)

Delay equations, including renewal and delay-differential equations, are increasingly used in the mathematical modelling of biological populations. In this context, the interest is mainly focused on the long-term dynamics of the system and their change when varying some parameters.
In this talk I will present the pseudospectral discretization of nonlinear delay equations with finite delay. This approach leads to an approximating nonlinear ODE system whose dynamical and bifurcation properties can be investigated by using well-established software for ODEs (e.g., MATCONT for MATLAB). The spectral convergence of the approach for approximating equilibria, their stability properties and the parameter values of bifurcations has been proved in [1] and allows to obtain reliable results with low-dimensional ODE systems. Moreover, I will present some test examples from [1, 2], which show the effectiveness of the method for investigating periodic solutions and the relevant stability and bifurcation properties.

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An extension of the Savage-Hutter equations for the modeling of gravity driven mass flows over arbitrary topography in one space dimension

Wolfgang Fellin (University of Innsbruck)
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Gravity driven mass flows (e.g. debris flows, snow or rock avalanches), pose a threat to human life and property. To predict their behavior, many models have been proposed in the last decades, where the Saint Venant model for shallow water flow and the Savage-Hutter model [2] for granular flow are of high importance.
Due to many simplifications, these models have difficulties on irregular terrain and are therefore only partially applicable on arbitrary topography, which we encounter in real world problems.
To get a better understanding of the additional forces induced by topographical changes, we focus on two space dimensions, the slope following coordinate X and perpendicular to it Z. Inspired by the paper by Bouchut et.al. [1], we have repeated the derivations, starting with the compressible Euler equations of gas dynamics. After linearization in the curvature $\partial_X \theta$, depth integration in Z and scaling analysis, we acquire a system of PDEs for mass and momentum balance in one space dimension X with additional force terms, which can be reduced to the classical Savage-Hutter equations after further simplification.
With numerical simulations we demonstrate the differences between the old and new equations, which are significant in strongly curved terrain and at higher speeds of the mass flow.

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Finite Element Heterogeneous Multiscale Method for Time-Dependent Maxwell's Equations

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Our goal is to simulate electromagnetic wave propagation through a highly oscillatory material. More precisely, let $\Omega \subset \mathbb{R}^d$ be a bounded Lipschitz domain ($d = 2, 3$), the final time T be positive, and $\boldsymbol{f} \in \boldsymbol{L}^2(0,T;L^2(\Omega))$, $\boldsymbol{E}_0 \in \boldsymbol{H}_0(\mathbf{curl};\Omega)$, and $\boldsymbol{E}_0' \in \boldsymbol{L}^2(\Omega)$ be the given source term, the initial state, and the initial velocity, respectively. We consider the second-order formulation of time-dependent Maxwell's equations, whose variational formulation is given as follows.

$$\begin{cases} \text{Find } \boldsymbol{E}^\eta : (0,T) \to \boldsymbol{H}_0(\mathbf{curl};\Omega), \text{ such that for all } \boldsymbol{v} \in \boldsymbol{H}_0(\mathbf{curl};\Omega) \\ \big(\partial_{tt}\varepsilon^\eta \boldsymbol{E}^\eta(t), \boldsymbol{v}\big)_{0,\Omega} + \big((\mu^\eta)^{-1}\,\mathbf{curl}\,\boldsymbol{E}^\eta(t), \mathbf{curl}\,\boldsymbol{v}\big)_{0,\Omega} = \big(\boldsymbol{f}(t), \boldsymbol{v}\big)_{0,\Omega}, \\ \boldsymbol{E}^\eta(0) = \boldsymbol{E}_0, \qquad \text{and} \qquad \partial_t \boldsymbol{E}^\eta(0) = \boldsymbol{E}_0', \end{cases} \tag{1}$$

where ε^η is the electric permittivity and μ^η is the magnetic permeability. We assume, that both material properties oscillate on a microscopic scale of size $\eta \ll \operatorname{diam}(\Omega)$, e.g. for periodic materials, η is the period.

The use of standard discretization methods, such as Finite Differences or Finite Element (FE), would lead to humongous degrees of freedom, since all the microscopic details must be resolved. To overcome these difficulties, we present a multiscale method for (1) within the framework of FE Heterogeneous Multiscale Methods (HMM), see [1] and references therein. This method is capable of approximating the effective behavior of $\boldsymbol{E}^\eta$ efficiently, e.g in the special case of periodic materials, the computational costs are independent of η.

Our FE-HMM for (1) can be seen as a non-conforming FE method for second-order hyperbolic equations. To prove an a-priori error bound we use a new generalization of the Strang-type lemma given in [2].

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Locally implicit time integration for linear Maxwell's equations

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An attractive feature of discontinuous Galerkin (dG) spatial discretizations of Maxwell's equations is their ability to handle complex geometries by using unstructured, possibly locally-refined meshes. Furthermore, dG methods lead to block diagonal mass matrices which in combination with an explicit time integration method allow for a fully explicit scheme. However, such explicit approaches require a constraint on the time step size related to the diameter of the smallest mesh element to ensure stability, the well-known CFL condition. This makes the simulation inefficient, in particular if the number of tiny mesh elements is small compared to the total number of elements. A natural way to overcome this restriction is using implicit time integrators but these come with the expense of having to solve a large linear system in each time step.
A more suitable approach consists in treating only the tiny mesh elements implicitly while retaining an explicit time integration for the remaining coarse elements. This results in so-called locally implicit methods. In this talk we consider a second order locally implicit method proposed by [1] and analyzed in [2, 3]. Both, the efficiency and the error analysis of this method strongly rely on the skew-adjointness of the Maxwell-operator and its (central fluxes) dG discretization. Unfortunately, this skew-adjointness does not hold for stabilized dG discretizations, i.e. for upwind fluxes dG methods. Thus, the construction and the analysis of the locally implicit method are so far restricted to dG methods with central fluxes. However, upwind fluxes dG methods exhibit many advantages such as a superior stability behavior and higher accuracy.
In this talk we present how the locally implicit method can be adapted to treat the upwind fluxes dG discretization. We show that the new method preserves the efficiency of the underlying locally implicit method. Moreover, we give an error analysis for the full discretization based on a variational formulation and energy techniques. We prove that the new method is again of second order in time and that it exhibits the higher spatial accuracy of an upwind dG method.

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Numerical solution of degenerate ultraparabolic equations for pricing of Asian options

Tatiana Chernogorova (University of Sofia)
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In this talk, we develop splitting methods for degenerate parabolic equations that arise naturally within the framework of PDE valuation of Asian options. Here we are concentrated on path-dependent options. First, using the energy method we discuss well-posed boundary conditions. Then, two splitting algorithms are proposed to transform the whole time-dependent problem into two unsteady subproblems of a smaller complexity. Convection-diffusion degenerate parabolic equation is involved on the first problem and it is discretized by fitted finite volume difference schemes. The second one is a transport problem and it is approximated by monotone weighted difference schemes. Numerical experiments show that the numerical algorithms are working appropriately to conserve the positivity of the price of options and are better than those provided in the existing literature.

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