Abstract

The Vlasov-Poisson and Vlasov-Maxwell equations are the most fundamental description of a (collisionless) plasma. The equations describe the evolution of a particle-probability distribution in 3+3 dimensional phase space coupled to an electromagnetic field. The difficulties in obtaining a numerical solution of those equations is summarized in the following three statements:

1. Due to the six dimensional phase space the amount of memory required to store the interpolation is proportional to the sixth power in the number of grid points.
2. The Vlasov equation is stiff (i.e. the time step size is limited by the CFL condition)
3. The coupling to the Maxwell/Poisson equation makes the system highly non-linear.

A numerical scheme based on Strang splitting has been proposed that translates the basis functions of some interpolation space and projects the translated basis function back onto the proper subspace. The before mentioned scheme has been used in a number of plasma simulations and some geometrical properties have been investigated. However, no error analysis of this scheme is available. Furthermore, even though high order interpolation methods in space have been proposed, for the fully magnetized case no high order splitting method in time can be found in the literature. Such methods would allow us to take larger time steps and thus improve upon the efficiency of the scheme.

Therefore the aim of this project is to:

1. supply an in-depth numerical analysis of Strang splitting for Vlasov type equations;
2. extend the achieved results to higher order splitting methods;
3. provide a convergence analysis of the fully discrete problems (using discontinuous Galerkin in space);
4. extend the previous results to higher order methods in space.

We will start to investigate the properties of an abstract initial value problem that includes the Vlasov-Poisson as well as the Vlasov-Maxwell equations as a special case. The error analysis so obtained can be extended to the fully discrete problem by considering the spatial discretization as a perturbation of the analytic problem.

To implement higher order splitting methods we have to evaluate the force term at certain intermediate steps. We will develop a strategy that leads to computationally efficient schemes. It is further of large interest to investigate how a high order splitting method behaves, if the number of grid points is reduced in favor of a higher order discontinuous Galerkin approximation in space. Due to the six dimensional phase space of the full Vlasov equation this could potentially reduce the memory footprint by some orders of magnitude.