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SEMINARVORTRAG

Das Institut für Mathematik lädt zu folgendem Vortrag im Rahmen des Forschungsseminars: Numerische Mathematik – aktuelle Forschung ein:

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über

Numerical instabilities of Inverse Scattering Transform

Zeit: **Donnerstag, den 23. Mai 2019 um 17:15 Uhr**

Ort: **Bauing.-Gebäude, Technikerstraße 13, HSB 6**

Gäste sind herzlich willkommen!

Alexander Ostermann

Numerical instabilities of Inverse Scattering Transform

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The 1D focusing Nonlinear Schrodinger equation (NLSE) is universal physical model of weakly nonlinear wave processes in different media. For example, NLSE describes propagation of light in optical fiber with Kerr nonlinearity, gravity waves on surface of deep water and waves in weakly interacting Bose gas with attraction between particles. The NLSE can be completely integrated using Inverse Scattering Transform (IST) technique [1]. The IST approach allows to find various exact solutions of the NLSE, such as multisoliton, multi-breather solutions and the so called rational rogue wave solutions, and implicitly (for example via integral equations) express general solution of the Cauchy problem.

The IST approach introduces the so-called scattering data which is in one-to-one correspondence with the wave field and, similarly to the Fourier harmonics in the linear wave theory, changes trivially during system evolution. With numerical methods, the scattering data can be analysed, that may bring some insights into the system behaviour. The main theoretical background of the IST theory for the NLSE had been developing during 70^{th} of the last century - see the monograph [2]. However, up to now there is no universal stable numerical implementation of the whole IST scheme in the case of arbitrary wave fields.

For example, recently we have demonstrated, that numerical implementation of the well known multisoliton formulas fails due to instabilities when the number of solitons reaches approximately thirty [3]. Meanwhile the ability to effectively construct multisoliton wave fields is of high necessity due to the recent surge of interest to statistical description of the integrable models [4,5], applications of the IST technique in optical fibre telecommunications [6,7] and rogue wave theory [8].

In this talk we discuss how multisoliton solutions are used in nonlinear wave studies. Then we focus on different IST schemes which face with numerical instabilities when the number of solitons is large. We examine the nature of this instability which is related to the ill-conditionality of the multisoliton formulas. Then we present our current approach to generate multisoliton wave fields based on optimisation of the multisoliton construction scheme and the usage of arbitrary precision arithmetic [3,6,8]. Finally, we discuss the open questions and perspectives regarding development of more stable and efficient numerical algorithms of multisoliton wave fields computation.

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