Perturbation Theory and Direct Velocity Feedback for Quadratic Eigenvalue Problem

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Abstract

We consider the two related problems connected with the quadratic eigenvalue problem (QEP):

$$(\mu^2 M + \mu C + K)x = 0\,.$$

The first problem refers to the relative perturbation bounds for the eigenvalues as well as for the eigensubspaces of the considered QEP, where $M, C, K \in \mathbb{C}^{n\times n}$ are given Hermitian matrices, such that all eigenvalues of QEP are real, that is the quadratic eigenvalue problem is so called hyperbolic. Using a proper linearization and the new relative perturbation bounds for obtained definite matrix pairs, we develop a novel relative perturbation bounds which are uniform and depend only on matrices M, C, K perturbations δM, δC and δK and the standard relative gaps.

Within the second problem, we present a novel approach to the problem of Direct Velocity Feedback (DVF) optimization of vibrational structures, which can be written as

$$M\ddot{x} + vbb^T\dot{x} + Kx = 0\,,$$

where $M, K \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices, $b \in \mathbb{R}^{n\times r}$ and $v \in \mathbb{R}$ (usually $v > 0$). The above problem leads to the parametric eigenvalue problem, thus we describe the behavior of the eigenvalues for a small and large gains separately, that is for a small and a large v. For the small gains, which are connected to a modal damping approximation, we present a standard approach based on Gerschgorin discs. For the large gains we present a new approach which allows us to approximate all eigenvalues very accurately and efficiently. Besides mentioned application the new bounds are also valuable for better understanding of damped vibration systems where eigenvalue behavior plays important role.